MAREES TERRESTRES

BULLETIN D'INFORMATIONS

109

15 JANVIER 1991

Association Internationale de Geodesie

Commission Permanente des Marees Terrestres

Editeur Prof. Paul MELCHIOR

Observatoire royal de Belgique

Avenue Circulaire 3

1180 Bruxelles

BIM 109

15 janvier 1991

A CONCISE HISTORY OF THE THEORIES OF TIDES, PRECESSION-NUTATION AND POLAR MOTION (FROM ANTIQUITY TO 1950)

MARTIN EKMAN

National Land Survey
Division of Geodetic Research
S - 801 82 Gävle, Sweden

Contents

1. HISTORY OF TIDAL THEORY

1.1 The first tidal discoveries

About 330 B.C. the Greek astronomer and explorer Pytheas made a long voyage. He sailed from the western part of the Mediterranean Sea, where he lived in a Greek colony, to the British Isles. Observing the great ocean tides there, he made a fundamental discovery: The tides were in some way controlled by the Moon. This discovery can be considered the starting point of tidal research; it was published in Pytheas' "On the Ocean", now lost but quoted by other antique authors. Pytheas discovered not only that there were two high tides per lunar day, but also that the amplitude depended on the phases of the Moon.

The Greek scientists had no possibilities to observe the tides at home because of their insignificance there. Nevertheless, around 150 B.C. Seleukos found out that the two tides per day had unequal amplitudes when the Moon was far from the equator; this is what we today call the diurnal inequality. This knowledge was based on observations he had made at the Red Sea, where the diurnal inequality happens to be fairly pronounced.

In the year 77 A.D. a comprehensive work called "Naturalis historia" appeared. It was written by the Roman scientist Gaius Plinius (23 - 79 A.D.; he was killed by the famous volcanic eruption of Vesuvius). In this work Plinius summarizes the tidal knowledge of his time, beginning as follows:

"Much has been said about the nature of waters; but the most wonderful circumstance is the alternate flowing and ebbing of the tides, which exists, indeed, under various forms, but is caused by the Sun and the Moon. The tide flows twice and ebbs twice between each two risings of the Moon."

Plinius further writes, about some places in southern Spain:

> "There are, however, some tides which are of a peculiar nature ... At Gades [Cadiz], which is very near the temple of Hercules, there is a spring inclosed like a well, which sometimes rises and falls with the ocean, and, at other times, in both respects contrary to it. ... On the shores of the Baetis [Guadalquivir], there is a town where the wells become lower when the tide rises, and fill again when it ebbs; while at other times they remain stationary. The same thing occurs in one well in the town of Hispalis [near Sevilla, now in ruins]."

This passage on reversed tides in wells represents, remarkably enough, the first observations of earth tides - in the form of tidal strain. But Plinius, of course, could have no idea about that.

More than two thousand years ago the most important characteristics of the tides were known, mainly due to Greek observations at the British Isles and in the Red Sea. But in what way were the tides created? It was to take one thousand six hundred years before one began to understand the origin of this strange phenomenon. Meanwhile, several unsuccessful attempts were made to explain it.

1.2 Search for the origin of tides: On the wrong tracks

The considerable tides around Britain interested Bede the Venerable (673 - 735), a learned English monk. In the beginning of the 8th century Bede discovered the phase lag of the ocean tides, realizing each port to have its own tidal phase. As to the origin of the tides he was of the opinion that the tide ebbed through the Moon blowing on the water, and flowed again when the Moon had moved a bit.

The first scientific attempt to explain the tides was made in the middle of the 13th century by the Arabian scientist Zakariya al-Qazwini (c. 1203 - 1283). In a book on the wonders of Creation he claims that the flowing tide is caused by the Sun and the Moon heating the waters, thereby making them expand. He describes this in the following way in the case of the Sun:

> "As to the rising of the waters, it is supposed that when the Sun acts on them it rarefies them, and they expand and seek a space ampler than that wherein they were before, and the one part repels the other in the five directions eastwards, westwards, southwards, northwards and upwards."

His hypothesis, however, failed to explain why the Moon, not the Sun, played the leading role.

The great difficulty in understanding how the Moon and the Sun could have an influence on the Earth made people look for completely new explanations of the tides. The whole idea of Moon and Sun acting in some way was distrusted. According to one idea the tides were caused by the great whirlpool, Malströmmen (the Maelstrom), outside the coast of northern Norway: Low tide was just the consequence of the sea water disappearing into the whirl, while high tide occurred when the water reappeared from the whirl. Today we know that there is, in fact, a connection between the wirlpool and the tide

- but it is the tide that causes the whirl, not the other way around!

After the (re)discovery of America it was suggested in 1557 by Julius Caesar Scaliger (1484 - 1558), an Italian scientist, that the tides were caused not only by the Moon, but also by the sea water oscillating between the coasts of America and Europe. The background to this suggestion might have been the resonance phenomena that by then were known to occur in some of the large lakes in Switzerland.

Johannes Kepler (1571 - 1630), the German astronomer, was convinced that the tides in some way depended on the Moon and the Sun. He claimed in 1609 in his "Astronomia nova" that the explanation was an attractive force of the Moon and the Sun, a force which he believed to be some kind of magnetism. Clearly, he was inspired by Gilbert's recent discovery of the magnetic field of the Earth.

However, Galileo Galilei (1564 - 1642), the Italian physicist and astronomer, was surprised that the great Kepler "became interested in the action of the Moon on the water, and in other occult phenomena, and similar childishness". Galilei himself believed, defending the Copernican theory of a rotating Earth in 1616 and 1632, that the tides were produced by the combined effect of the Earth's rotation around its axis and its orbital motion around the Sun. These motions would set the water on the Earth into oscillations, observed as tides:

"It must happen that in coupling the diurnal motion with the annual, there results an absolute motion of the parts of the surface which is at one time very much accelerated and at another retarded by the same amount. ... Now if it is true (as is indeed proved by experience) that the acceleration and retardation of motion of a vessel makes the contained water run back and forth along its length, and rise and fall at its extremities, then who will make any trouble about granting that such an effect may - or rather, must - take place in the ocean waters?"

The French mathematician René Descartes (1596 - 1650) - often known by his Latin name Renatus Cartesius - supported a lunar origin of the tides, presenting in 1644 his own idea on how it all worked: The Moon and the Earth were each surrounded by a large vortex. The pressure exerted by the vortex of the Moon on that of the Earth was transmitted down to the Earth's surface, giving rise to the tides. However, the theory of vortices erroneously predicted a low tide when there was in reality a high tide, although it must be admitted that the picture was quite complicated because of the phase lag of the ocean tides.

An extended version of Galilei's theory was given in 1666 by John Wallis (1616 - 1703), an English mathematician. He suggested the tidal oscillations to result from the Earth's rotation combined, not only with the Earth's motion around the Sun, but also with its motion around the centre of gravity of the Earth-Moon system. Thereby Wallis tried to include the influence of the Moon into the theory.

The whole thing was very confusing: If Moon and Sun did not control the tides, how did one explain all the observations? If the observations were correct, how did one explain that Moon and Sun could control the tides on the Earth?

1.3 Aha! Gravitation and tides

The solution to the problem was given in 1687 when Isaac Newton (1642 - 1727), the English mathematician, physicist and astronomer, published the theory of gravitation in his "Philosophiae naturalis principia mathematica". The origin of the tides was the hitherto unknown attractional force of Moon and Sun (and all other celestial bodies) - the gravitation. The tides were created by the gravitation being different at different distances from the celestial body. Newton writes:

"But let the body S come to act upon it [the globe], and by its unequable attraction the water will receive this new motion. For there will be a stronger attraction upon that part of the water that is nearest to the body, and a weaker upon that part which is more remote."

With his theory Newton could explain the three fundamental properties of the tides: the main period of 12 lunar hours, the dependence of the amplitude on the lunar phases, and the diurnal inequality. To clarify the situation Newton constructed the figure which is shown in our Figure 1:

"For the whole sea is divided into two hemispherical floods, one in the hemisphere KHk on the north side, the other in the opposite hemisphere Khk, which we may therefore call the northern and the southern floods. These floods being always opposite the one to the other, come by turns to the meridians of all places, after an interval of 12 lunar hours. And seeing the northern countries partake more of the northern flood, and the southern countries more of the southern flood, thence arise tides, alternately greater and less in all places without the equator."

Newton also was able to calculate the tidal force of the Sun and the Moon, respectively. Newton considered this "force to move the Sea" in the sense of producing in Syzygiis Solſtitialibus quàm in Æquinoctialibus. In Quadraturis autem Solſtitialibus majores ciebunt æſtus quàm in Quadraturis Æquinoctialibus; eò quod Lunæ jam in æquatore conſtitutæ effectus maximè ſuperat effectum Solis. Incidunt igitur æſtus maximi in Syzygias & minimi in Quadraturas Luminarium, circa tempora Æquinoctii utriuſque. Et æſtum maximum in Syzygiis comitatur ſemper minimus in Quadraturis, ut experientiâ compertum eſt. Per minorem autem diſtantiam Solis à Terra, tempore hyberno quàm tempore æſtivo, fit ut æſtus maximi & minimi ſæpius præcedant Æquinoctium vernum quàm ſequantur, & ſæpius ſequantur autumnale quàm præcedant.

Pendent etiam effectus Luminarium ex locorum latitudine. Deſignet *ApEP* Tellurem aquis profundis undique coopertam; *C* centrum ejus; *Pp*, polos; *AE* Æquatorem; *F* locum quemvis extra Æquatorem; *Ff* parallelum loci; *Dd* parallelum ei reſpondentem ex altera parte æquatoris; *L* locum quem Luna tribus ante horis occupabat; *H* locum Telluris ei perpendiculariter ſubjectum; *h* locum huic oppoſitum; *K*, *k* loca inde gradibus 90 diſtantia, *CH*, *Ch* Maris altitudines maximas menſuratas à centro Telluris; & *CK*, *Ck* altitudines minimas: & ſi axibus *Hh*, *Kk* deſcribatur Ellipſis, deinde Ellipſeos hujus revolutione circa axem majorem *Hh* deſcribatur Sphærois *HPKhpk*; deſignabit hæc figuram Maris quam proximè, & erunt *CF*, *Cf*, *CD*, *Cd* altitudines Maris in locis *F*, *f*, *D*, *d*. Quinetiam ſi in præfata Ellipſeos revolutione punctum quodvis *N* deſcribat circulum *NM*, ſecantem parallelos *Ff*, *Dd* in locis quibuſvis *R*, *T*, & æquatorem *AE* in *S*; erit *CN* altitudo Maris in locis omnibus *R*, *S*, *T*, ſitus in hoc circulo. Hinc in revo-

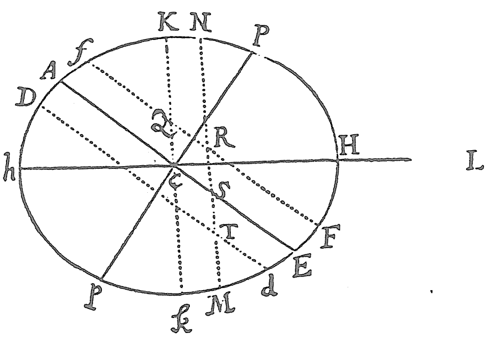


Figure 1. A page from Newton's "Principia" (1687), with the figure to which the second quotation on the preceding page refers. The figure shows the tidally deformed ocean surface of the Earth: Pp denotes the axis of rotation, AE the equator, and L the direction to the Moon.

the full rise from low tide to high tide. For the Sun he found, by using the Sun's disturbing influence on the lunar orbit, that the tidal force was 1 / 12 900 000 of the force of gravity in case the Sun was at zenith or nadir, and at its mean distance. This is an excellent value. By analysing English tidal observations with respect to the ratio between spring tides and neap tides Newton then found the tidal force of the Moon to be 4.5 times that of the Sun. The true value, however, is 2.2. Thus Newton overestimated the lunar tidal force by a factor of 2, approximately. This factor will pop up again in section 2.3, where the luni-solar gravitation is discussed within the theory of precession.

Newton's epoch-making discoveries meant that a foundation had been constructed for a mathematical treatment of tides. The Swiss mathematician and physicist Daniel Bernoulli (1700 - 1782) in 1740 wrote an essay on tides based on Newton's theory, but this was still not a great advance. However, Bernoulli was the one who found out that Newton had overestimated the ratio between the lunar and solar tides. Using French tidal observations he found a ratio of 2.5, close to the modern value.

The break-through for a mathematical theory of tides was made by Pierre de Laplace (1749 - 1827), the French mathematician and astronomer. He introduced the tidal potential, in a theory presented to the Royal Academy of Sciences in Paris 1775. Later on he extended it considerably and included it in his "Traité de mécanique céleste" of 1799. Here we find "Laplace's tidal formula", expressing the tidal potential as a function of latitude, declination and hour angle - see Figure 2. Let us listen to the description of Laplace:

202 MECANIQUE CÉLESTE,

La partie de $\alpha V'$ dépendante de l'action de l'astre L, est par le n°. 4, égale à

$$\frac{L}{4r^3}.\{\sin.^2 v - \tfrac{1}{2}.\cos.^2 v\}.(1+3.\cos.2\theta)$$
$$+\frac{3L}{r^3}.\sin.v.\cos.v.\sin.\theta.\cos.\theta.\cos.(nt+\varpi-\psi)$$
$$+\frac{3L}{4r^3}.\cos.^2 v.\sin.^2\theta.\cos.(2nt+2\varpi-2\psi).$$

Supposons que la partie correspondante de αy, soit égale à cette quantité multipliée par une indéterminée Q; ce produit étant de la forme $Y^{(2)}$, ou satisfaisant pour $Y^{(2)}$, à l'équation aux différences partielles,

$$0=\left\{\frac{d.\left\{(1-\mu\mu).\left(\frac{dY^{(2)}}{d\mu}\right)\right\}}{d\mu}\right\}+\frac{\left(\frac{ddY^{(2)}}{d\varpi^2}\right)}{1-\mu\mu}+6.Y^{(2)};$$

la partie de $\alpha V'$ correspondante à l'action de la couche fluide dont le rayon intérieur étant l'unité, le rayon extérieur est $1+\alpha y$, sera par le n°. 2, $\frac{4\pi}{5}.Y^{(2)}$, ou $\frac{3}{5\rho}.g.Y^{(2)}$; l'équation $\alpha g y=\alpha V'$, donnera donc,

$$\alpha y=\frac{L}{4r^3.g.\left(1-\frac{3}{5\rho}\right)}.\{\sin.^2 v-\tfrac{1}{2}.\cos.^2 v\}.(1+3.\cos.2\theta)$$
$$+\frac{3L}{r^3.g.\left(1-\frac{3}{5\rho}\right)}.\sin.v.\cos.v.\sin.\theta.\cos.\theta.\cos.(nt+\varpi-\psi)$$
$$+\frac{3L}{4r^3.g.\left(1-\frac{3}{5\rho}\right)}.\cos.^2 v.\sin.^2\theta.\cos.(2nt+2\varpi-2\psi).$$

Figure 2. Laplace's tidal formula as it appears in the "Mécanique céleste" (1799). His interpretation of it is quoted on the next page.

"The three preceding terms give rise to three different types of oscillations. The periods of the oscillations of the first type are very long; they are independent of the rotational motion of the Earth, and depend only on the motion of the celestial body L in its orbit. The periods of the oscillations of the second type depend mainly on the rotational motion t of the Earth; they are one day approximately. Finally, the periods of the oscillations of the third type depend mainly on the angle 2t; they are about half a day."

Thus Laplace showed the tide to be mathematically separable into three different kinds of tides: long-periodical, diurnal and semi-diurnal. This separation has since then been a corner-stone in tidal theory.

Moreover, with the same works Laplace was the first one to treat ocean tides as a problem of water in motion instead of water in equilibrium. His hydrodynamical equations, describing the propagation of tidal waves through the ocean, could not be solved in practice until man had invented the computer. Meanwhile, co-tidal charts were constructed using more or less unreliable methods.

1.4 Tidal friction and the length of the day

A completely new aspect of the tides - that of tidal friction - was introduced by the German scientist and philosopher Immanuel Kant (1724 - 1804). In 1754 he wrote an article in the Königsberg weekly magazine called "Ob die Erde in ihrer Umdrehung um die Achse einige Veränderung erlitten habe". Kant here realizes that the friction caused by the tidal motion of the ocean relative to the earth might cause a marked retardation of the Earth's rotation. He finds that this will go on until the Earth always turns the same side towards the Moon, i.e. until the length of the day is equal to a month. Kant writes:

"One can no longer doubt that the everlasting motion of the ocean from evening towards morning [from east towards west], a real and considerable force, will also always contribute something to decreasing the rotation of the Earth around its axis. This effect must inevitably become noticeable after a long period of time."
"As the Earth gradually approaches the standstill of its rotation, the period of this change will come to an end when the Earth's surface comes to a rest in relation to the Moon, i.e. when the Earth turns around its axis in the same time as that in which the Moon moves around the Earth."

Kant admits that he cannot present any evidence to support his hypothesis but leaves this as a task for others.

Although Kant claimed that it would be "a most shameful prejudice" not to bother about tidal friction, almost no one did so until a hundred years later, 1853. Then William Ferrel (1817 - 1891), an American oceanographer and meteorologist, pointed out that tidal friction causing a lengthening of the day, our unit of time, would lead to an apparent acceleration in the motions of celestial bodies. He tried to calculate this effect for the motion of the Moon, assuming the semi-diurnal ocean tide to have a mean phase lag of 30° (2 hours).

A small acceleration of the Moon - observed through the study of ancient records of solar eclipses - had been detected already by Halley (1693). It was known, however, to be caused by the disturbing gravitational forces of the Sun and the planets. How, then, did Ferrel explain why his effect was not observed? Here a popular view at that time came in handy. The Earth was thought to cool down and, thereby, to contract. This would make the Earth rotate faster. Ferrel assumed that the effects of tidal friction and the Earth's cooling happened to balance each other, so that nothing could be observed!

At the same time as Ferrel published his paper an error was discovered in the complicated computations of the gravitational perturbations of the Moon's motion. When correcting this error it was found that half of the observed acceleration of the Moon no longer could be accounted for. This made Ferrel claim, in 1864 (Figure 3), that the residual acceleration could be explained by tidal friction causing a lengthening of the day amounting to 1 second in 300 000 years. This would require a phase lag of only 2° (8 minutes).

Soon after, in 1866, the English astronomer and geodesist George Airy (1801 - 1892) commented upon the same problem. Airy found that tidal friction, in addition to lengthening the day, should cause a growing distance of the Moon from the Earth. But the difficulties in handling these problems were overwhelming; this is nicely illustrated by Airy in a simple example:

"Conceive, for instance (as a specimen of a large class), a tide-mill for grinding corn. The water, which has been allowed to rise with the rising tide, is not allowed to fall with the falling tide, but after a time is allowed to fall, thereby doing work, and producing heat in the meal formed by grinding the corn. I do not doubt that this heat is the representative of vis viva [kinetic energy], lost somewhere, but whether it is lost in the rotation of the Earth or in the revolution of the Moon, I am quite unable to say."

Since this time the phenomenon of tidal friction has been the subject of more or less continuous scientific discussion; we will encounter it again in section 1.6.

Five hundred and forty-third Meeting.

December 13, 1864. — MONTHLY MEETING.

The PRESIDENT in the chair.

The Corresponding Secretary read letters relative to exchanges.

Mr. Ferrel read the following paper.

Note on the Influence of the Tides in causing an Apparent Secular Acceleration of the Moon's Mean Motion.

As the unit of time depends upon the time of the earth's rotation upon its axis, any slight secular change in the time of its rotation, must cause an apparent secular acceleration or retardation of the moon's mean motion. There are two circumstances which may affect the time of the earth's rotation, first, the effect of the attractions of the sun and moon upon the tidal wave retarded by friction, secondly, a gradual decrease of the earth's volume from a loss of heat.

Figure 3. Ferrel's introduction to his idea on the effect of tidal friction (1864).

1.5 The discovery of earth tides

During the second half of the 19th century a vivid debate was going on concerning the internal constitution of the Earth. Was it fluid or solid? The English physicist William Thomson (1824 - 1907) - later known as Lord Kelvin - developed a theory of the Earth as an elastic solid. It appeared under the title "On the Rigidity of the Earth" in the Philosophical Transactions of the Royal Society of London in 1863. From his calculations Thomson arrives at the following conclusion:

"Hence it is obvious that, unless the average substance of the earth is more rigid than steel, its figure must yield to the distorting forces of the moon and sun, not incomparably less than it would if it were fluid."

Thus Thomson proposes the existence of earth tides; cf. Figure 4. According to Thomson the earth tides could be discovered and measured by observations of long period ocean tides. His idea was that the earth tides would reduce the observed amplitude of the ocean tides. Of these only the long period ones could be calculated theoretically, the diurnal and semi-diurnal ones being too disturbed by resonance phenomena.

At about the same time, in 1868, Thomson introduced the powerful tool of harmonic analysis into tidal theory. This led him to invent (four years later) the first tide prediction machine; it could handle 10 tidal constituents.

It was George Darwin (1845 - 1912; a son of Charles Darwin) who applied Thomson's ideas. Darwin was one of Thomson's students, specialising in astronomy and geophysics. He analysed tidal observations from 14 ports in England, France and India, together comprising 33 years of observations. Using the lunar fortnightnly and monthly tides he was able to find the ratio of the height of the ocean tide on the elastic Earth to that on a rigid Earth, i.e. the

II. "On the Rigidity of the Earth." By Professor WILLIAM THOMSON, F.R.S. Received April 14, 1862.

(Abstract.)

The author proves that unless the solid substance of the earth be on the whole of extremely rigid material, more rigid for instance than steel, it must yield under the tide-generating influence of sun and moon to such an extent as to very sensibly diminish the actual phenomena of the tides, and of precession and nutation. Results of a mathematical theory of the deformation of elastic spheroids, to be communicated to the Royal Society on an early occasion, are used to illustrate this subject. For instance, it is shown that a homogeneous incompressible elastic spheroid of the same mass and volume as the earth, would, if of the same rigidity as glass, yield about $\frac{7}{9}$, or if of the same rigidity as steel, about $\frac{2}{3}$ of the extent that a perfectly fluid globe of the same density would yield to the lunar and solar tide-generating influence. The actual phenomena of tides (that is, the relative motions of a comparatively light liquid flowing over the outer surface of the solid substance of the earth), and the amounts of precession and nutation, would in the one case be only $\frac{2}{9}$, and in the other $\frac{2}{5}$ of the amounts which a perfectly rigid spheroid of the same dimensions, the same figure, the same homogeneous density, would exhibit in the same circumstances. The close

Figure 4. The first part of the abstract of Thomson's theory (1863) claiming the existence of earth tides.

number which we today denote γ. He obtained $\gamma = 0.68 \pm 0.11$. His value happens to agree very well with modern values, but the main point is that Darwin showed it to be significantly smaller than 1. Thereby the existence of earth tides was proved. Darwin's historical result was first published in 1882 in "A Numerical Estimate of the Rigidity of the Earth" in Nature (see Figure 5); the full account of the tidal analysis was given in the next year. There he concludes:

"These results really seem to present evidence of a tidal yielding of the earth's mass, showing that it has an effective rigidity about equal to that of steel."

A NUMERICAL ESTIMATE OF THE RIGIDITY OF THE EARTH[1]

ABOUT fifteen years ago Sir William Thomson pointed out that, however it be constituted, the body of the earth must of necessity yield to the tidal forces due to the attraction of the sun and moon, and he discussed the rigidity of the earth on the hypothesis that it is an elastic body.

If the solid earth were to yield as much as a perfect fluid to these forces, the tides in an ocean on its surface would necessarily be evanescent, and if the yielding be of smaller amount, but still sensible, there must be a sensible reduction in the height of the oceanic tides.

Sir William Thomson appealed to the universal existence of oceanic tides of considerable height as a proof that the earth, as a whole, possesses a high degree of rigidity, and maintained that the previously received geological hypothesis of a fluid interior was untenable. At the same time he suggested that careful observation would afford a means of arriving at a numerical estimate of the average modulus of the rigidity of the earth's mass as a whole. The semi-diurnal and diurnal tides present phenomena of such complexity, that it is quite beyond the power

[1] Paper read by G. H. Darwin, F.R.S., at the British Association Southampton meeting.

Figure 5. The beginning of Darwin's announcement (1882) of the discovery of the earth tides.

1.6 Tides and dynamical properties of the Earth

The concept of earth tides opened up new prospects for the research on tidal friction. Considering the Earth to be a viscous fluid, George Darwin studied tidal friction in the Earth's interior, instead of in the oceans. Darwin found, like Airy had done in the ocean case, that tidal friction will not only have the effect of retarding the Earth's rotation, but also cause the Moon to recede from the Earth:

"The moon-earth system is, from a dynamical point of view, continually losing energy from the internal tidal friction. One part of this energy turns into potential energy of the moon's position relatively to the earth, and the rest developes heat in the interior of the earth."

Darwins computations, published in 1879 as the first of a long series of papers on the subject, showed that in the early days of the Earth's history the Moon must have been very much closer to the Earth than now. Thus tidal friction was found to play a fundamental role in the evolution of the the Earth-Moon system; it even was found to raise questions as to the very origin of the Earth-Moon system.

Three years later, in 1882, Darwin tried to calculate the effect of ocean loading on the elastic crust. This led him to predict the existence of loading tides, i.e. earth tides due to loading from ocean tides. Darwin's illustration of the loading tides is shown in Figure 6.

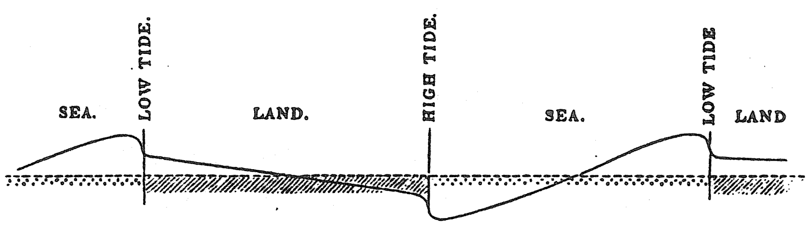
A vertical tidal displacement of the crust should be accompanied by a corresponding tidal tilt.
Many years of search using horisontal pendulums had elapsed when finally Oskar Hecker (1864 - 1938), a German geodesist and seismologist, succeeded in observing this tilt of only a hundredth of a seond of arc, at Potsdam in 1907. Hecker's instrumentally obtained value of γ agreed with Darwin's old result, obtained in a

quite different way. Furthermore, Hecker found a difference between the values in the N-S direction and the E-W direction, a difference which he could ascribe to the ocean loading tides suggested by Darwin.

A powerful method of describing tidal deformations of the elastic Earth was given in 1909 by the English geophysicist Augustus Love (1863 - 1940), and included in his book "Some Problems of Geodynamics" in 1911. Here we find what we now know as "Love's numbers" h and k, h characterizing the height of the deformation caused by the tidal potential, k characterizing the additional potential caused by the deformation. Love established the relation

$$\gamma = 1 + k - h$$

Furthermore, he - as well as Larmor - derived a relation containing k only, between the polar motion period of the elastic Earth and that of a rigid Earth (see section 3.3). From the combination of tidal observations and polar motion observations Love was able to calculate



The straight line is a section of the undisturbed level, the shaded part being land, and the dotted sea. The curve shows the distortion, when warped by high and low tide as indicated.

Figure 6. Ocean loading tides as illustrated by Darwin (1882).

the values of h and k: $h = 0.6$, $k = 0.3$. The value of h indicated a maximum vertical tidal displacement of 0.5 m.

A little later, in 1914, one of Hecker's geodetic colleagues, Wilhelm Schweydar (1877 - 1959), was the first to observe earth tides with a gravimeter. For the enhancement of the gravity variations due to the elasticity of the Earth he obtained a factor of $\delta = 1.20$, only slightly larger than the modern value ($\delta \approx 1.16$). Schweydar derived the relation

$$\delta = 1 + h - \frac{3}{2} k$$

From then on the combination of tilt and gravity observations, i.e. γ and δ, were to be fundamental for the determination of Love numbers.

An important contribution to the foundations of tidal theory was the harmonic expansion of the tidal potential made by the English mathematician and oceanographer Arthur Doodson (1890 - 1968) in 1921. It consisted of no less than 386 components of different periods and amplitudes.

Tidal friction, that had started as a problem of ocean tides, had been turned by Darwin into a problem of earth tides. By 1920 it was turned back into an ocean tidal problem by Harold Jeffreys (1891 - 1989), the English astronomer and geophysicist. Jeffreys brought forward evidence of the secular retardation of the Earth's rotation being caused mainly by tidal friction in shallow seas, a theory which occupied a strong position for quite a long time. It was an extension of a theory published the year before by an English meteorologist, Geoffrey Taylor (1886 - 1975), whose idea, curiously enough, originated from a dynamical similarity between tidal friction on an ocean bottom and wind friction along a ground covered with grass.

A different tidal effect on the Earth's rotation was introduced by Jeffreys in 1928. He found that the long period earth tides, forming small periodical variations in the flattening of the Earth, must cause corresponding variations in the rotational velocity large enough to affect accurate time-keeping.

A problem that had been unsolved ever since the days of Plinius was that of tides in wells. How did they arise, and why did they usually have a phase opposite to that of the tidal potential? The problem was solved in 1940 by the American geophysicist Chaim Leib Pekeris (1908 -):

"It is clear that at the time of the moon's transit, when the earth tide is high, the region underneath the station is under tension and is dilated, while six hours later, when the earth-tidal displacement is downwards, it is under compression. Compression in the water-bearing region would squeeze the water into the well and would thus bring about a rise of the water level at a time the displacement due to the earth tide is downwards."

Pekeris calculated that the volume strain, or dilatation, should be of the order of 10^{-8}. This formed the beginning of tidal strain research.

Already in 1905 a theory for the elastic deformations of a non-homogeneous Earth had been developed by a German mathematician, Gustav Herglotz (1881 - 1953). It ended in a differential equation of the sixth order which was, at that time, beyond the powers of anybody to solve. However, in 1950 the Japanese geophysicist Hitoshi Takeuchi turned the problem into three differential equations of the second order and succeeded in solving them numerically. Thereby he obtained, for the first time, tidal Love numbers for a realistic Earth model as given by seismological evidence.

The year before, Jeffreys pointed out that precession-nutation is caused by the diurnal tidal forces. A resonance in the liquid core of the Earth had been found by him to noticeably affect the nutation (section 2.6). Consequently, the liquid core should affect the diurnal earth tides, too, causing the corresponding Love numbers to deviate from those computed by Takeuchi. This initiated the great challenge of finding, by theory as well as observations, the response to tidal forces of an elastic Earth, partly covered by oceans, with a liquid core.

2. HISTORY OF PRECESSION-NUTATION THEORY

2.1 The discovery of precession

During many years the Greek astronomer Hipparchos (c. 180 - c. 120 B.C.) made observations of stars on the island of Rhodos. From these observations he created the first extensive star catalogue, containing nearly 1000 stars with their coordinates. When comparing some of his coordinates with those determined one and a half century earlier, he found a systematic decrease of the ecliptical longitudes of the stars. Hipparchos correctly interpreted this as a continuous motion of the equinoxes along the ecliptic. Thus Hipparchos had made a remarkable discovery: the precession. This phenomenon meant that the equator was not stable but moved in such a way that the celestial pole moved in a circle around the ecliptical pole.

Hipparchos' discovery was published about 125 B.C. in "On the Displacement of the Solsticial and Equinoctial Points". In this work he also calculated the precessional constant, i.e. the value of the annual precession. For the star Spica he knew the coordinates from Timocharis' old observations in 294 and 283 B.C. and his own observations in (at least) 146 and 135 B.C. Hipparchos writes:

"Spica, for example, was formerly 8°, in zodiacal longitude, in advance of the autumnal [equinoctial] point, but is now 6° in advance."

This makes a precession of 2° in 148 years, i.e. 49"/year. The value is very close to the modern one, 50"/year. However, Hipparchos was well aware of its uncertainty. From all his calculations he considered 1° in 100 years, or about 36"/year, to be a minimum value of the precessional constant.

Hipparchos' work is no longer preserved but it is quoted by the Greek mathematician and astronomer

Klaudios Ptolemaios (c. 100 - c. 175 A.D.) in his famous book "Almagest". This book was written about 150 A.D. under the Greek name "Mathematike syntaxis" (Mathematical Systematic Treatise), later "Megale syntaxis" (Great Systematic Treatise). When translated into Arabic only the first word was retained, now in the form "megiste" (greatest), to which was added the Arabic definite article "al", finally leading to today's "Almagest". Ptolemaios here confirms the existence of the precession. But he does not accept Hipparchos' view of the precession as a movement of the equinoxes, probably because this view is related to the Earth as a rotating body. Ptolemaios was convinced that there was no such thing as a rotation of the Earth. To him, therefore, the precession is a movement of a celestial sphere onto which the stars seem to be fixed:

"The sphere of the fixed stars also performs a motion of its own in the opposite direction to the revolution of the Universe."
"The sphere of the fixed stars has a movement towards the rear with respect to the solsticial and equinoctial points ... This motion takes place about the poles of the ecliptic."

Ptolemaios also determined the precessional constant. However, the value which Hipparchos had considered to be the smallest possible one, 1^{o} in 100 years, is adopted by Ptolemaios as the most probable one. He refers to own observations leading to this value, but these observations seem to have been suffering from a systematic error. His star catalogue suffers from the same systematic error all the way through.

The "Almagest" was in many respects a master-piece, serving as a text-book during one thousand four hundred years. Consequently, Ptolemaios' view of the character of the precession and his value of the precessional constant were to mislead and confuse scientists during the same long period of time.

2.2 Precession and the rise and fall of trepidation

In the 9th century an Arabian mathematician, Thabit ibn Qurra (836 - 901), studied the "Almagest", even making a revision of one of the Arabic translations. The values of the precessional constant obtained up till then, including Ptolemaios' erroneous one, led ibn Qurra to the conclusion that this constant was not constant at all. So he had to introduce a large variation in the precession, known as trepidation. To account for this he suggested an additional moving celestial sphere.

The idea that the precession might be variable was not new, but from now on the trepidation was accepted as a real phenomenon by most scientists. One who expressed strong doubts about its reality was, however, the Arabian astronomer Mohammed al-Battani (858 - 929), in connection with making a redetermination of the precessional constant and a new star catalogue.

The machinery of moving celestial spheres adopted to explain precession and trepidation was still not questioned for many centuries - not until 1543. That year marks the (re)discovery of the Earth's rotation. It was one of the basic ideas put forward by Nicolaus Copernicus (1473 - 1543), the Polish astronomer and priest, in his "De revolutionibus orbium coelestium" (the title page of which ended with the words: "Buy it, read it, enjoy it"!). This had important consequences for the view on the precession. Copernicus realizes that the precession is a movement of the Earth itself:

"From the time of Ptolemaios to ours there has been a precession of the equinoxes and solstices of about 21^{o}."
"The equinoxes seem to arrive before their time - not that the sphere of the fixed stars is moved eastward, but rather that the equator is moved westward, as it is inclined obliquely to the plane of the ecliptic in proportion to the amount of deflexion of the axis of the terrestrial globe."

Thus the precession now became a phenomenon associated with the axis of rotation of the Earth. According to Copernicus' theory the Earth moves in such a way that the rotational axis describes a conical motion around the normal of the ecliptic, with a period of 25 800 years.

Copernicus still believed, however, that the precession was accompanied by a trepidation. Using the works of Ptolemaios and al-Battani he found a period for the trepidation of 1 700 years. This result can be seen to be produced mainly by the ancient systematic error of Ptolemaios.

Before we leave Copernicus we should mention that he discovered, as he himself calls it, "an additional surprise of nature": the decrease of the obliquity of the ecliptic. Also this effect he believed to show a kind of variation, which he suspected was closely related to the trepidation.

The end of the deep-rooted notion of trepidation came with the Danish astronomer Tycho Brahe (1546 - 1601). He had erected an impressive observatory on the small island of Ven. By using his extremely accurate observations made here, and by critically going through the ancient observations, he arrived in 1588 at the definite conclusion that the trepidation did not exist.

So, one was now left with a uniform precession of the Earth. The cause of the precession was, however, still hidden in the dark. No one even seems to have made an attempt to find it.

2.3 Aha! Gravitation, flattening, and precession-nutation

In 1687 Isaac Newton (1642 - 1727), the English mathematician, physicist and astronomer, published the "Philosophiae naturalis principia mathematica" containing, among other things, his fundamental law of gravitation. In section 1.3 we saw how this discovery enabled Newton to find the origin of the tides and explain their main characteristics. An even more remarkable achievement was that Newton found the origin of the precession. The precession turned out to be caused by gravitational forces of the Moon and the Sun in combination with a deformation of the Earth. It was namely also realized by Newton that the Earth should be flattened at the poles as a consequence of its rotation. Thus the Moon's and the Sun's gravitation acting on the inclined Earth's equatorial bulge created the precession. A series of complicated arguments was presented by Newton to prove this result, which he himself expresses with the words:

"Redundant matter in the aequatorial regions of a globe causes the nodes to go backwards."

It is interesting to note an immediate application of the precession theory. We let Newton speak again:

"And thence from the motion of the nodes is known the constitution of the globe. That is if ... the motion (of the nodes) be in antecendentia, there is a redundance of the matter near the equator; but if in consequentia, a deficiency."

Since the precession of the equinoxes (or the nodes) was observed to be a motion backwards ("in antecendentia") the Earth must necessarily be flattened at the poles, not at the equator. Yet, as we know, the Earth's flattening was to be a matter of great controversy for half a century.

Prop. XXXIX. Prob. XIX.

Invenire Præcessionem Æquinoctiorum.

Motus mediocris horarius Nodorum Lunæ in Orbe circulari, ubi Nodi sunt in Quadraturis, erat 16″. 35‴. 16^iv. 36^v. & hujus dimidium 8″. 17‴. 38^iv. 18^v. (ob rationes supra explicatas) est motus medius horarius Nodorum in tali Orbe; fitque anno toto sidereo 20 *gr.* 11′. 46″. Quoniam igitur Nodi Lunæ in tali Orbe conficerent annuatim 20 *gr.* 11′. 46″. in antecedentia; & si plures essent Lunæ motus Nodorum cujusque, per Corol. 16. Prop. LXVI. Lib. I. forent reciprocè ut tempora periodica; & propterea si Luna spatio diei siderei juxta superficiem Terræ revolveretur, motus annuus Nodorum foret ad 20 *gr.* 11′. 46″. ut dies sidereus horarum 23. 56′. ad tempus periodicum Lunæ dierum 27. 7 hor. 43′; id est ut 1436 ad 39343. Et par est ratio Nodorum annuli Lunarum Terram ambientis; sive Lunæ illæ se mutuò non contingant, sive liquescant & in annulum continuum formentur, sive denique annulus ille rigescat & inflexibilis reddatur.

Fingamus igitur quod annulus iste quoad quantitatem materiæ æqualis sit Terræ omni *PapAPepE*, quæ globo *PapE* superior est; & quoniam globus iste est ad Terram illam superiorem ut *aC qu.* ad *AC qu.* — *aC qu.* id est (cum Terræ diameter minor *PC* vel *aC* sit ad diametrum majorem *AC* ut 689 ad 692) ut 4143 ad 474721 seu 1000 ad 114584; si annulus iste Terram secundum æquatorem cingeret, & uterque simul circa diametrum annuli revolveretur, motus annuli esset ad motum globi interioris (per hu-

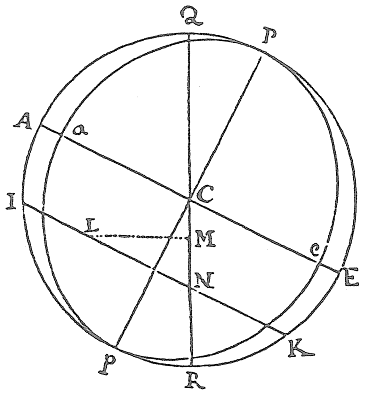


Figure 7. The beginning of Newton's calculation of precession given in the "Principia" (1687). The figure shows the flattened Earth: Pp denotes the axis of rotation, or minor axis, AE the major axis, and QR the normal to the ecliptic.

Newton also was able to make a theoretical calculation of the precessional constant. The beginning of this is shown in Figure 7. Assuming a hydrostatical equilibrium of the Earth he had found the Earth's flattening to be 1/231. Using this value he computed the solar precession. He then calculated the lunar precession in an ingenious way. He had shown that the gravitational attractions of the Moon and the Sun produce the precession - but he had also shown that the same forces produce the tides. So he simply uses the result of his analysis of ocean tidal observations (see section 1.3) to find also the lunar precession:

> "The remaining motion will now be $9'' \; 7''' \; 20^{iv}$ which is the annual precession of the equinoxes, arising from the force of the Sun. But the force of the Moon to move the sea was to the force of the Sun nearly as 4.4815 to 1. And the force of the Moon to move the equinoxes is to that of the Sun in the same proportion. Whence the annual precession of the equinoxes, proceeding from the force of the Moon, comes out $40'' \; 52''' \; 52^{iv}$, and the total annual precession, arising from the united forces of both, will be $50'' \; 00''' \; 12^{iv}$, the quantity of which motion agrees with the phaenomena."

Newton's results of 41" and 9" for the lunar and solar precession, respectively, agree pretty well with the actual figures, 34" and 16". Since Newton's ratio between the lunar and solar tides is about twice the real ratio the same thing applies to his lunar and solar precessions. Thus the excellent agreement between his total precessional constant and the observed one is somewhat illusory, being caused by different errors in numerical quantities cancelling each other.

Although Newton touched upon the existence of a small nutation, it was the English astronomer James Bradley (1692 - 1762) who discovered and explained the principal nutation of the Earth. Bradley had made repeated observations of the declinations of stars during 20 years. These observation series revealed a periodic variation of the declinations, the period being 18 1/2 years, the amplitude 9", and the phase depending on the right ascension of the star. Bradley explained the variation by a nutation of the Earth, i.e. a movement of the Earth involving a periodic variation of the inclination of the Earth's axis. The nutation, he found out, was closely related to that part of the precession which was caused by the Moon. Bradley announced this discovery in 1748 in the Royal Society of London:

"I suspected, that the Moon's Action upon the Equatorial Parts of the Earth might produce these Effects: For, if the Precession of the Equinox be, according to Sir Isaac Newton's Principles, caused by the Actions of the Sun and Moon upon those Parts; the Plane of the Moon's Orbit being at one time above ten Degrees more inclined to the Plane of the Equator than at another; it was reasonable to conclude, that the Part of the whole annual Precession, which arises from her Action, would in different Years be varied in its Quantity."
"I perceived that something more, than a mere Change in the Quantity of the Precession, would be requisite to solve this Part of the Phaenomenon. Upon comparing my Observations of Stars near the Solsticial Colure, that were almost opposite to each other in Right Ascension, I found ... that this apparent Motion, in both those Stars, might proceed from a Nutation in the Earth's Axis."

When being informed by Bradley about his results, John Machin (c. 1680 - 1751), an English mathematician, suggested to him a useful geometrical method of describing the nutation. The true pole of the celestial equator moved in a little circle - the nutation circle - of radius 9" with a period equal to that of the nodes of the Moon's orbit, 18.6 years, around a mean pole. This mean pole, in its turn, moved in the large precession circle of radius 23°.5 with the period of 25 800 years around the pole of the ecliptic.

Bradley suspected that the nutation circle might be an ellipse instead, but considered that this problem could be solved only within the framework of a mathematical theory of precession-nutation. The time was now ripe for such theories.

2.4 The mathematicians enter the scene

Mathematical attacks on precession-nutation were made already the year after Bradley's discovery of the nutation. One was performed by the French mathematician Jean le Rond d'Alembert (1717 - 1783; as a new-born baby he was found abandoned at the church S:t Jean le Rond, hence his name). In 1749 he presented his "Recherches sur la précession des équinoxes et sur la nutation de l'axe de la Terre", which contains a detailed mathematical theory of precession and nutation. Here he makes use of the dynamical principle that he himself had found six years earlier. In particular d'Alembert shows that the nutation circle must be replaced by a nutation ellipse, the major axis of which is directed towards the pole of the ecliptic - cf. Figure 8. His computation of the semi-axes of the nutation ellipse yields 9" and 6", in close agreement with the modern values (9" and 7").

The other mathematical attack on precession-nutation made in the same year appeared in a paper with the same title as d'Alembert's book. The author this time was Leonhard Euler (1707 - 1783), the Swiss mathematician working in Russia and Germany. Euler arrived at principally the same results as d'Alembert. But in addition Euler finds a new nutation term: one depending on the Sun, with a period of 1/2 year. This is the nutation already mentioned briefly by Newton which Euler now is able to calculate.

Nine years later, in 1758, Euler developed a general theory for rotating rigid bodies. Introducing the concepts of torque and moment of inertia Euler derived the equations of motion which today bear his name. This made it possible to deal with precession and nutation as a special solution of the Euler equations. Furthermore, an important consequence of these equations was the discovery of the phenomenon of polar motion; see further section 3.1.

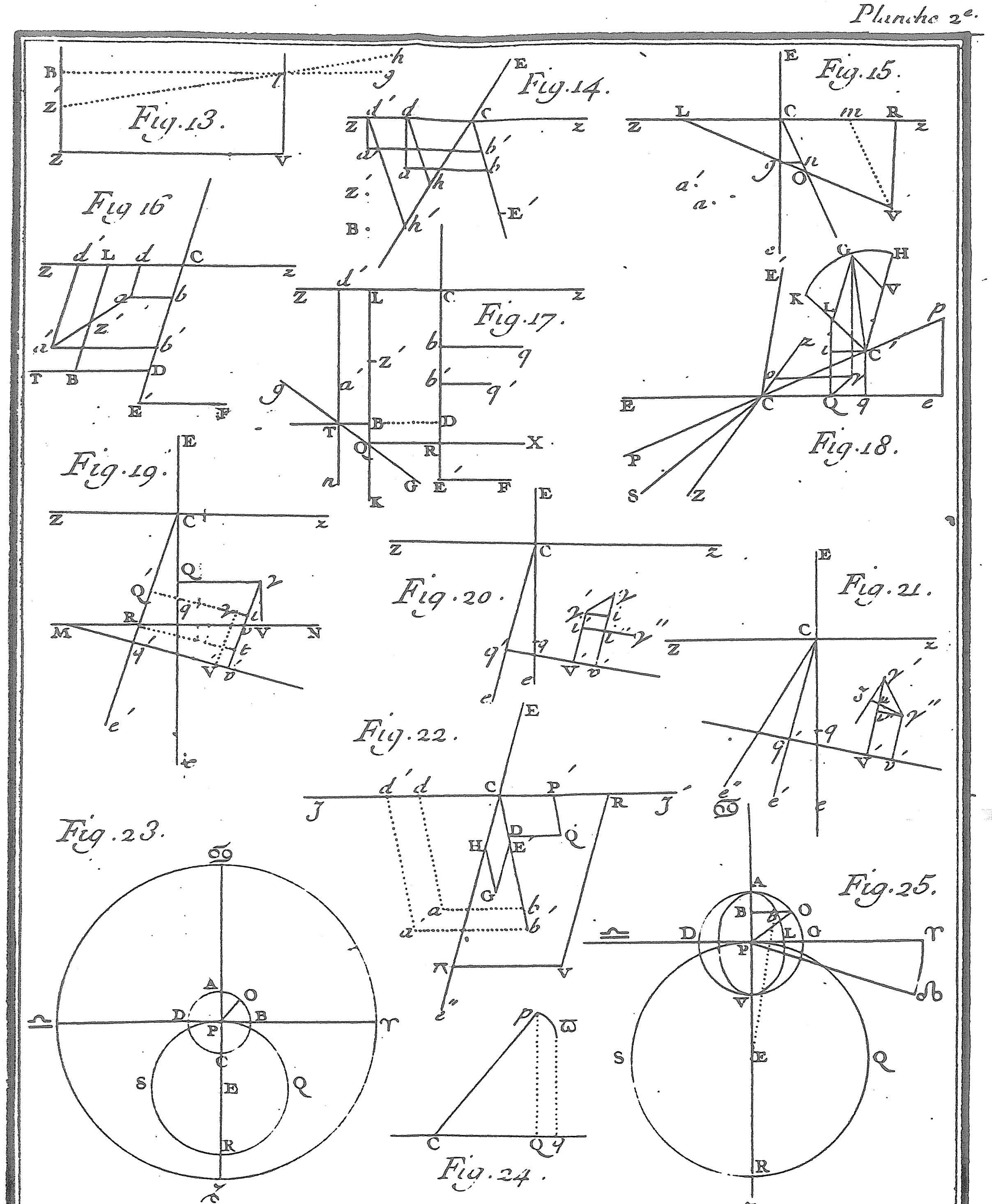


Figure 8. A page with figures from d'Alembert's book (1749). His Fig. 25 shows the nutation ellipse, the major axis of which is APV, as well as the precession circle, with the diameter PER.

In 1799 Pierre de Laplace (1749 - 1827), the French mathematician and astronomer, published the first two volumes of his "Traité de mécanique céleste". Here he developed the mathematical theory of tides (section 1.3). In the same work we find an extensive treatment of the theory of precession-nutation. Laplace applies Euler's equations mentioned above. In this way he derives precession-nutation formulae of the type illustrated in Figure 9. They show the central role played by the moment of inertia ratio (C - A)/C, reflecting the flattening of the earth ellipsoid. Furthermore, they show the possibility of expressing precession-nutation as a harmonic series, a possibility that could not be used very much until our own century.

312 MÉCANIQUE CÉLESTE,

lesquels P peut se développer, et par $\Sigma.k'.\sin.(it+\epsilon)$, la somme des termes dans lesquels P' peut se développer, Σ étant la caractéristique des intégrales finies; on aura

$$\frac{d\theta}{dt} = \left(\frac{A+B-2C}{2n.C}\right).\Sigma.k'.\sin.(it+\epsilon); \qquad (H)$$

$$\frac{d\psi}{dt}.\sin.\theta = \left(\frac{2C-A-B}{2n.C}\right).\Sigma.k.\cos.(it+\epsilon).$$

En intégrant ces équations, sans avoir égard aux constantes arbitraires; on aura les parties de θ et de ψ qui dépendent de l'action de l'astre L. Pour avoir les valeurs complètes de ces variables, il faut

Figure 9. Laplace's precession-nutation formulae as they appear in the "Mécanique céleste" (1799). For the Earth A = B.

2.5 Precession and the Ice Age

Around the middle of the 19th century a great interest in precession arose in an unexpected context: the Earth's climate. The background was the recent discovery of the Ice Age.

Searching for a cause of the Ice Age, Joseph Adhémar (1797 - 1862), a French mathematician, came up with the idea of the precession playing an essential role. He did so in a book called "Révolutions de la mer" which was published in 1842, five years after Agassiz' discovery of the Ice Age. Adhémar argues in the following way: Because of the eccentricity of the Earth's orbit around the Sun and the direction of the Earth's inclined rotational axis, the winters are longer in the southern hemisphere than in the northern one. The precession of the Earth - cf. Figure 10 - will cause this situation to change to the opposite and back again in 26 000 years, thus causing periodical climatic variations. At the same time the major axis of the Earth's orbit is moving, too, making the true period of these climatic variations 21 000 years. The present situation with longer winters in the southern hemisphere explains the large southern polar ice cap. Half the period ago, i.e. 10 500 years ago, the situation must have been the opposite - hence the Ice Age in the northern hemisphere at that time. Thus, according to Adhémar, the precession caused repeated glaciations, alternately in the northern and the southern hemispheres.

Adhémar's theory was modified in 1875 by James Croll (1821 - 1890), a Scottish geologist. Croll meant that the precession was insufficient to cause glaciations. However, the precession in combination with the periodical variations of eccentricity and inclination of the Earth's orbit could produce the glaciations, or at least trigger them. This hypothesis, which became more or less discarded towards the end of the century, has in our days received quite a lot of support, mainly through the theory of Milankovitch.

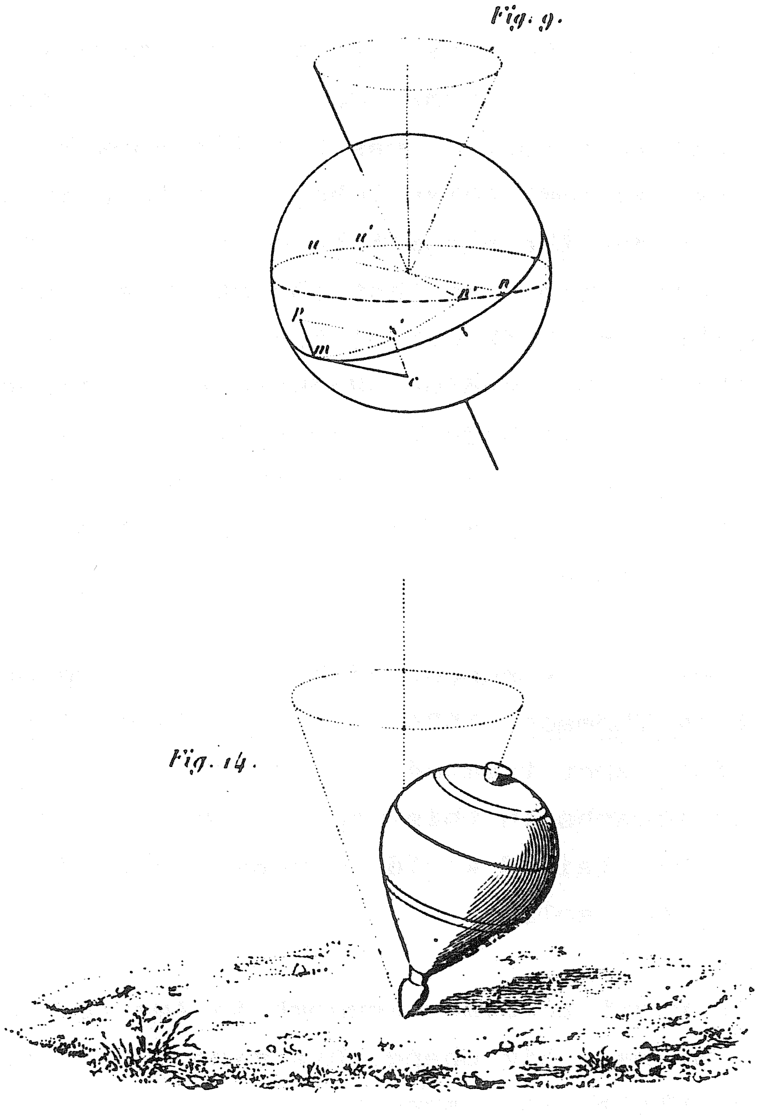


Figure 10. The precession of the Earth compared to the precession of a spinning top. Illustrations used by Adhémar to explain his theory on precession and climate (1842).

2.6 Precession-nutation and dynamical properties of the Earth

As early as 1839 the first attempt was made to investigate the effects of the internal constitution of the Earth on precession and nutation. It was William Hopkins (1793 - 1866), an English mathematician, who tried to compute these phenomena under the assumption of a fluid interior of the Earth. His conclusion was that the observed amount of precession required a thickness of the crust exceeding one fifth of the Earth's radius. Hopkins was a pioneer in applying mathematical methods to investigate a non-rigid Earth, complaining that he could not get geologists to understand mathematics nor mathematicians to take an interest in his geology.

Hopkins' result was supported by further arguments of his pupil William Thomson (1824 - 1907) - later Lord Kelvin - in the 1863 tidal paper treated in section 1.5. Thomson devoted some paragraphs in this paper to precession and nutation, which he claimed would be drastically diminished if the Earth was not solid.

However, a visit to Simon Newcomb in America in 1876 (cf. section 3.3) made him change his mind. On returning to England Thomson stated in a speech:

"Admitting fully my evidence for the rigidity of the earth from the tides, he [Newcomb] doubted the argument from precession and nutation ... I could only stammer out that I had convinced myself that so-and-so and so-and-so, at which I had arrived by a non-mathematical short cut, were true ... But doubt entered my mind regarding the so-and-so and so-and-so; and I had not completed the night journey to Philadelphia which hurried me away from our unfinished discussion before I had convinced myself that they were grievously wrong. So now I must request as a favour that each one of you on going home will instantly turn up his or her copies of the Transactions of the Royal Society for 1863 ... and draw the pen through §§ 21-32 of my paper on the 'Rigidity of the Earth'."
"So far nothing can be considered as absolutely proved with reference to the interior solidity of the earth from precession and nutation."

Inspired by the discovery of polar motion several scientists tried to develop theories for a rotating ellipsoidal Earth containing liquid, but no very successful theory with respect to precession-nutation emerged until 1910. Then Henri Poincaré (1854 - 1912), the French mathematical physicist, solved this complicated problem. Nine years earlier he had been able to modify Lagrange's equations so that they could be used not only for solids but also for liquids. Applying these equations Poincaré discovered that in a liquid core of about the same flattening as the Earth a resonance could occur which might perturb the nutation.

Since the beginning of the century there was seismological evidence for a core surrounded by a mantle with a thin crust, the radius of the core being close to half the Earth's radius. Poincaré's theory combined with this knowledge formed the foundation upon which Harold Jeffreys (1891 - 1989), the English astronomer and geophysicist, built his first analyses of the dynamic effects of a liquid core. They were published in a series of papers starting 1948. Jeffreys is here concerned with the significant discrepancy between the observed amplitude of the principal lunar nutation (9".21) and the theoretical amplitude (9".23), based on the assumption of a rigid Earth. He finds that the liquid core really will reduce the theoretical amplitude, but that it is reduced too much:

"The main conclusion is that the theoretical nutation, obtained by taking the Earth as rigid, is probably too large; allowance for fluidity of the core while taking the shell as rigid reduces it to well below the observed value; but allowance for elasticity of the shell at the same time might result in agreement."

Jeffrey's suspicion that elasticity might bring the liquid core theory in better agreement with observations was confirmed by himself the following year (1949), but there was still not full agreement:

"My previous conjecture that elasticity of the shell would reduce the effect of fluidity of the core is verified."
"Even with these adjustments the effect remains too great, though the excess is possibly not greater than might be due to the simplifications made in the adopted model [of the Earth]."

The work by Jeffreys formed a major step towards understanding precession-nutation of an elastic Earth with a liquid core. Moreover, it served as a starting point for a liquid core theory for earth tides (section 1.6), and it will turn up again in connection with polar motion (section 3.3).

3. HISTORY OF POLAR MOTION THEORY

3.1 The mathematical discovery of polar motion

The 18th century was a golden age for the application of mathematical methods to dynamical problems. Leonhard Euler (1707 - 1783), the Swiss mathematician working in Russia and Germany, was the one who derived equations for the rotation of rigid bodies. These equations are the "Euler equations" that we encountered in connection with precession-nutation in section 2.4. They were announced in 1758 in one of Euler's many papers in the Transactions of the Royal Academy of Sciences in Berlin, "Du mouvement de rotation des corps solides autour d'un axe variable". By putting the right-hand sides of his equations equal to zero Euler obtains the equations for a freely rotating body, i.e. the case with no gravitational torques present. The equations then reveal that the axis of rotation is not fixed in the body - in other words, that the poles on the surface of the body, e.g. the Earth, must be moving.

A very elegant method for finding the equations of motion of, among other things, a rotating body was given by the Italian-French mathematician Joseph Lagrange (1736 - 1813) in his "Méchanique analitique", appearing in 1788. Euler's equations turned out to be a simple example of Lagrange's equations, which were based on the calculus of variations developed by Euler. It is in Lagrange's book we find Euler's equations for polar motion in the form that we use them today - see Figure 11. They show that the frequency, or period, is governed by the flattening of the Earth through the moment of inertia ratio (C - A)/A.

Neither Euler nor Lagrange seems to have discussed the resultant motion of the pole very much. Lagrange, whose successful ambition was to create an analytical dynamics free from geometrical methods, even declares in his preface: "One cannot find any figures in this work."

In contrast to this, Louis Poinsot (1777 - 1859), another French mathematician, concentrated on a purely geometrical method to study the motion of the rotational axis of a freely rotating body. His solution, published in two versions in 1834 and 1851, revealed that the axis of rotation could be looked upon as describing two cones closely associated with each other: One cone was formed by the motion of the rotational axis around the axis of maximum moment of inertia, or the symmetry axis of the body. The other cone was formed by the motion of the rotational axis around the axis of angular momentum fixed in space. Poinsot found that the body cone rolls without slipping around the space cone, the line of contact being the instantaneous axis of rotation. The body cone produces the Eulerian motion of the pole; the space cone represents a small nutation.

48. En ſuppoſant F, G, H nulles, on a, comme on l'a vu dans l'article 42,

$$\frac{dT}{dp} = Ap, \quad \frac{dT}{dq} = Bq, \quad \frac{dT}{dr} = Cr,$$

& ces valeurs étant ſubſtituées dans les trois équations différentielles (A), il vient celles-ci,

$$dp + \frac{C-B}{A} qr\,dt = 0, \quad dq + \frac{A-C}{B} pr\,dt = 0, \quad dr + \frac{B-A}{C} pq\,dt = 0;$$

leſquelles s'accordent avec celles que M. Euler a employées dans la ſolution qu'il a donnée le premier de ce problême

Figure 11. Euler's equations for a freely rotating body as presented by Lagrange in the "Méchanique analitique" (1788). For the Earth A = B.

3.2 The great surprise

During the second half of the 19th century an intense activity was going on at several observatories to confirm the existence of a polar motion of the Earth. From Euler's equations it could be predicted that its period would be 306 days. The polar motion should manifest itself as a periodic variation of the latitude of the observatory.

Meanwhile William Thomson (1824 - 1907) - the English physicist we already know from earth tides and precession - pointed out, in 1876, that the motion of the pole might be more complicated than generally believed. Possible redistributions of matter in and on the Earth would influence the position of the pole, thereby preventing polar motion from damping. The seasonal redistribution of masses in the ocean and the atmosphere should cause a polar motion of its own. And the motion of the pole should raise a tide in the ocean. Altogether, Thomson expected the whole thing to be a quite irregular phenomenon.

Besides, the German geodesist Friedrich Helmert (1843 - 1917) claimed that a secular drift of the pole could go on. Its main cause would be, he argued in 1884, postglacial rebound.

The search for a variation of latitude went on for many years without success. When it finally was detected it was by a German astronomer, Friedrich Küstner (1856 - 1936), who was not looking for this effect at all. Küstner's purpose was to determine carefully the constant of aberration, by observing stars at the Berlin observatory. When his results turned out to be inconsistent he made a detailed investigation of them. It ended in the detection of a latitude variation amounting to a few tenths of a second of arc in about one year. His announcement of this in 1888 aroused a great international interest.

Seth Carlo Chandler (1846 - 1913), a private astronomer in America, had been making stellar observations coincident in time with those of Küstner in Germany, the two of them, however, not knowing about each other. Chandler had observed the same effect as Küstner but, unlike him, had not dared to publish it. When reading Küstner's report Chandler immediately recognized the effect and started upon a thorough investigation not only of their own data but of all appropriate observation series made in the world during the last 50 years. In 1891, when he had analysed two long simultaneous series from America and Russia as well as the two series already mentioned, he was ready to publish a great surprise, in "On the Variation of Latitude" in the Astronomical Journal:

"Before entering upon the details of the investigations ... it is convenient to say that the general result of a preliminary discussion is to show a revolution of the earth's pole in a period of 427 days, from west to east, with a radius of thirty feet, measured at the earth's surface."

The period was 427 days instead of 306! This was a most unexpected result and so contradictory to theory that many found it hard to believe.

A year later (1892) Chandler had completed investigations comprising material from no less than 17 observatories and was able to announce, in a later part of his above-mentioned paper:

"The observed variation of the latitude is the resultant curve arising from two periodic fluctuations superposed upon each other. The first of these, and in general the more considerable, has a period of about 427 days ... The second has an annual period."

Chandler's curve of the latitude variation is shown in Figure 12. The annual period was the one predicted by Thomson sixteen years earlier. But what was the 427-day-period? Chandler could offer no explanation.

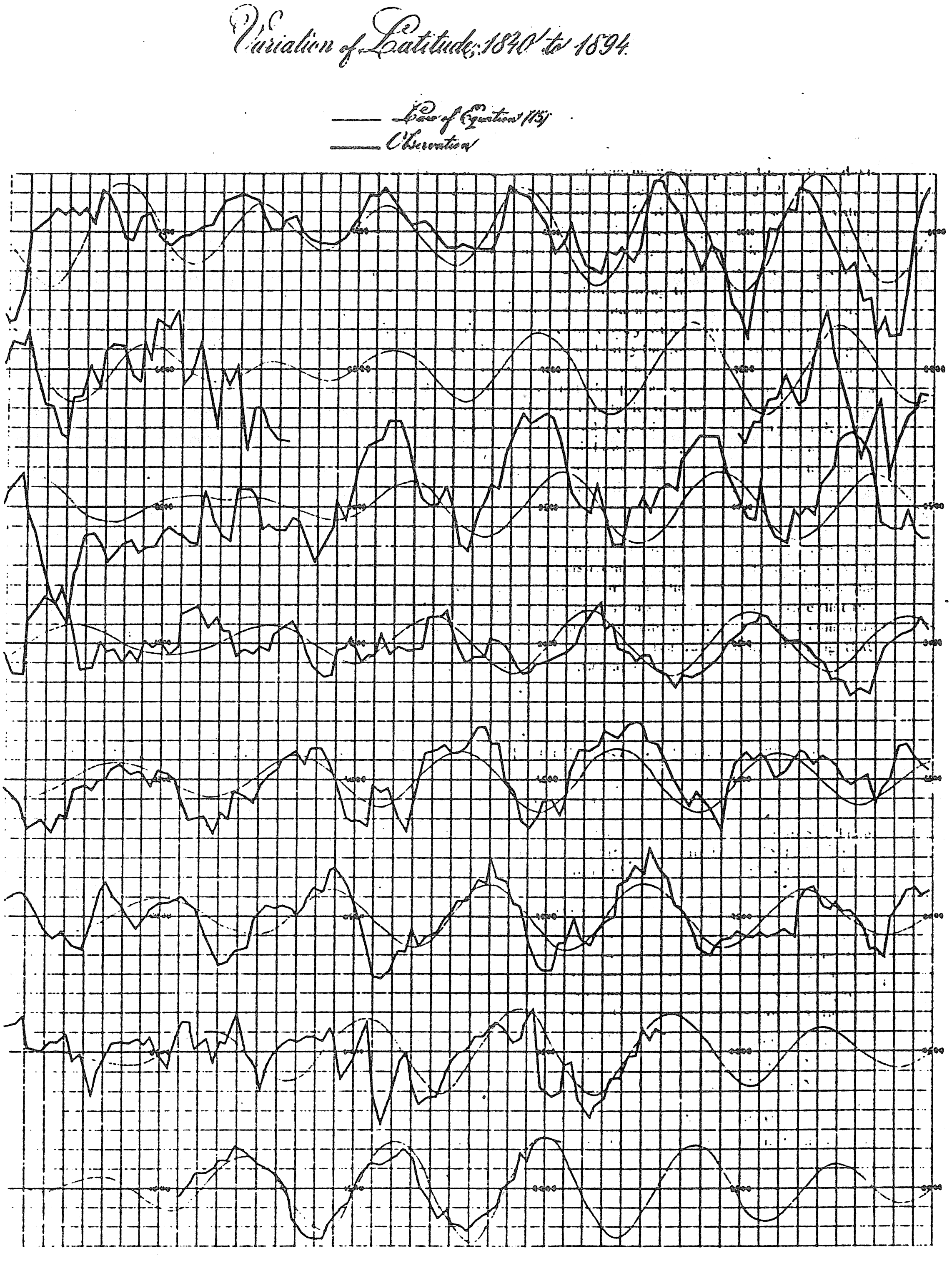


Figure 12. The periodic latitude variations found by Chandler (1892).

Let us for a moment look back a little. During 50 years scientists had tried to find the polar motion - with no result. And then, suddenly, one man succeeds - using the same data which had led nowhere when in the hands of others. How could this come about? The answer is that people before Chandler were so convinced about the theoretical 306-day-period that they never looked for anything else. Chandler, on the other hand, had no respect for existing theories:

> "I am not much dismayed by the argument of conflict with dynamic laws, since all that such a phrase means must refer merely to the existent state of the theory at any given time."

3.3 Polar motion and dynamical properties of the Earth

Already the year after Chandler's discovery the American astronomer Simon Newcomb (1835 - 1909), in the Monthly Notices of the Royal Astronomical Society, presented an explanation of the surprisingly long period of polar motion. In an earlier paper Newcomb had admitted that the Chandler period was "in such disaccord with the received theory of the earth's rotation that, at first, I was disposed to doubt its possibility". But in his paper of 1892, "On the Dynamics of the Earth's Rotation, with respect to the Periodic Variations of Latitude" (see Figure 13), Newcomb writes:

> "Mr. Chandler's discovery gives rise to the question whether there can be any defect in the theory which assigns 306 days as the time of rotation. The object of this paper is to point out that there is such a defect - namely, the failure to take account of the elasticity of the Earth itself, and of the mobility of the ocean."

The main point here was the elasticity of the Earth. Only ten years earlier Darwin - inspired by Thomson - had made the first numerical estimation of the Earth's elasticity, based on his discovery of the earth tides; see section 1.5. Making use of this Newcomb showed, with a fairly simple way of reasoning, that the effect of the elasticity is to lengthen the period of polar motion by about 100 days or somewhat more. This was in good accordance with the observations.

As we can see from the quotation above Newcomb also paid attention to the mobility of the ocean. He found that the ocean pole tide with an amplitude of the order of one cm will have the effect of lengthening the polar motion period by some 30 days. Putting the two effects together Newcomb arrived at a theoretical period of 443 days, only slightly exceeding Chandler's observed period of 427 days.

On the Dynamics of the Earth's Rotation, with respect to the Periodic Variations of Latitude. By Simon Newcomb.

The recent remarkable discovery of Mr. S. C. Chandler, that the axis of rotation of the Earth revolves around the axis of maximum moment of inertia in a period of about 427 days, is worthy of special attention.* At first sight it seems in complete contradiction to the principles of dynamics, which show that the ratio of the time of such a rotation to that of the Earth's revolution should be equal to the ratio of the polar moment of inertia of the Earth to the difference between the equatorial and the polar moments. Representing these moments by A and C, it is well known that the theory of rotation of a rigid body gives the equation

$$\tau = \frac{A}{C-A},$$

τ being the period of rotation of the pole in sidereal days.

Now the ratio in question is given with an error not exceeding a few hundredths of its total amount by the magnitude of the precession and nutation. The value found by Oppolzer is $\frac{1}{305}$, giving the time of rotation as 305 days.

This result has long been known, and several attempts have been made to determine the distance between the two axes, especially at Poulkova and Washington. A series of observations was made with the Washington Prime Vertical Transit during the years 1862–1867, including six complete periods of the inequality. Thus the determination of the coefficient and zero of the argument is completely independent of all sources of error having an annual or diurnal period. Such errors are

* *Astronomical Journal*, Numbers 248. 249.

Figure 13. The beginning of Newcomb's paper announcing the explanation for Chandler's period of the polar motion (1892).

In 1893 Francois Folie (1833 - 1905), a Belgian astronomer, claimed that an increase of the period should be caused by a partial fluidity of the Earth's interior. A closer examination of this view was undertaken in 1895 by Sydney Samuel Hough (1870 - 1923), an English astronomer working in South Africa for many years, and at the same time also by a Russian mathematician, Fedor Sludsky (1841 - 1897). Hough arrived at the unexpected result that a fluid core will shorten the period of polar motion - not lengthen it. Accepting Newcomb's explanation of the Chandler period he, therefore, concluded that a fluid core could not be very large. Thus Hough found that polar motion theory produced constraints on the dimensions of a fluid interior of the Earth, just as Hopkins earlier had suggested that precession theory did.

At the turn of the century elastic deformations of the Earth were known through two different phenomena: earth tides and polar motion. In section 1.6 the tidal Love numbers h and k were treated. Their numerical use was made possible by an important formula for polar motion. It was derived in 1909 by two British scientists, the physicist Joseph Larmor (1857 - 1942) and the geophysicist Augustus Love (1863 - 1940). Larmor and Love worked separately but simultaneously. Their two papers were read in the Royal Society of London on the same day. The formula they both had found relates the Chandler period T of the elastic Earth to the Euler period T_o of a rigid Earth through the Love number k:

$$1 - \frac{T_o}{T} = k \frac{\omega^2 a/\gamma}{2f - \omega^2 a/\gamma}$$

Their derivations, however, differed: While Love's presupposed a certain internal structure of the Earth, Larmor's was free from such a hypothesis and, hence, more generally valid. The formula yielded k = 0.3, a value which immediately could be used in tidal theory to calculate also h.

Through the polar motion observations performed at the International Latitude Service a secular effect, as anticipated by Helmert, was revealed in 1922 by Walter Lambert (1879 - 1968), an American geodesist. The mean pole drifted slowly towards North America, at a rate of about half a second of arc per century. The possibility of this being a crustal movement in the opposite direction was ruled out by Lambert as being too large. The origin of the polar drift was unknown; it would take another half a century before it turned out that Helmert's postglacial rebound idea was, in fact, relevant.

The liquid core effect studied by Hough was taken up again in 1948 by Harold Jeffreys (1891 - 1989), the English astronomer and geophysicist we already know from his theories of tides and nutation. He now could give a numerical estimate: The liquid core will shorten the period by about 30 days. This is about as much as the ocean will lengthen the period, as found by Newcomb, so that these two effects nearly cancel out.

With Jeffreys' liquid core paper, dealt with also in the context of nutation (section 2.6), and with Takeuchi's elastic tidal model (section 1.6), the time had come for bringing the three related geodynamic phenomena of earth tides, precession-nutation and polar motion together into a common theory.

References

A. Original works (in chronological order)

Ptolemaios K: Mathematike syntaxis ("Almagest"). Alexandria c. 150. (English translation with annotations by G J Toomer, Duckworth & Co. 1984.)

Copernicus N: De revolutionibus orbium coelestium libri VI. Nürnberg 1543. (English translation in Great Books of the Western World, 16, 1952.)

Galilei G: Dialogo sopra i due massimi sistemi del mondo, Tolemaico e Copernicano. Firenze 1632. (English translation, University of California Press 1953.)

Wallis J: Hypothesis about the Flux and Reflux of the Sea. Philosophical Transactions of the Royal Society, 1, London 1666.

Newton I: Philosophiae naturalis principia mathematica. London 1687.

Bernoulli D: Traité du flux et reflux de la mer. 1740. (Referenced in d'Alembert, 1749; see below.)

Bradley J: A Letter to the Right honourable George Earl of Macclesfield concerning an apparent Motion observed in some of the fixed Stars. Philosophical Transactions of the Royal Society, 45, London 1748.

d'Alembert J: Recherches sur la précession des équinoxes, et sur la nutation de l'axe de la Terrè, dans le systeme newtonien. Paris 1749.

Euler L: Recherches sur la précession des équinoxes, et sur la nutation de l'axe de la Terre. Histoire de l'Académie Royale des sciences et belles lettres, Berlin 1749.

Kant I: Untersuchung der Frage: Ob die Erde in ihrer Umdrehung um die Achse, wodurch sie die Abwechselung des Tages und der Nacht hervorbringt, einige Veränderung seit den ersten Zeiten ihres Ursprunges erlitten habe, welches die Ursache davon sey, und woraus man sich ihrer versichern könne? Königsbergische wöchentliche Frag- und Anzeigungs-Nachrichten, 23-24, Königsberg 1754.

Euler L: Du mouvement de rotation des corps solides autour d'un axe variable. Histoire de l'Académie Royale des sciences et belles lettres, Berlin 1758.

Laplace P S: Recherches sur plusieurs points du système du monde. Histoire de l'Académie Royale des sciences, Paris 1775.

Lagrange J L: Méchanique analitique. Paris 1788.

Laplace P S: Traité de mécanique céleste, II. Paris 1799.

Poinsot L: Théorie nouvelle de la rotation des corps. L'Institut, Journal général des sociétés et travaux scientifiques, 2, Paris 1834.

Hopkins W: Researches in Physical Geology. Philosophical Transactions of the Royal Society, 129, 130, 132, London 1839, 1840, 1842.

Adhémar J: Révolutions de la mer - déluges périodiques. Paris 1842. (2nd edition, Paris 1860.)

Poinsot L: Théorie nouvelle de la rotation des corps. Journal de mathématiques pures et appliquées, 16, Paris 1851.

Ferrel W: On the Effect of the Sun and Moon upon the Rotatory Motion of the Earth. Astronomical Journal, 3, Cambridge 1853.

Thomson W: On the Rigidity of the Earth. Philosophical Transactions of the Royal Society, 153, London 1863.

Ferrel W: Note on the Influence of the Tides in Causing an Apparent Secular Acceleration of the Moon's Mean Motion. Proceedings of the American Academy of Arts and Sciences, 6, Washington 1864.

Airy G B: On the Supposed Possible Effect of Friction in the Tides, in influencing the Apparent Acceleration of the Moon's Mean Motion in Longitude. Monthly Notices of the Royal Astronomical Society, 26, London 1866.

Thomson W: Committee for the Purpose of Promoting the Extension, Improvement and Harmonic Analysis of Tidal Observations. Report of the British Association for the Advancement of Science, 1868.

Croll J: Climate and Time in their Geological Relations - A Theory of Secular Changes of the Earth's Climate. London 1875.

Thomson W: Opening Address to Section A of the British Association for the Advancement of Science. Report of the British Association for the Advancement of Science, 1876.

Darwin G H: On the Precession of a Viscous Spheroid, and on the remote History of the Earth. Philosophical Transactions of the Royal Society, 170, London 1879.

Darwin G H: A Numerical Estimate of the Rigidity of the Earth. Nature, 27, 1882. (Extended version in § 848 in Thomson & Tait: Treatise on Natural Philosophy, 2nd edition. Oxford 1883.)

Darwin G H: On Variations in the Vertical due to Elasticity of the Earth's Surface. Philosophical Magazine and Journal of Science, 5/14, London 1882.

Helmert F R: Die mathematischen und physikalischen Theorieen der höheren Geodäsie, II. Leipzig 1884.

Küstner F: Neue Methode zur Bestimmung der Aberrations-Constante nebst Untersuchungen über die Veränderlichkeit der Polhöhe. Beobachtungs-Ergebnisse der Königlichen Sternwarte, 3, Berlin 1888.

Chandler S C: On the Variation of Latitude. Astronomical Journal, 11, 12, Boston 1891, 1892.

Newcomb S: On the Periodic Variation of Latitude, and the Observations with the Washington Prime-Vertical Transit. Astronomical Journal, 11, Boston 1891.

Newcomb S: On the Dynamics of the Earth's Rotation, with respect to the Periodic Variations of Latitude. Monthly Notices of the Royal Astronomical Society, 52, London 1892.

Folie F: Expression complète et signification véritable de la nutation initiale. Acta Mathematica, 16, Stockholm 1893.

Hough S S: The Oscillations of a Rotating Ellipsoidal Shell containing Fluid. Philosophical Transactions of the Royal Society, A 186, London 1895.

Sludsky F: De la rotation de la terre supposée fluide à son intérieur. Bulletin de Société Imperiale des Naturalistes, 9, Moscow 1895.

Herglotz G: Ueber die Elastizität der Erde bei Berücksichtigung ihrer variablen Dichte. Zeitschrift für Mathematik und Physik, 52, 1905.

Hecker O: Beobachtungen an Horizontalpendeln über die Deformation des Erdkörpers unter dem Einflusz von Sonne und Mond. Veröffentlichung des Königl. Preuszischen Geodätischen Institutes, 32, Berlin 1907.

Love A E H: The Yielding of the Earth to Disturbing Forces. Proceedings of the Royal Society, A 82, London 1909.

Larmor J: The Relation of the Earth's Free Precessional Nutation to its Resistance against Tidal Deformation. Proceedings of the Royal Society, A 82, London 1909.

Poincaré H: Sur la précession des corps déformables. Bulletin Astronomique, 27, Paris 1910.

Love A E H: Some Problems of Geodynamics. Cambridge 1911.

Schweydar W: Beobachtung der Änderung der Intensität der Schwerkraft durch den Mond. Königl. Preuszische Akademie der Wissenschaften, Physikalisch-mathematische Classe, Berlin 1914.

Taylor G I: Tidal Friction in the Irish Sea. Philosophical Transactions of the Royal Society, A 220, London 1919.

Jeffreys H: Tidal Friction in Shallow Seas. Philosophical Transactions of the Royal Society, A 221, London 1920.

Doodson A T: The Harmonic Development of the Tide-generating Potential. Proceedings of the Royal Society, A 100, London 1921.

Lambert W D: The Interpretation of Apparent Changes in Mean Latitude, at the International Latitude Stations. Astronomical Journal, 34, Albany 1922.

Jeffreys H: Possible Tidal Effects on Accurate Time-keeping. Monthly Notices of the Royal Astronomical Society, Geophysical Supplement, 2, London 1928.

Pekeris C L: Note on Tides in Wells. Travaux de l'Association Internationale de Géodésie, 16, Paris 1940.

Jeffreys H: The Earth's Core and the Lunar Nutation. Monthly Notices of the Royal Astronomical Society, 108, London 1948.

Jeffreys H: Dynamic Effects of a Liquid Core. Monthly Notices of the Royal Astronomical Society, 109, London 1949.

Takeuchi H: On the Earth Tide of the Compressible Earth of Variable Density and Elasticity. Transactions of the American Geophysical Union, 31, 1950.

B. Works including history of tidal theory, precession-nutation theory, or polar motion theory

Berry A: A Short History of Astronomy from Earliest Times through the Nineteenth Century. London 1898.

Birett H: Zur Vorgeschichte der Newtonschen Theorie der Gezeiten. In Birett et al. (eds.): Zur Geschichte der Geophysik. Springer Verlag, Berlin 1974.

Cartwright D E: The Historical Development of Tidal Science, and the Liverpool Tidal Institute. In Sears & Merriman (eds.): Oceanography the Past. Springer Verlag, Berlin 1980.

Darwin G H: The Tides and Kindred Phenomena in the Solar System. London 1898.

Harris R A: Manual of Tides, I. Report of the United States Coast and Geodetic Survey, Washington 1898.

Lambeck K: The Earth's Variable Rotation: Geophysical Causes and Consequences. Cambridge University Press, Cambridge 1980.

Lambert W D: The Variation of Latitude. Bulletin of the National Research Council, 78, Washington 1931.

Lisitzin E: Sea Level Changes. Elsevier Scientific Publishing Co., Amsterdam 1974.

Marmer H A: The Tide. New York 1926.

Melchior P: The Tides of the Planet Earth. Pergamon Press, Oxford 1978.

Neugebauer O: A History of Ancient Mathematical Astronomy. Springer Verlag, Berlin 1975.

Ronan C: The Cambridge Illustrated History of the World's Science. Cambridge University Press, Cambridge 1983.

Todhunter I: A History of the Mathematical Theories of Attraction and the Figure of the Earth. London 1873.

Vicente R O: The Earth's Constitution and the Nutations. In Fedorov et al. (eds.): Nutation and the Earth's Rotation. D. Reidel Publishing Co., Dordrecht 1980.

Wood R M: The Dark Side of the Earth: The Battle for the Earth Sciences, 1800 - 1980. Allen & Unwin, London 1985.

Acknowledgements: I would like to thank Paul Melchior for suggesting some valuable improvements. I am also grateful for the kind assistance given at the library of the Uppsala Astronomical Observatory.

"I do not know what I may appear to the world; but to myself I seem only to have been like a boy, playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

Isaac Newton (1727)

In Situ Calibration of LaCoste–Romberg Earth Tide Gravity Meter ET19 at BFO Schiltach

H.–G. Wenzel[1)], W. Zürn[1)] and T.F. Baker[2)]

1) Black Forest Observatory (BFO), Heubach 206, D–7620 WOLFACH, F.R. Germany
2) Proudman Oceanographic Laboratory, Bidston Observatory, BIRKENHEAD U.K.

Abstract

The LaCoste–Romberg (LCR) earth tide gravity meter ET19 is continuously operating at Black Forest Observatory (BFO) Schiltach. The electrostatic feedback of the instrument has been calibrated by parallel recording using two LCR model G gravity meters equipped with an SRW electrostatic feedback. The electrostatic feedbacks of the model G gravity meters have been calibrated before and after the parallel recording at Gravimeter–Eichsystem Hannover with an accuracy of about 10^{-3}. Changes of the SRW feedback calibration of one instrument of about $4 \cdot 10^{-3}$ between the pre– and postcalibration campaigns limits the achieved accuracy of the ET19 feedback calibration to about $3 \cdot 10^{-3}$. The calibration factor of the ET19 micrometer screw determined by the parallel recording experiment is 0.46 % lower than the value given by the manufacturer. The tidal parameters for M2 and O1 after correction for oceanic tidal effects are compared to other results in Central Europe and agree within the errors with the WAHR–DEHANT model calculations.

1. Introduction

The LCR earth tide gravity meter ET19, equipped with a WEBER– LARSON electrostatic feedback (WEBER and LARSON 1966, LARSON 1968) is operating continuously at BFO Schiltach for earth tide and free oscillation recording. The calibration of the gravity meter signal has been carried out up to now by comparing the feedback output with the micrometer screw of the instrument on a regular basis. Thus, the feedback calibration is based on the manufacturer calibration of the micrometer screw. BAKER et al. 1981 have reported an apparent change of 0.7 % in the LCR calibration for earth tide gravity meters at Austin/Texas, and about 0.6 % too high manufacturer calibration factor of the micrometer screw of gravity meter ET15. This was the reason to investigate the manufacturer's calibration of our instrument. The gravity meter ET19 could not be removed from the observatory in order to calibrate the micrometer screw on a suitable gravimeter calibration line, because it is operating as a sensitive ultralong–period seismometer waiting for earth quakes large enough to excite the free oscillations of the earth to measurable amplitudes. Thus, a parallel recording with two well calibrated LCR model G gravimeters has been performed, using the earth tide gravity variation as a common input to all three instruments. This method can also be applied to the calibration of superconducting gravity meters (e.g. RICHTER 1987), because those instruments cannot be transported due to technical reasons.

2. Data Recording and Preprocessing

The gravity meter ET19 is installed in the gravimeter vault of the BFO underground observatory behind the air lock with about 150 m rock coverage (Fig. 1). The gravimeter vault is not thermostatized, but the temperature is stable to about ± 0.1° C. The gravimeter is protected against humidity by operating it in a box with continuously dried air.

In order to check the manufacturer calibration of ET19, the LaCoste–Romberg model G gravity meters G156F and G249F, both equipped with an SRW electrostatic feedback (SCHNÜLL et al. 1984, RÖDER et al. 1988) have been operated in parallel to ET 19 between December 8, 1988 and April 11, 1989. Both gravimeters have been installed on the floor in the electronic vault in front of the air lock with about 120 m rock coverage, because humidity is moderate at about 50 % due to the operation of an air dryer. The temperature in the electronic vault is only stable to about ± 1° C due to the operation of a number of electronic devices. There was an electronic failure in the recording channel of gravity meter G156F from February 7 to March 23; the data of G156F for this period will not be considered in the following.

The signals of all three gravity meters and of an air pressure sensor named BMG6, installed in front of the air lock, have been recorded after analog antialias filtering. As antialias filters, 8th order Butterworth lowpass filters with 150 s cutoff period have been used for all four channels. The filtered signals have been sampled at 5 minute intervals using a Hewlett–Packard 3421A 10 channel AD converter and a Hewlett–Packard HP41CX computer with casette tape recorder, placed in the electronic vault. The resolution of the recorded signals was 0.1 nm/s^2 for the gravity meters (1 nm/s^2 = 0.1 μgal = 10^{-10} g) and better than 0.1 Pascal for the air pressure. The 5 minute samples have been condensed to hourly sample values by fitting a 3rd degree least squares polynomial to ± 30 minutes around the full hour, yielding a formal standard deviation of about ± 0.2 nm/s^2 for the hourly gravity values and about ± 3 Pascal for the hourly air pressure values. One basic feature of this smoothing method was the detection of periods of excessive noise in the standard deviations, e.g. due to earth quakes; those periods have subsequently been edited.

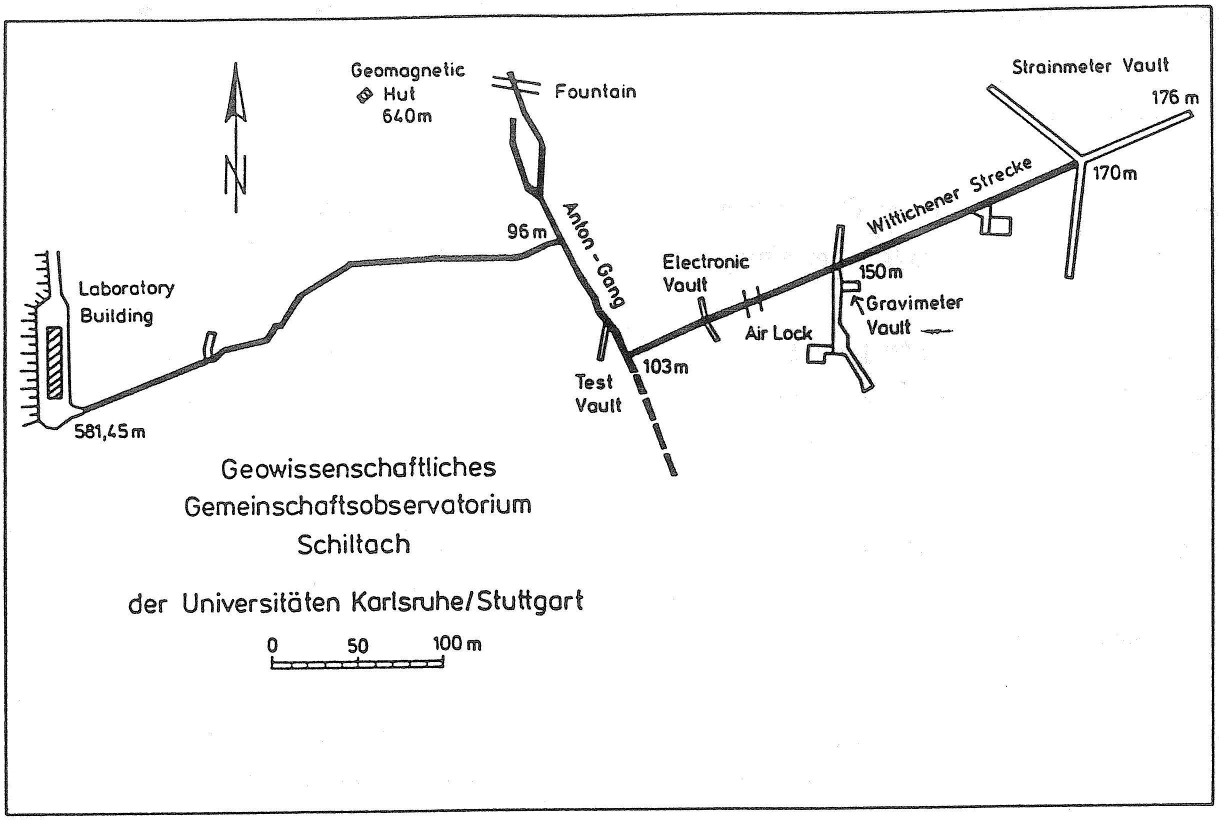


Fig 1. : Sketch of BFO underground observatory

3. Calibration of the SRW Feedbacks

Both model G gravity meters have been calibrated at the vertical gravimeter calibration line Hannover (KANNGIESER et al. 1983) before and after the parallel recording with a precision of better $1 \cdot 10^{-3}$. The results of the Hannover calibration are given in Tab. 1 together with the calibration values obtained in connection with a fifth force experiment (MÜLLER et al. 1990). Between the different calibration campaigns at Hannover, the linear calibration factor of the feedbacks differed by less than 10^{-4} for G156F but by $4 \cdot 10^{-3}$ for G249F. In the evaluation of the above mentioned fifth force experiment, the gravimeter G249F gave the largest deviation of 6 instruments from Newtonian gravitation of about $6 \cdot 10^{-3}$, which could only be explained by unknown calibration uncertainities (MÜLLER et al. 1990). After having finished the parallel calibration experiment, the gravity meter G249F has been examined and a possible source of the feedback calibration instability has been detected in the form of loose connections to the capacitive position indicator board.

The calibration instability of the G249F feedback of $4 \cdot 10^{-3}$ and the calibration accuracy at the Hannover vertical gravimeter calibration line limit the achievable overall accuracy of the ET19 feedback calibration to about $3 \cdot 10^{-3}$. For the calibration of the signals recorded at the BFO parallel experiment, the averages of the pre– and postcalibration results have been used (Tab. 1).

Table 1: Results of SRW Feedback Calibration at Hannover Calibration Line

Gravimeter : LCR–G156F

Date	Linear Calibration [nm/s^2 per Volt]	Quadratic Calibration [nm/s^2 per $Volt^2$]
880728	10502.1 ± 2.5	+15.2 ± 1.4
880831	10502.4 ± 2.8	+12.0 ± 1.6
890426	10501.7 ± 2.6	+07.7 ± 1.5
average used:	10502.0	+09.9

Gravimeter : LCR–G249F

Date	Linear Calibration [nm/s^2 per Volt]	Quadratic Calibration [nm/s^2 per $Volt^2$]
880728	10613.7 ± 3.1	–27.6 ± 2.0
880831	10597.8 ± 2.7	–26.1 ± 1.6
890426	10635.0 ± 2.7	–10.9 ± 1.6
average used :	10616.4	–18.5

4. ET19 Feedback Calibration by Direct Comparison of Parallel Recordings

The calibration factor of the electrostatic feedback of ET19 gravity meter has been computed by comparing the recorded signal of ET19 with the calibrated signal of gravimeters G156F and G249F in a least squares adjustment procedure. Linear calibration factors for the ET19 feedback and 3rd degree drift polynomials have been adjusted using data subsets, each covering a time span of about one week. The linear calibration factors for the ET19 feedback and standard deviations of the fit are given in Tab. 2. The relative precision of the adjusted calibration factors is about $3 \cdot 10^{-4}$ for the individual data subsets, the standard deviation of the fit is about 1 nm/s^2 on the average. The average calibration factors for the ET19 feedback derived from gravimeters G156F and G249F differ by $1.2 \cdot 10^{-3}$; this discrepancy is assumed to be mainly related to the calibration instability of G249F feedback.

During the parallel recording experiment, micrometer screw calibrations of ET19 feedback have been performed on December 8, 1988 and April 4, 1989, resp. Using the manufacturer calibration of the micrometer screw of 1.281 nm/s^2 per counter unit, the calibration of the ET19 feedback results in 2918.8 ± 1.9 and 2919.7 ± 2.3 nm/s^2 per Volt resp., which is 0.46 % too high compared to the average calibration from the parallel recording experiment.

Table 2: Adjusted Linear Calibration Factors of ET19 from Direct Comparison of Parallel Recordings (Weekly Intervals).

Calibration Factors in nm/s^2 per Volt, Standard deviations s in nm/s^2

Subset	Hours	Reference is G156F Cal.fact.	s	Reference is G249F Cal.fact.	s
881207...881216	206	2907.7 ± 1.3	± 4.6	2906.4 ± 0.8	± 2.7
881216...881224	191	2908.7 ± 1.6	± 5.5	2906.8 ± 0.6	± 2.0
881224...881229	113	2909.9 ± 1.8	± 4.0	2905.2 ± 1.0	± 2.2
881229...890102	109	2904.3 ± 1.0	± 1.2	2904.4 ± 1.1	± 1.2
890102...890107	112	2908.3 ± 0.3	± 0.9	2906.2 ± 0.2	± 0.5
890110...890117	164	2907.8 ± 0.4	± 0.9	2905.5 ± 0.3	± 0.6
890117...890126	214	2907.6 ± 0.4	± 1.2	2904.4 ± 0.4	± 1.4
890126...890203	188	2907.8 ± 0.2	± 0.6	2903.8 ± 0.3	± 0.6
890203...890210	167	failure		2904.0 ± 0.3	± 0.8
890210...890217	168	failure		2903.9 ± 0.7	± 1.7
890217...890224	172	failure		2903.0 ± 0.4	± 0.9
890224...890303	162	failure		2902.6 ± 0.6	± 1.3
890303...890310	173	failure		2903.1 ± 0.3	± 0.8
890310...890317	167	failure		2903.4 ± 0.6	± 1.4
890317...890323	143	failure		2902.8 ± 0.6	± 1.0
890323...890331	189	2907.8 ± 0.4	± 1.0	2904.2 ± 0.5	± 1.2
890331...890407	171	2906.8 ± 0.4	± 0.9	2903.2 ± 0.3	± 0.8
890407...890412	090	2907.2 ± 0.2	± 0.6	2903.0 ± 0.2	± 0.5
Average :		2907.6 ± 0.4		2904.2 ± 0.3	

5. ET19 Feedback Calibration by Comparison of Earth Tide Parameters

For the three data sets recorded in course of the parallel experiment, tidal parameters of the main waves have been computed using standard tidal analysis software. The tidal analysis has been carried out applying the program ETERNA, which uses a least squares adjustment algorithm proposed by CHOJNICKI 1973 and modified by WENZEL 1976 to perform the error estimation from the amplitude spectrum of residuals. The TAMURA 1987 tidal potential development has been used including the 4th degree and order tidal potential, because all three data sets significantly display the errors of CARTWRIGHT–TAYLER—EDDEN 1973 tidal potential development (WENZEL and ZÜRN 1990). The program has been implemented and operated on an IBM–AT compatible personal computer. The instrumental phase lag of the different recording channels, which is resulting to about 90 % from the used antialias lowpass filters, have been determined by Fourier transform of the differentiated step response (e.g. WENZEL 1976). From repeated experiments, the accuracy of the instrumental phase lag determination is estimated to about 0.01^{o} and 0.02^{o} at the 1 and 2 cpd tidal bands resp. The gravitational and deformational influence of the air pressure has been partially taken into account by adjusting a linear regression coefficient between the highpass filtered observed gravity signals and the highpass filtered air pressure.

The adjusted tidal parameters using the manufacturer's calibration for ET19 and the Hannover calibration for G156F and G249F are listed in Tab. 4, 5 and 6. The achieved standard deviations of the adjustments of 0.77, 0.91 and 0.79 nm/s^2 for the gravity meters ET19, G156F and G249F resp. demonstrate the high performance of the electrostatic feedback systems and the low noise environment at BFO Schiltach. The linear calibration factors for the ET19 feedback relative to the manufacturer calibration has been derived from the amplitude factors of the waves O1, K1 and M2 in Tab.3, which yields again a 0.46 % too high manufacturer calibration.

Table 3 : Calibration Factors of ET19 Feedback Relative to the Manufacturer Calibration Derived from Tidal Amplitude Factors

Wave	Reference is G156F Cal.fact.	Reference is G249F Cal.fact.
O1	0.9960 ± 0.0004	0.9949 ± 0.0004
K1	0.9956 ± 0.0003	0.9951 ± 0.0003
M2	0.9961 ± 0.0003	0.9947 ± 0.0002
Average :	0.9959	0.9949

Table 4 : Tidal Analysis Results of Gravity Meter ET19 Using the Manufacturer Calibration

BLACK FOREST OBSERVATORY, GEODETIC AND GEOPHYSICAL INSTITUTES,
UNIVERSITIES KARLSRUHE AND STUTTGART, FEDERAL REPUBLIC OF GERMANY.
48.3306N 8.3300E H589M P150M VERTICALCOMPONENT
GRAVIMETER LACOSTE-ROMBERG NO. ET19 ELECTRONIC FEEDBACK
1988.12.08 - 1989.04.11 121 DAYS
INSTALLATION W. ZUERN, H. OTTO, BFO
MAINTENANCE W. ZUERN, H. OTTO, BFO
CALIBRATED WITH LACOSTE SCREW CALIBRATION 2919 NM/S**2 PER VOLT.
INSTRUMENTAL PHASE LAG CORRECTED FOR 0.521 DEG O1 AND 1.083 DEG M2
TAMURA 1987 TIDAL POTENTIAL USED.

NUMBER OF DAYS 121.0

ESTIMATION OF NOISE BY FOURIER-SPECTRUM OF RESIDUALS

0.1 CPD BAND	0.0054 NM/S**2	1.0 CPD BAND	0.0872 NM/S**2
2.0 CPD BAND	0.0572 NM/S**2	3.0 CPD BAND	0.0266 NM/S**2
4.0 CPD BAND	0.0161 NM/S**2		

ADJUSTED TIDAL PARAMETERS

NR.	FROM	TO	WAVE	AMPL. NM/S**2	SIGNAL/ NOISE	AMPL.FAC.	PHASE LAG DEG
1	282	424	Q1	67.866	778.0	1.1485	-0.3579
						0.0015	0.0736
2	425	482	O1	355.631	4076.8	1.1523	0.0293
						0.0003	0.0141
3	483	530	M1	28.093	322.0	1.1574	-0.0329
						0.0036	0.1779
4	531	585	K1	495.031	5674.8	1.1405	0.2717
						0.0002	0.0101
5	586	626	J1	28.192	323.2	1.1616	-0.0558
						0.0036	0.1773
6	627	728	OO1	15.408	176.6	1.1601	-0.0869
						0.0066	0.3244
7	729	830	2N2	11.800	206.2	1.1619	2.4771
						0.0056	0.2778
8	831	880	N2	75.203	1314.3	1.1826	2.4836
						0.0009	0.0436
9	881	936	M2	395.354	6909.7	1.1903	1.9865
						0.0002	0.0083
10	937	975	L2	11.238	196.4	1.1970	-0.1520
						0.0061	0.2917
11	976	1108	S2	184.371	3222.3	1.1931	0.5383
						0.0004	0.0178
12	1109	1190	M3	4.647	174.5	1.0690	-0.0191
						0.0061	0.3283
13	1191	1200	M4	0.013	0.8	0.2505	92.1660
						0.3052	69.8080

STANDARD DEVIATION 0.768 NM/S**2 DEGREE OF FREEDOM 2775

METEOROLOGICAL OR HYDROLOGICAL PARAMETERS

NR.	REGR.COEFF.	STD	SYMBOL
1	-0.03256	0.00019	AIRPRES.PASCAL

Table 5 : Tidal Analysis Results of Gravity Meter G156F Using the Gravimeter–Eichsystem Hannover Calibration

GRAVIMETRIC EARTH TIDE STATION SCHILTACH NR. 716 W.GERMANY
BLACK FOREST OBSERVATORY, GEODETIC AND GEOPHYSICAL INSTITUTES,
UNIVERSITIES KARLSRUHE AND STUTTGART, FEDERAL REPUBLIC OF GERMANY.
48.3306N 8.3300E H589M P150M VERTICALCOMPONENT
GRAVIMETER LACOSTE-ROMBERG NO. G156F ELECTRONIC FEEDBACK SRW
1988.12.08 - 1989.04.11 78 DAYS
INSTALLATION K. LINDNER, KARLSRUHE, H. OTTO, BFO
MAINTENANCE H. OTTO, BFO
CALIBRATED TO GRAVIMETER-EICHSYSTEM HANNOVER
INSTRUMENTAL LAG CORRECTED FOR 0.548 DEG O1 AND 1.140 DEG M2
TAMURA 1987 TIDAL POTENTIAL USED.

NUMBER OF DAYS 77.5

ESTIMATION OF NOISE BY FOURIER-SPECTRUM OF RESIDUALS

0.1 CPD BAND	0.0097 NM/S**2	1.0 CPD BAND	0.1185 NM/S**2
2.0 CPD BAND	0.0794 NM/S**2	3.0 CPD BAND	0.0452 NM/S**2
4.0 CPD BAND	0.0326 NM/S**2		

ADJUSTED TIDAL PARAMETERS

NR.	FROM	TO	WAVE	AMPL. NM/S**2	SIGNAL/ NOISE	AMPL.FAC.	PHASE LAG DEG
1	282	424	Q1	67.637	570.7	1.1446	-0.3064
						0.0020	0.1004
2	425	482	O1	354.192	2988.4	1.1477	0.1039
						0.0004	0.0192
3	483	530	M1	28.041	236.6	1.1553	-0.1058
						0.0049	0.2422
4	531	585	K1	492.851	4158.4	1.1355	0.3562
						0.0003	0.0138
5	586	626	J1	28.133	237.4	1.1591	0.1256
						0.0049	0.2414
6	627	728	OO1	15.338	129.4	1.1549	-0.0947
						0.0089	0.4427
7	729	830	2N2	11.765	148.2	1.1585	2.5554
						0.0078	0.3867
8	831	880	N2	74.993	944.4	1.1793	2.6195
						0.0012	0.0607
9	881	936	M2	393.839	4959.8	1.1857	2.1155
						0.0002	0.0116
10	937	975	L2	11.128	140.1	1.1853	0.0156
						0.0085	0.4088
11	976	1108	S2	183.601	2312.2	1.1881	0.6227
						0.0005	0.0248
12	1109	1190	M3	4.570	101.0	1.0514	0.2691
						0.0104	0.5671
13	1191	1200	M4	0.036	1.1	0.6842	92.2800
						0.6182	51.7676

STANDARD DEVIATION 0.914 NM/S**2 DEGREE OF FREEDOM 1680

METEOROLOGICAL OR HYDROLOGICAL PARAMETERS

NR.	REGR.COEFF.	STD	SYMBOL
1	-0.03135	0.00032	AIRPRES.PASCAL

Table 6 : Tidal Analysis Results of Gravity Meter G249F Using the Gravimeter–Eichsystem Hannover Calibration

GRAVIMETRIC EARTH TIDE STATION SCHILTACH NR. 716 W.GERMANY
BLACK FOREST OBSERVATORY, GEODETIC AND GEOPHYSICAL INSTITUTES,
UNIVERSITIES KARLSRUHE AND STUTTGART, FEDERAL REPUBLIC OF GERMANY.
48.3306N 8.3300E H589M P150M VERTICALCOMPONENT
GRAVIMETER LACOSTE-ROMBERG NO. G249F ELECTRONIC FEEDBACK SRW
1988.12.08 - 1989.04.11 120 DAYS
INSTALLATION K. LINDNER, KARLSRUHE, H. OTTO, BFO
MAINTENANCE H. OTTO, BFO
CALIBRATED TO GRAVIMETER-EICHSYSTEM HANNOVER
INSTRUMENTAL LAG CORRECTED FOR 0.530 DEG O1 AND 1.101 DEG M2
TAMURA 1987 TIDAL POTENTIAL USED.

NUMBER OF DAYS 121.0

ESTIMATION OF NOISE BY FOURIER-SPECTRUM OF RESIDUALS

0.1 CPD BAND	0.0057 NM/S**2	1.0 CPD BAND	0.0981 NM/S**2
2.0 CPD BAND	0.0572 NM/S**2	3.0 CPD BAND	0.0296 NM/S**2
4.0 CPD BAND	0.0196 NM/S**2		

ADJUSTED TIDAL PARAMETERS

NR.	FROM	TO	WAVE	AMPL. NM/S**2	SIGNAL/ NOISE	AMPL.FAC.	PHASE LAG DEG
1	282	424	Q1	67.531	688.1	1.1429	-0.3299
						0.0017	0.0833
2	425	482	O1	353.811	3605.3	1.1464	0.0791
						0.0003	0.0159
3	483	530	M1	28.004	285.4	1.1537	0.0583
						0.0040	0.2008
4	531	585	K1	492.578	5019.3	1.1349	0.3174
						0.0002	0.0114
5	586	626	J1	28.001	285.3	1.1537	-0.0078
						0.0040	0.2008
6	627	728	OO1	15.324	156.2	1.1538	-0.0531
						0.0074	0.3669
7	729	830	2N2	11.740	205.4	1.1560	2.5999
						0.0056	0.2790
8	831	880	N2	74.794	1308.3	1.1761	2.5446
						0.0009	0.0438
9	881	936	M2	393.271	6879.3	1.1840	2.0816
						0.0002	0.0083
10	937	975	L2	11.178	195.5	1.1906	0.1735
						0.0061	0.2930
11	976	1108	S2	183.356	3207.4	1.1865	0.6230
						0.0004	0.0179
12	1109	1190	M3	4.600	155.6	1.0584	-0.0814
						0.0068	0.3683
13	1191	1200	M4	0.030	1.6	0.5777	92.2440
						0.3719	36.8804

STANDARD DEVIATION 0.794 NM/S**2 DEGREE OF FREEDOM 2775

METEOROLOGICAL OR HYDROLOGICAL PARAMETERS

NR.	REGR.COEFF.	STD	SYMBOL
1	-0.03382	0.00020	AIRPRES.PASCAL

6. Comparison of Major Waves with Results in Central Europe and Model Calculations

BAKER et al. 1989 recently reported results of gravity tide measurements for O1 and M2 tides at five stations in Europe, four of which are at reasonably small distance to BFO Schiltach. Their records were obtained with LaCoste–Romberg Earth Tide meters equipped with electrostatic feedbacks essentially identical to the ET19 at BFO Schiltach. Their instruments were also carefully calibrated on the vertical gravimeter calibration line Hannover. The observed gravimetric factors and phases were corrected for ocean attraction and loading using SCHWIDERSKI's 1980 models for the global oceans with improvements concerning the European shelves (BAKER 1980). The results agree within the errors with the predictions of DEHANT 1987 for anelastic earth models based on the theory of WAHR 1981 for an elastic, rotating, elliptical, seismological constrained earth for both waves (Figs. 2 – 5), while the corrected observations in the gravity tide data bank of the International Center for Earth Tides show excessively large scatter in addition to a baseline shift.

Here we add another set of observations of high quality (as demonstrated by the residual noise) for BFO Schiltach for comparison. We average the gravimetric factors for the instruments G156F and G249F (since the calibration of ET19 is now based on gravimeters G156F and G249F) and the phases of all three instruments for the tides O1 and M2 :

O1 :	1.1470 ± 0.0004	$+ 0.071^{\circ} \pm 0.02^{\circ}$
M2 :	1.1848 ± 0.0003	$+ 2.061^{\circ} \pm 0.01^{\circ}$

The uncertainties given above are derived formally from the residual spectrum after tidal analysis. A few other sources of errors have to be taken into account. We estimate the accuracy of the Hannover calibration line to be about 0.1 %, the precision of the calibration process was about 0.2 % (basically because of the change of G249F that we experienced between the two base line calibrations; the precision of an individual calibration is much better). These two errors affect gravimetric factors only and the result are uncertainties of 0.23 % for both O1 and M2. Timing uncertainties were estimated to 0.01° and 0.02° for O1 and M2, respectively. The errors of the instrumental phase lag determination were estimated to be 0.01° for O1 and 0.02° for M2. These have to be added to the statistical errors and we obtain phase uncertainties of 0.03° for both O1 and M2.

The oceanic contributions by attraction and loading were computed for BFO Schiltach using the models employed before by BAKER et al. 1989. The results are 1.4 nm/s^2 with a local phase lead of 167.5^o for O1 and 16.5 nm/s^2 with 59.6^o lead for M2. Correction of the observed values for the oceanic effects leads to the following values, where the standard deviations do not include the errors of oceanic contributions :

O1 :	1.1515 ± 0.0026	$+ 0.022^o \pm 0.03^o$
M2 :	1.1589 ± 0.0026	$- 0.012^o \pm 0.03^o$

In Figures 2 – 5 these results are plotted against latitude together with the results from BAKER et al. 1989 for the stations Bad Homburg, Bruxelles, Chur and Zürich. They are also compared with the predictions by DEHANT 1987 and DEHANT and ZSCHAU 1989. Note that these models disagree at the level of 0.1% for the gravimetric factors.

Our results at BFO Schiltach for O1 and M2 agree well within the error bars with the Wahr–Dehant–Zschau models of an (an–)elastic, seismologically constrained, elliptical and rotating earth with global oceans described by the models of SCHWIDERSKI 1980 after improvements on the shelves in NW–Europe. They also compare well with the measurements of BAKER et al. 1989 at other stations of Central Europe. The above results for BFO Schiltach therefore lend further support to their conclusions.

7. Conclusions

We have shown that the calibration of stationary earth tide gravity meters can be performed with a relative precision of better than 10^{-3} within a time span of a few weeks by parallel recording using LaCoste–Romberg gravity meters with well calibrated electrostatic feedbacks. However, the instability of the feedback calibration of one of our instruments limited the achieved overall calibration accuracy of our stationary gravity meter ET19 to about $3 \cdot 10^{-3}$. After correction for oceanic tidal effects, the gravimetric tidal parameters for the waves O1 and M2 observed at BFO Schiltach agree within 0.2 % and 0.04^o with the WAHR–DEHANT predictions for an (an–)elastic, rotating, elliptical earth.

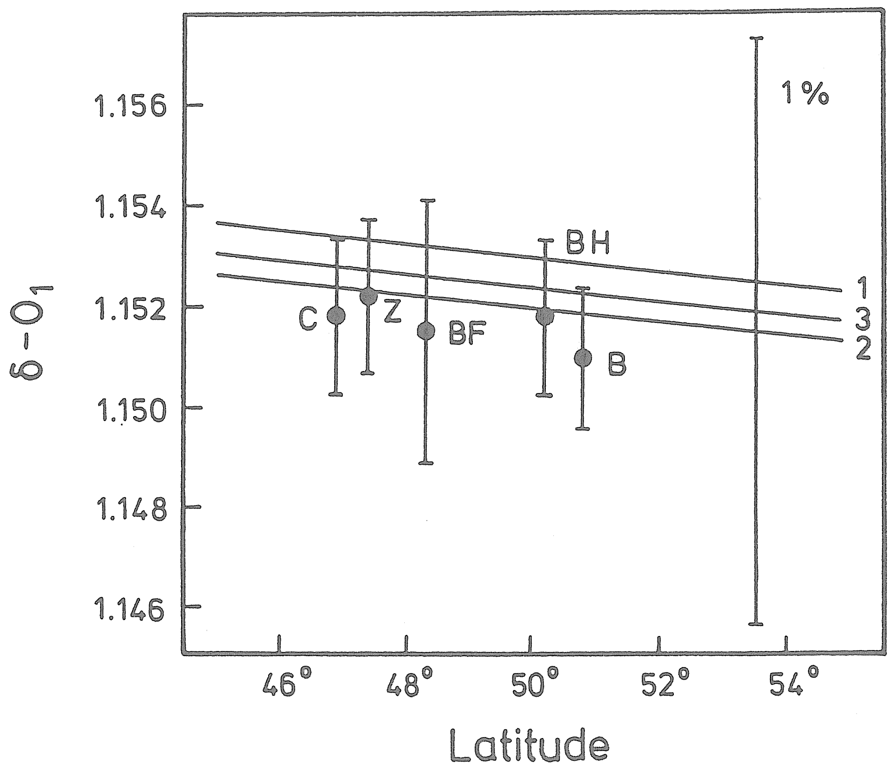


Fig. 2: Gravimetric factors for O1 as a function of latitude for selected European stations (C = Chur, Z = Zürich, BF = Schiltach, BH = Bad Homburg, B = Bruxelles). Solid lines represent earth models : 1 = DEHANT 1987, inelastic; 2 = DEHANT and ZSCHAU 1989, elastic; 3 = DEHANT and ZSCHAU 1989, inelastic.

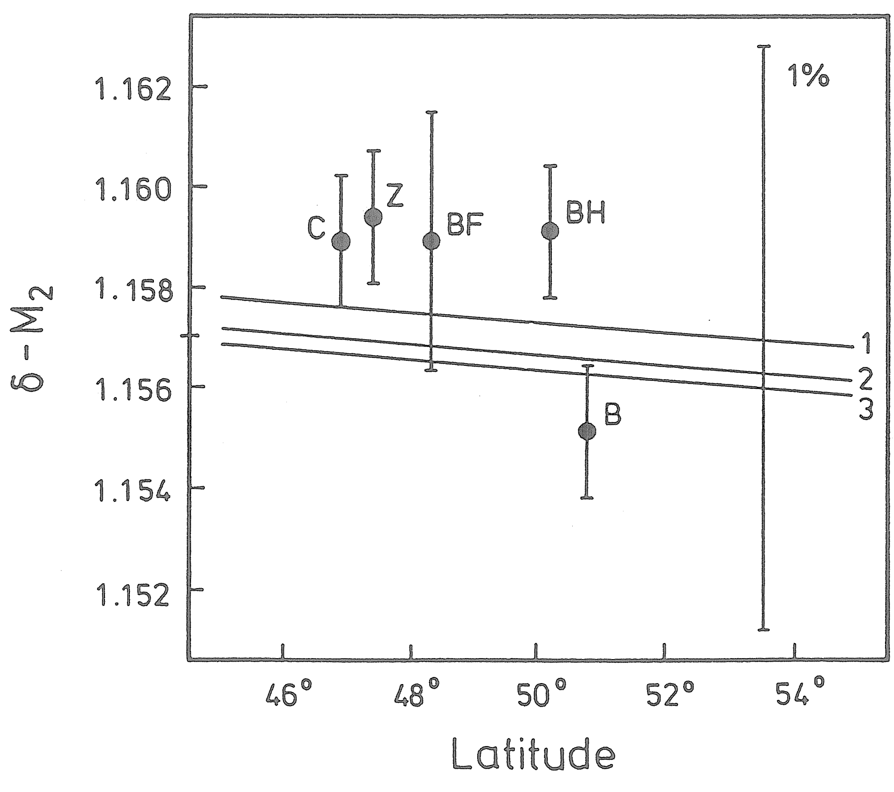


Fig. 3: Same as Fig. 2 for M2.

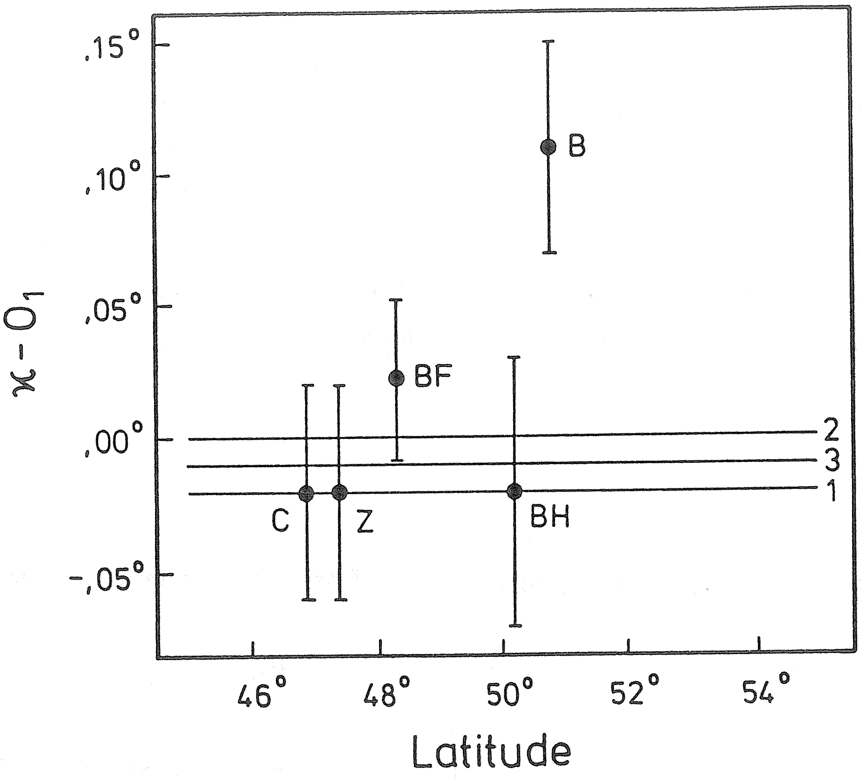


Fig. 4: Phases for O1 as a function of latitude for selected European stations (C = Chur, Z = Zürich, BF = Schiltach, BH = Bad Homburg, B = Bruxelles). Solid lines represent earth models : 1 = DEHANT 1987, inelastic; 2 = DEHANT and ZSCHAU 1989, elastic; 3 = DEHANT and ZSCHAU 1989, inelastic.

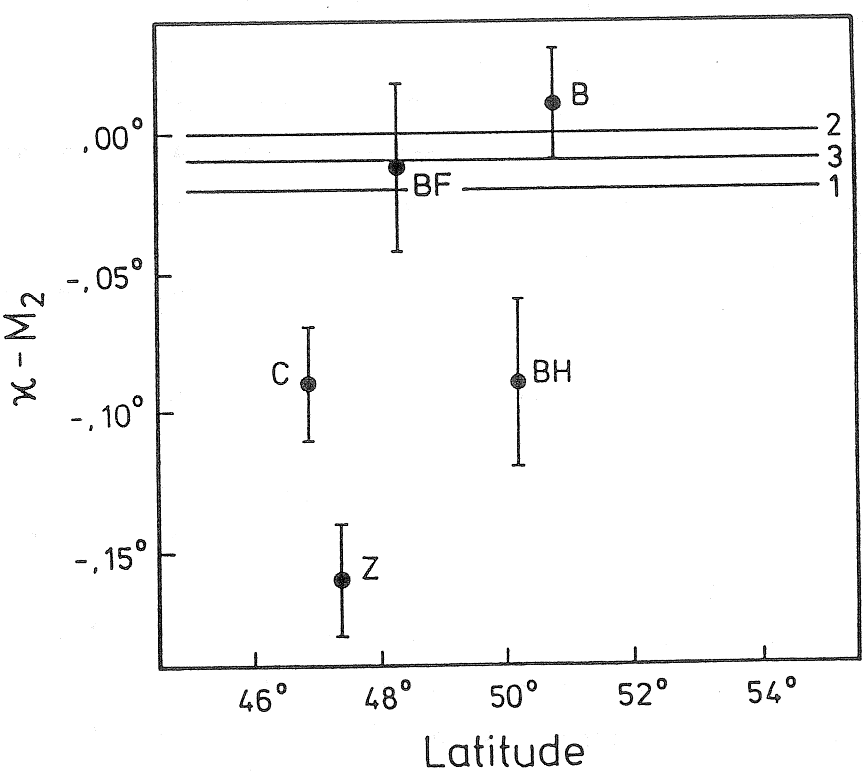


Fig. 5: Same as Fig. 2 for M2.

Acknowledgements

The vertical baseline calibration of the LCR gravity meters, their installation at BFO, and the installation of the Hewlett–Packard data acquisition system by K. Lindner and H. Otto is greatfully acknowledged. The maintenance of all the instruments during the parallel recording experiment has been carried out by H. Otto. The preprocessing of the recorded data has partly been carried out by K. Lindner. Many thanks to M. Schnüll, Institut für Erdmessung, Universität Hannover, for the help with gravity meter LCR–G249F.

References

BAKER, T.F. 1980 : Tidal Gravity in Britain : Tidal Loading and the Spatial Distribution of the Marine Tide. Geophysical Journal of the Royal Astronomical Society, Vol. 62, 249 – 267, 1980.

BAKER, T.F., R.J. EDGE and G. JEFFRIES 1981 : High Precision Tidal Gravity. Interim Scientific Report, Institute of Oceanographic Sciences, Bidston Obseratory, Birkenhead, Merseyside U.K. and US Air Force Geophysics Laboratory, AFGL–TR–81–03, 26 pp, Hanscom AFB, Massachusetts, 1981.

BAKER, T.F., R.J. EDGE and G. JEFFRIES 1989 : European Tidal Gravity : An Improved Agreement between Observations and Models. Geophysical Research Letters, Vol. 16, No. 10, 1109–1112, 1989.

CARTWRIGHT, D.E. and A.C. EDDEN 1973 : Corrected Tables of Tidal Harmonics. The Geophysical Journal, Vol. 33, No. 3, Oxford 1973.

CARTWRIGHT, D.E. and R.J. TAYLER 1971 : New Computations of the Tide Generating Potential. The Geophysical Journal, Vol. 23, No.1, Oxford 1971.

CHOJNICKI, T. 1973: Ein Verfahren zur Erdgezeitenanalyse in Anlehnung an das Prinzip der kleinsten Quadrate. Mitteilungen aus dem Institut für Theoretische Geodäsie der Universität Bonn No. 15, Bonn 1973.

DEHANT, V. 1987 : Tidal Parameters for an Inelastic Earth. Phys. Earth Planet. Inter., 49, 97–116, 1987.

DEHANT, V. and J. ZSCHAU 1989 : The Effect of Mantle Inelasticity on Tidal Gravity : A Comparison between the Spherical and the Elliptical Earth Model. Geophysical Journal, Vol. 97, 549 – 555.

DUCARME, B. 1985 : A Data Bank for Earth Tides. Bulletin d'Informations Marées Terrestres No. 94, 6387, Bruxelles 1985.

KANNGIESER, E., K. KUMMER, W. TORGE and H.–G. WENZEL 1983 : Das Gravimeter–Eichsystem Hannover. Wissenschaftliche Arbeiten der Fachrichtung Vermessungswesen der Universität Hannover No. 120, Hannover 1983.

LARSON, J.V. 1968 : A Cross Correlation Study of the Noise Performance of Electrostatically Controlled LaCoste and Romberg Gravimeters. Ph.D. Thesis, Maryland University, 1968.

MÜLLER, G., W. ZÜRN, K. LINDNER and N. RÖSCH 1990 : Search for Non–Newtonian Gravitation – A Gravimetric Experiment in a Hydroelectric Lake. Geophysical Journal International, Vol.101, No. 2, 329 – 344, 1990.

RICHTER, B. 1987 : Das supraleitende Gravimeter. Deutsche Geodätische Kommission, Reihe C No. 329, Frankfurt a.M. 1987.

RÖDER, R.H., M. SCHNÜLL and H.–G. WENZEL 1988 : SRW Feedback for LaCoste–Romberg Gravimeters with Extended Range. Bureau Gravimetrique International, Bulletin d'Informations No. 62, 46–50, Toulouse 1988.

SCHNÜLL, M., R.H. RÖDER and H.–G. WENZEL 1984 : An Improved Electronic Feedback for LaCoste–Romberg Gravity Meters. Bureau Gravimetrique International, Bulletin d'Informations No. 55, 227–36, Toulouse 1984.

SCHWIDERSKI, E. 1980 : Ocean Tides, Part I : Global Ocean Tidal Equations; Part II : A Hydrodynamical Interpolation Model. Marine Geodesy, Vol. 3, 161 – 255, 1980.

TAMURA, Y. 1987 : A Harmonic Development of the Tide Generating Potential. Bulletin d'Informations Marées Terrestres No. 99, 6813–6855, Bruxelles 1987.

WAHR, J. 1981 : Body Tides on an Elliptical, Rotating, Elastic and Oceanless Earth. Geophys. Journal of the Royal Astronomical Society, Vol. 64, 677–703, Oxford 1981.

WEBER, J. and J.V. LARSON 1966 : Operating of LaCoste and Romberg Gravimeter at Sensitivity Approaching the Thermal Fluctuation Limits. Journal of Geophysical Research Vol 71, 6006–6009, 1966.

WENZEL. H.–G. 1976 : Zur Genauigkeit von gravimetrischen Erdgezeiten–beobachtungen. Wissenschaftliche Arbeiten der Lehrstühle für Geodäsie,: Photogrammetrie und Kartographie an der Technischen Universität Hannover No. 67, Hannover 1976.

WENZEL, H.–G. and W. ZÜRN 1990 : Errors of CARTWRIGHT–TAYLER–EDDEN 1973 Tidal Potential Displayed by Gravimetric Earth Tide Observations at BFO Schiltach. Bulletin d'Informations Marées Terrestres, in press.

Traduction

Sur la variation temporelle des amplitudes et des phases des inclinaisons et déformations de marées.

S.M. Molodenskii
(Institut de Physique Terrestre de l'Académie des Sciences d'URSS., Moscou)

Prévision des tremblements de Terre N° 8 pp. 54-60 - 1988

On sait que l'un des signes précurseurs des tremblements de Terre serait la variation des vitesses des ondes longitudinales et transversales V_p, V_s dans le temps [1, 2]. Une estimation numérique des variations des amplitudes des inclinaisons et des déformations de marées déterminées par la variation des modules élastiques dans le volume du semi espace élastique a été faite en [3]. Les résultats de [3] ont été basés sur l'intégration directe des équations à deux dimensions aux dérivées partielles décrivant les déformations de marées horizontalement dans un milieu non homogène. Il est évident que cette méthode de solution exige une très grande quantité de calculs même pour obtenir une solution qualitative du problème. En outre pour évaluer la dépendance des solutions en fonction des paramètres (coordonnées, dimensions et géométrie de la zone dilatante, son orientation par rapport à l'onde de marée) il faut effectuer des calculs numériques pour toutes les valeurs possibles des paramètres libres.

Etant donné que les variations temporelles de V_p, V_s sont assez faibles, pour une solution qualitative du problème il convient d'appliquer la méthode des perturbations selon les petits paramètres $\delta V_p/V_p$, $\delta V_s/V_s$ [4 à 6]. Nous donnons ici les résultats de certains calculs modèles des inclinaisons et des déformations de marées pour la géométrie la plus simple du domaine dans lequel varient les modules élastiques. De pair avec les variations des amplitudes on examine aussi les déphasages.

Lors de l'examen de l'influence des hétérogénéités locales du milieu sur les inclinaisons et les déformations de marées, on peut négliger les effets de sphéricité de la Terre, la variation de la densité et des modules élastiques avec la profondeur et également l'influence des forces gravitationnelles sur les déformations de marées et se limiter à une approximation d'un demi espace élastique presque homogène. En outre, il résulte de [3] que nous nous limiterons ensuite à l'examen du problème à deux dimensions en supposant que l'étendue de la zone dilatante dans une des directions horizontales dépasse sensiblement, par ses dimensions, son étendue dans les deux autres directions. Dans cette approximation les relations générales [4] entre les hétérogénéités des modules élastiques $\delta\lambda$, $\delta\mu$, pour le déplacement vertical de l'élément de la surface h (x_0) et pour le déplacement horizontal du même élément dans la direction de l'axe x $t_x(x_0)$ ont la forme

$$h(x_0) = \frac{\operatorname{div} u^0}{F_z} \iint_\tau \delta\lambda \operatorname{div} v \, d\tau +$$

$$+ \frac{2}{F_z}\left[\frac{\partial u_x^0}{\partial x_0} \iint_\tau \delta\mu \frac{\partial v_x}{\partial x} d\tau + \frac{\partial u_z^0}{\partial z_0} \iint_\tau \delta\mu \frac{\partial v_z}{\partial z} d\tau\right];$$

$$t_x(x_0) = \frac{\operatorname{div} u^0}{F_x} \iint_\tau \delta\lambda \operatorname{div} w \, d\tau + \qquad (1)$$

$$+ \frac{2}{F_x}\left[\frac{\partial u_x^0}{\partial x_0} \iint_\tau \delta\mu \frac{\partial w_x}{\partial x} d\tau + \frac{\partial u_z^0}{\partial z_0} \iint_\tau \delta\mu \frac{\partial w_z}{\partial z} d\tau\right],$$

où τ est la projection de la région dans laquelle se situent les hétérogénéités δx, $\delta\mu$ sur le plan (x, z), u^0 est la valeur non perturbée du vecteur déplacement de marée dans un milieu homogène, v est la solution du problème à deux dimensions de Boussinesq décrivant le vecteur déplacement dans le demi espace homogène sous l'effet de la force répartie suivant l'axe $z = 0$, $x = x_0$, $-\infty < y < \infty$ avec la densité de l'unité de longueur F_z et agissant dans la direction verticale, w est la solution du même problème pour la force d'intensité F_x et agissant dans la direction de l'axe x. Les expressions sous intégrales (1) ont la forme

$$\frac{1}{F_z} \operatorname{div} v = -\frac{z}{\pi(\lambda+\mu)r^2}; \qquad \frac{1}{F_x} \operatorname{div} w = \frac{x - x_0}{\pi(\lambda+\mu)r^2}; \qquad (2)$$

$$\frac{1}{F_z}\frac{\partial v_x}{\partial x} = \frac{z}{\pi r^2 \mu}\left[\frac{\lambda}{2(\lambda+\mu)} - \frac{(x-x_0)^2}{r^2}\right]; \quad \frac{1}{F_x}\frac{\partial w_x}{\partial x} = \frac{x-x_0}{\pi r^2 \mu}\left[\frac{\lambda+2\mu}{2(\lambda+\mu)} - \frac{z}{r^2}\right];$$

$$\frac{1}{F_z}\frac{\partial v_z}{\partial z} = \frac{z}{\pi r^2 \mu}\left[\frac{\lambda}{2(\lambda+\mu)} - \frac{z^2}{r^2}\right]; \quad \frac{1}{F_x}\frac{\partial w_z}{\partial z} = \frac{x-x_0}{\pi r^2 \mu}\left[\frac{\lambda+2\mu}{2(\lambda+\mu)} - \frac{(x-x_0)^2}{r^2}\right];$$

$$r^2 = (x-x_0)^2 + z^2.$$

La substitution de (2) en (1) détermine complètement h (x_0) et

$t_x(x_o)$ pour le domaine arbitraire τ et les valeurs arbitraires $\delta\lambda$, $\delta\mu$. Pour le domaine τ donné sous la forme d'un rectangle

$$x_1 \leq x \leq x_2$$

$$z_1 \leq z \leq z_2$$

l'intégration de (1) suivant x, z donne

$$\frac{\partial h(x_0)}{\partial x_0} = A \ln r^2 \Big\| + B \frac{(x-x_0)^2}{r^2} \Big\| ;$$

$$\frac{\partial t_x(x_0)}{\partial x_0} = C \operatorname{arctg} \frac{z}{x-x_0} \Big\| + B \frac{(x-x_0)z}{r^2} \Big\| , \tag{3}$$

où

$$A = \frac{1}{2\pi\mu(\lambda+\mu)} \left\{ \operatorname{div} u^0 \left(\mu\delta\lambda - \frac{\lambda^2}{2\mu}\delta\mu - \lambda\delta\mu \right) - \lambda\delta\mu \frac{\partial u_x^0}{\partial x_0} \right\} ;$$

$$B = -\frac{\delta\mu}{\mu\pi} \left\{ \frac{\partial u_x^0}{\partial x_0} + \frac{\lambda}{2\mu} \operatorname{div} u^0 \right\} ; \tag{4}$$

$$C = \frac{1}{\pi(\lambda+\mu)} \left\{ \operatorname{div} u^0 \left(\frac{\lambda\delta\mu}{2\mu} - \delta\lambda \right) - \delta\mu \frac{\partial u_x^0}{\partial x_0} \right\} ;$$

le symbole II désigne le résultat de la double substitution de x_1 à x_2 suivant x et de z_1 à z_2 suivant z : ainsi par exemple,

$$\ln r^2 \| = \ln((x_2-x_0)^2 + z_2^2) - \ln((x_1-x_0)^2 + z_2^2) -$$
$$- \ln((x_2-x_0)^2 + z_1^2) + \ln((x_1-x_0)^2 + z_1^2)$$

etc

Les expressions (3) sont encore plus simples dans les cas limites de la demi-bande infinie où soit $x_2 \to \infty$ soit $z_2 \to \infty$.

Ainsi, pour $z_1 = 0$ et $z_2 \to \infty$

$$\frac{\partial h(x_0)}{\partial x_0} = A \ln \frac{(x_0-x_1)^2}{(x_0-x_2)^2} ;$$

$$\frac{\partial t_x(x_0)}{\partial x_0} = C\Pi(x_0, x_1, x_2) , \tag{5}$$

où

$$\Pi(x_0,x_1,x_2)=\begin{cases}\pi & \text{при } x_1 < x_0 < x_2 \\ 0 & \text{при } x_0 < x_1 \text{ и } x_0 > x_2 .\end{cases}$$

Pour $z_2 \to 0$, $x_2 \to \infty$

$$\frac{\partial h(x_0)}{\partial x_0} = A\, ln \frac{(x_0 - x_1)^2}{(x_0 - x_1)^2 + z_2^2} ; \qquad (6)$$

$$\frac{\partial t_x(x_0)}{\partial x_0} = -C\, arctg \frac{z_2}{x_1 - x_0} - B \frac{(x_1 - x_0) z_2}{(x_1 - x_0)^2 + z_2^2} .$$

Il convient de noter que l'allure asymptotique de ces expressions est différente : pour $|x_0| \to \infty$, $\partial h/\partial x_0 \approx O(x_0^{-2})$ alors que $\partial t_x(x_0)/\partial x_0 \sim O(x_0^{-1})$.

Les valeurs $\partial u_{0x}/\partial x_0$ et div u^0 entrant en (1) sont déterminées par les paramètres de l'onde de marée non perturbée. En fait, elles ont une phase différente et c'est pourquoi la phase des corrections (1) peut ne pas coïncider avec la phase de l'onde de marée non perturbée. Pour obtenir des corrections relatives aux inclinaisons et aux déformations de marées il faut exprimer les valeurs $\partial u_x/\partial_0$, div u^0 par les amplitudes et les phases non perturbées des déformations et des inclinaisons. En prenant pour les nombres de Love et Shida les valeurs numériques $k = 0,3$, $h = 0,6$, $\ell = 0,08$ et en posant

$$V = V_0\, g \frac{r^2}{a} S$$

où V est le potentiel générateur de marées, r est le rayon vecteur, a est le rayon de la Terre, g est l'accélération de la pesanteur à la surface de la Terre,

$$S = P_2^m(\cos\theta)\cos(\sigma t - m\varphi)$$

est la fonction sphérique du second ordre, P_2 est le polynome associé de Legendre, θ est la colatitude, φ est la longitude, σ est la fréquence angulaire et t est le temps, nous obtiendrons :

$$div\, u^0 = 0{,}48\, V_0 S ;$$

$$e^0_{xx} = \frac{\partial u^0_x}{\partial x_0} = cos^2\alpha\, e_{\theta\theta} + sin^2\alpha\, e_{\varphi\varphi} + sin2\alpha\, e_{\theta\varphi} ; \qquad (7)$$

$$\gamma_0 = \frac{\partial u^0_z}{\partial x_0} = -0{,}7 V_0 \left(cos\alpha \frac{\partial S}{\partial\theta} + \frac{sin\alpha}{sin\theta}\frac{\partial S}{\partial\varphi}\right),$$

où α est l'angle entre l'axe x et le méridien

$$e_{\theta\theta} = 0{,}6\, V_0 S + 0{,}08\, V_0 \frac{\partial^2 S}{\partial \theta^2}\ ; \tag{8}$$

$$e_{\varphi\varphi} = 0{,}6\, V_0 S + 0{,}08\, V_0 \left(ctg\theta \frac{\partial S}{\partial \theta} + \frac{1}{sin^2\theta} \frac{\partial^2 S}{\partial \varphi^2} \right);$$

$$e_{\theta\varphi} = \frac{0{,}08}{sin\theta} V_0 \left(\frac{\partial^2 S}{\partial\theta \partial\varphi} - ctg\theta \frac{\partial S}{\partial \varphi} \right)$$

sont les composantes du tenseur des déformations dans le système de coordonnées sphériques. La substitution de ces expressions en (4) détermine les corrections aux amplitudes et aux phases des inclinaisons et des déformations avec des valeurs arbitraires x_o, x_1, x_2, z_1, z_2 et des variations arbitraires $\delta\lambda$, $\delta\mu$. La relation des corrections aux amplitudes et aux phases dépend sensiblement de α. Ainsi par (7), (8) et (4) on constate que pour $\alpha = o$ la phase $\partial h / \partial x_o$ concorde toujours avec la phase de α_o, et la phase $\partial t_x / \partial x_o$ avec la phase de e_{xx}. En posant en (4), (7) $\alpha = o$, $\lambda = \mu$, V_s = const. c'est à dire en prenant

$$\delta\mu = o \qquad \frac{\delta\lambda}{\lambda} = 6\,\frac{\delta\lambda_p}{V_p}$$

nous obtiendrons les corrections suivantes aux amplitudes :

$$\frac{\delta\gamma}{\gamma_0} = -\frac{0{,}65 S}{\partial S/\partial\theta} \frac{\delta V_p}{V_p} \ln r \|; \tag{9}$$

$$\frac{\delta e_{xx}}{e^0_{xx}} = -\frac{0{,}46 S}{0{,}6 S + 0{,}08\, \partial^2 S/\partial\theta^2} \frac{\delta V_p}{V_p} arctg \frac{z}{x - x_0} \Big\| .$$

Dans le cas $\alpha = \pi/2$, e_{xx} ne change également qu'en amplitude :

$$\frac{\delta e_{xx}}{e^0_{xx}} = \frac{\delta e_{\varphi\varphi}}{e^0_{\varphi\varphi}} = -\frac{0{,}46 S\, \delta V_p / V_p}{0{,}6 S + 0{,}08 \left(ctg\theta \frac{\partial S}{\partial\theta} + \frac{1}{sin^2\theta} \frac{\partial^2 S}{\partial\varphi^2} \right)} arctg \frac{z}{x - x_0} \Big\| . \tag{10}$$

Quant à l'inclinaison γ_p elle ne change qu'en phase. Le déphasage, en radians, est de

$$\delta\psi_\gamma = -\frac{0{,}65\, S \sin\theta}{\partial S/\partial\varphi} \frac{\delta V_p}{V_p} \ln r \Big\| . \tag{11}$$

Pour comparer la solution analytique (1) avec les résultats de l'intégration numérique directe de Beaumont et Berger [3] nous donnons sur la figure 1 les graphiques des fonctions $\delta\gamma / \gamma_o$ et $\delta e_{xx}/e_{xx}$ calculées d'après les formules (9) pour α = 0

$\delta V_p/V_p = -0{,}15$, $\delta V_s = 0$, $z_1 = 0{,}5$, $z_2 = 2$, $x_2 - x_1 = 3$ (lignes continues) et les graphiques des mêmes fonctions tirés du travail [3] (pointillé). On a décrit par un rectangle hachuré la région dans laquelle les valeurs V_p changent. On constate sur cette figure que même avec des variations très grandes des modules élastiques (pour $\delta V_p/V_p = -0{,}15 | \delta\lambda | \approx \lambda_o$) la solution analytique (3) concorde tout à fait avec [3].

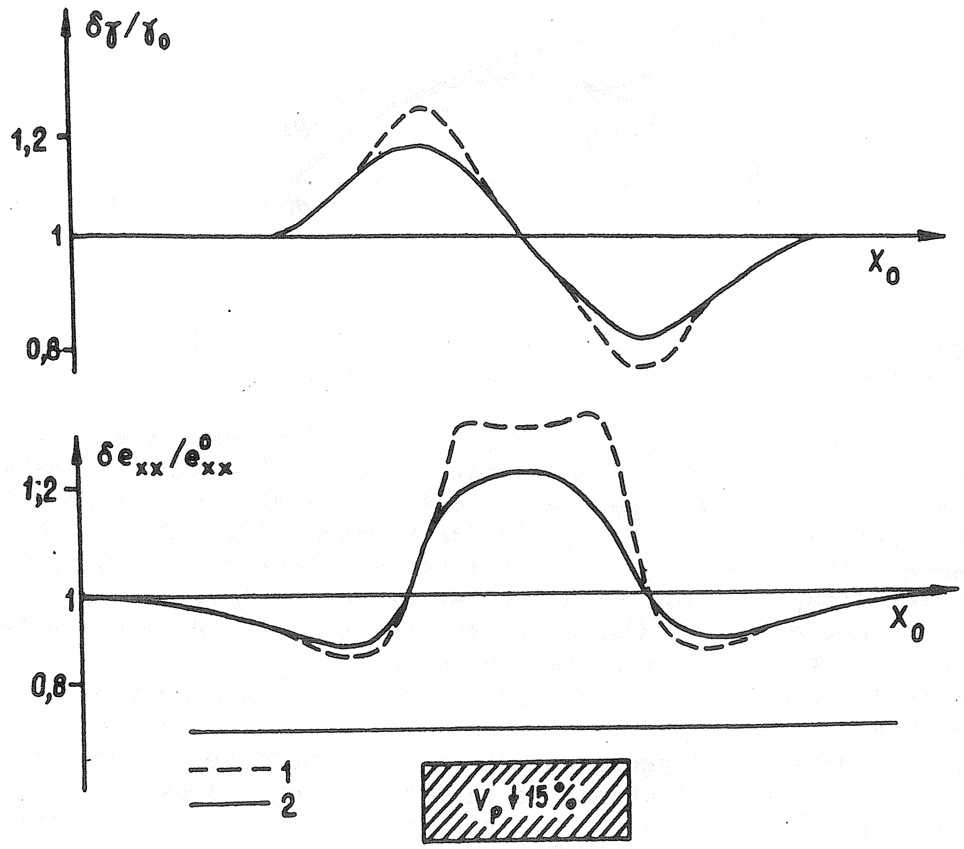


Fig.1 Comparaison des résultats des solutions numérique et analytique du problème de l'influence de la zone dilatante sur les inclinaisons et les déformations de marées. 1.- solution de Beaumont et Berger; 2.- solution analytique.

L'aspect caractéristique des dépendances $\delta\gamma$, δe_{xx}, $\delta\psi_\sigma$, $\delta\psi_e$ en fonction de α est représenté sur la figure 2. Il convient de noter qu'à la suite des déformations provoquées par les charges provenant de la marée océanique, les axes principaux du tenseur des déformations e_{ik} (8) se manifestent dans le fait que tous les graphiques de la figure 2 se déplacent suivant l'axe des abscisses d'une seule et même valeur de α_o.
L'angle α_o peut avoir une valeur importante dans les zones proches de l'océan.

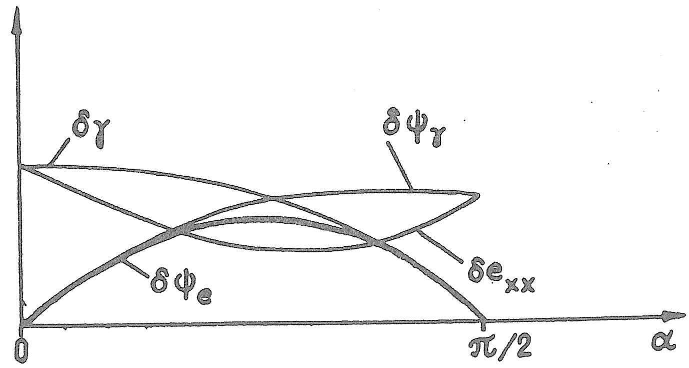


Figure 2 Dépendance des amplitudes et des phases des inclinaisons de marées et des déformations en fonction de α

En conclusion nous comparerons les relations des anomalies des amplitudes $\delta\gamma_{EW}$, $\delta\gamma_{NS}$ et des phases $\delta\psi_{EW}$, $\delta\psi_{NS}$ des inclinaisons de marées déterminées par les formules (3 à 8) avec les résultats des observations à la station de Kondara [7]. On a observé d'après les données [7] à proximité immédiate de la fracture de Kondara d'importantes anomalies des paramètres γ_{NS} et ψ_{EW}; quant aux valeurs de γ_{EW} et ψ_{NS} elles étaient voisines de leurs valeurs normales. En utilisant les formules (3 - 4) et (7 - 8) il est facile de calculer la projection de $\delta\gamma_x$ sur les directions nord-sud et est-ouest. Considérant pour la simplicité que les inclinaisons dans les directions est-ouest et nord-sud à Kondara on a mesuré à la même distance de la fracture en supposant $\theta = 52°$, $\delta\mu = 0$, $\alpha = 20° \div 35°$ (ce qui correspond à l'angle d'inclinaison de la fracture de Kondara dans la direction est-ouest) nous obtiendrons pour les ondes semi diurnes :

$$\delta\psi_{NS} = 0 \; ; \; \delta\gamma_{EW} / \delta\gamma_{NS} = -0.046 \div -0.014$$

$$\delta\gamma_{NS} / \delta\psi_{EW} = \begin{cases} 2.5 & \text{pour } \alpha = 20° \\ 2.0 & \text{pour } \alpha = 25° \\ 1.6 & \text{pour } \alpha = 30° \\ 1.3 & \text{pour } \alpha = 35° \end{cases} \qquad (12)$$

Des résultats voisins ont été obtenus aussi pour $\delta\mu \neq 0$. Les relations (12) montrent que les plus grandes perturbations sont effectivement supportées par les composantes γ_{NS} et ψ_{EW}. En supposant, en conformité avec [7], $\delta\gamma_{NS} = -0,35$ et $\psi_{EW} = -15° = -0,27$ rad., nous obtiendrons $\delta\gamma_{NS} / \psi_{EW} = 1,3$ ce qui concorde aussi avec (12). Ainsi la relation observée entre les amplitudes et les phases à Kondara peut être déterminée par la présence de la fracture.

L'auteur remercie L.A. Latinina, A.E. Ostrovoskii et V.Y. Starkov pour leur commentaires fructueux.

BIBLIOGRAPHIE

1. S a v a r e n s k y E.F. Tectonophysics, v. 6, No 17, 1968.

2. Р и к и т а к и Т. Предсказание землетрясений. М.:Мир,1979, с.370.

3. B e a u m o n t C., B e r g e r J. Earthquake prediction:Modification of tide tilts and strains by dilatancy. Geoph. Journ. Roy. Astr.Soc.v.39,1974, p. 111-121.

4. М о л о д е н с к и й С.М. Влияние горизонтальных неоднородностей мантии на амплитуды приливных волн. - Изв. АН СССР, Физика Земли, № 2, 1977, с.3-11.

5. М о л о д е н с к и й С.М. Влияние локальных неоднородностей коры и верхней мантии на приливные наклоны земной поверхности. -В сб.: Вращение и приливные деформации Земли. Киев: Наукова думка, 1981, № 13, с.10-13.

6. Д о б р о в о л ь с к и й И.П., З у б к о в С.И., М я ч к и н В.И. Об оценке размеров зоны проявления предвестников землетрясений. - В сб.: Моделирование предвестников землетрясений.М.: Наука, 1980, с.7-44.

7. О с т р о в с к и й А.Е., С т а р к о в В.И., С т а р к о в а Э.Я. Аномалии в амплитудах и фазах приливных наклонов. - В сб.: Изучение земных приливов. М.: Наука, 1980, с. 182-187.

Traduction

Influence du relief des régions plates sur les inclinaisons de marées et les déformations de la surface de la Terre

S.M. MOLODENSKII

Institut de Physique Terrestre de l'Académie des Sciences d'U.R.S.S., Moscou.

Prévision des tremblements de Terre, N° 8, pp 61-64 - 1988

Le problème de l'influence du relief sur les inclinaisons et les déformations de marées de la surface de la Terre a été examiné dans les travaux [1, 2]. En [1] le problème a été résolu dans une variante bidimensionnelle par la méthode des éléments finis, par intégration numérique directe des équations aux dérivées partielles; en [2] ce même problème a été résolu par la méthode des représentations conformes de N.Y. Mouskhelichvili. Les résultats de [1] et [2] étaient très concordants; cependant, à cause des difficultés de calcul, on les a limités uniquement à quelques modèles les plus simples du relief.

Dans les régions plates où le relief se caractérise par des angles assez faibles d'inclinaison des éléments de la surface de la Terre par rapport à l'horizon, le problème de l'influence du relief sur les inclinaisons de marées et des déformations peut être résolu par la méthode des perturbations suivant un petit paramètre égal au sinus de l'angle d'inclinaison des éléments de la surface par rapport à l'horizon. Dans cette communication on montre que cette méthode permet d'obtenir des expressions analytiques simples pour un relief arbitraire convenant pour l'utilisation pratique.

En se limitant à l'examen des hétérogénéités locales du relief nous résoudrons le problème de l'approximation du demi espace élastique homogène. Nous introduirons le système des coordonnées cartésiennes (x, y, z) dont les axes (x, y) se trouvent dans le plan limitant le demi espace et l'axe z est dirigé vers l'intérieur du demi espace. L'écart du relief de la surface terrestre s par rapport au plan z = o se caractérise dans ce cas par le vecteur normale extérieure

$$n = (\epsilon_x (x, y), \epsilon_y (x, y), -1)$$

où

$$\epsilon_x = n_x \qquad \epsilon_y = n_y$$

sont les deux petits paramètres fonctions de x, y. Si on se limite à l'examen du problème bidimensionnel alors $\epsilon_y = 0$.

Suivant la méthode du petit paramètre nous représenterons le vecteur des déplacements de marées sous la forme

$$u = u^o + \delta u$$

où u^o est le vecteur déplacement dans le milieu homogène limité par le plan $z = 0$, et δu est la variation de u^o à cause de l'influence du relief. Après avoir désigné par $\sigma^o{}_{ik}$ et $\delta\sigma_{ik}$ les tenseurs des tensions correspondant aux champs des déplacements de u^o et δu nous écrirons les conditions de l'absence de charges extérieures sur la surface s. Tenant compte que $\sigma^o{}_{xz} = \sigma^o{}_{yz} = \sigma^o{}_{zz} = 0$ et en négligeant les termes d'ordre ϵ_x^2, ϵ_y^2, nous obtiendrons :

$$\begin{aligned} &\sigma^0_{xx}\varepsilon_x + \sigma^0_{xy}\varepsilon_y - \delta\sigma_{xz}|_{z=0} = 0 ; \\ &\sigma^0_{xy}\varepsilon_x + \sigma^0_{yy}\varepsilon_y - \delta\sigma_{yz}|_{z=0} = 0 ; \\ &\delta\sigma_{zz}|_{z=0} = 0. \end{aligned} \qquad (1)$$

Les relations (1) sont équivalentes aux conditions aux limites déterminant les déplacements élastiques (δu_x, δu_y, δu_z) du demi espace homogène sous l'action de la charge appliquée sur le plan $z = 0$ avec la densité de surface $(\sigma^o{}_{xx}\epsilon_x + \sigma^o_{xy}\epsilon_y ; \sigma^o_{xy}\epsilon_x + \sigma^o_{yy}\epsilon_y ; 0)$

Dans la variante bidimensionnelle, la relation (1) a la forme

$$\delta\sigma_{xz}|_{z=0} = \sigma^0_{xx}\varepsilon_x ; \quad \delta\sigma_{yz}|_{z=0} = \sigma^0_{xy}\varepsilon_x ; \quad \delta\sigma_{zz}|_{z=0} = 0. \qquad (2)$$

La solution générale des équations de l'équilibre élastique du demi espace homogène pour les conditions aux limites (2) peut être présentée sous la forme :

$$\delta u_i = \int_{-\infty}^{\infty} \varepsilon_x(x_0)\left(\frac{\sigma^0_{xx}}{F_x} w_i^{(1)}(x - x_0, z) + \frac{\sigma^0_{xy}}{F_y} w_i^{(2)}(x - x_0, z)\right) dx_0 , \qquad (3)$$

où $w_i{}^{(1)}$, $w_i{}^{(2)}$ sont les fonctions source déterminées par les équations

$$(\lambda + \mu)\, grad\, div\, w^{(1)} + \mu \Delta w^{(1)} = 0 ; \qquad (4a)$$

$$(\lambda + \mu)\, grad\, div\, w^{(2)} + \mu \Delta w^{(2)} = 0 \qquad (4b)$$

aux conditions aux limites

$$\sigma_{xz}^{(1)}\Big|_{z=0} = F_x\,\delta(x-x_0);\quad \sigma_{yz}^{(1)}\Big|_{z=0} = \sigma_{zz}^{(1)}\Big|_{z=0} = 0; \tag{5a}$$

$$\sigma_{yz}^{(2)}\Big|_{z=0} = F_y\,\delta(x-x_0);\quad \sigma_{xz}^{(2)}\Big|_{z=0} = \sigma_{zz}^{(2)}\Big|_{z=0} = 0, \tag{5b}$$

où $\sigma_{ik}^{(1)}$, et $\sigma_{ik}^{(2)}$ sont les tenseurs des tensions correspondant aux déplacements w_i et w_i respectivement, δ $(x - x_0)$ - δ est la fonction de Dirac, λ, μ sont les paramètres de Lamé. Les équations (4a), (5a) décrivent le vecteur déplacement dans le demi espace homogène sous l'effet de charge à densité constante linéaire F_x appliquée à la droite infinie $x = x_0$, $z = 0$ et agissant le long de l'axe x et les équations (4b), (5b) sont les déplacements lors de la présence d'une charge de densité linéaire F_y appliquée à la même droite $x = x_0$, $z = 0$ mais agissant le long de l'axe y. Nous ne nous arrêterons pas dans cette communication sur la méthode de solution des problèmes aux limites (4), (5). Par substitution directe en (4) (5) il est facile de s'assurer que les vecteurs $w^{(1)}$ et $w^{(2)}$ peuvent être représentés sous la forme analytique suivante :

$$w_x^{(1)} = \frac{F_x}{2\pi\mu}\left[\frac{\lambda+2\mu}{2(\lambda+\mu)}\, ln((x-x_0)^2+z^2) + \frac{z^2}{(x-x_0)^2+z^2}\right];\quad w_y^{(1)} = 0; \tag{6a}$$

(6b)

$$w_z^{(1)} = \frac{F_x}{2\pi\mu}\left[\frac{\mu}{\lambda+\mu}\, arctg\,\frac{x-x_0}{z} + \frac{z(x-x_0)}{(x-x_0)^2+z^2}\right];$$

$$w_x^{(2)} = w_z^{(2)} = 0;\quad w_y^{(2)} = -\frac{F_y}{2\pi\mu}\, ln((x-x_0)^2+z^2) + const.$$

Après avoir substitué (6) en (3) en faisant la différenciation des résultats selon x et en faisant tendre ensuite z vers zéro nous obtiendrons les relations cherchées entre ϵ_x (x) et les corrections aux inclinaisons et aux déformations de marées dans la direction de l'axe x $\partial\mu_z / \partial x$ et $\partial\mu_x / \partial x$. En tenant compte que

$$\lim_{z\to 0} \frac{\partial}{\partial x}\, arctg\,\frac{x-x_0}{z} = \pi\,\delta(x-x_0), \tag{7}$$

nous obtiendrons pour les inclinaisons

$$\frac{\partial\delta u_z}{\partial x}\Big|_{z\to 0} = \frac{\sigma_{xx}^{0}}{2(\lambda+\mu)}\,\varepsilon_x(x). \tag{8}$$

De la relation (8) il résulte que la correction pour le relief aux inclinaisons de marées au point x dépend uniquement de la valeur ϵ_x en ce même point x c'est à dire en fonction de l'angle d'inclinaison de l'élément infiniment petit de la surface de la Terre par rapport à l'horizon au point d'observation. En fait, puisque les observations clinométriques se font toujours dans des galeries de mine de profondeur z_0 de l'ordre de dizaines de mètres, l'expression sous intégrée (7) est différente de zéro sur l'intervalle fini $(x - z_0, x + z_0)$. C'est pourquoi dans $\epsilon_x(x)$ en (8) il convient d'entendre le sinus de l'angle d'inclinaison de la surface de la Terre par rapport à l'horizon, moyenné sur un intervalle de l'ordre de la profondeur de la galerie.

Pour les déformations nous obtiendrons ensuite :

$$\frac{\partial \delta u_y}{\partial y} = 0; \quad \frac{\partial \delta u_x}{\partial x} = \frac{\sigma^0_{xx}}{2\pi\mu}\,\frac{\lambda + 2\mu}{\lambda + \mu}\,\lim_{z\to 0}\int_{-\infty}^{\infty} \frac{\epsilon_x(x_0)(x - x_0)}{(x - x_0)^2 + z^2}\,dx_0. \qquad (9)$$

Cette formule montre que les corrections aux déformations, à l'inverse des corrections aux inclinaisons dépendent des valeurs $\epsilon_x(x_0)$ sur tout l'intervalle infini $-\infty < x_0 < \infty$.

Les valeurs relatives des corrections aussi bien aux inclinaisons qu'aux déformations de marées sont de l'ordre de ϵ_x et c'est pourquoi il faut même en tenir compte dans des domaines plats dont l'inclinaison de la surface sur l'horizon est de l'ordre du degré.

En suivant [1, 2] nous nous approcherons de la projection de la surface de la Terre sur le plan (x, z), brisé, avec des valeurs constantes par sections $\epsilon_x(x_0)$ c'est à dire que nous poserons

$$\epsilon_x(x_0) = \epsilon_i \quad \text{pour} \quad x_i < x_0 < x_{i+1},$$

où ϵ_i (i = 1, ...N) sont certaines valeurs constantes. En intégrant (9) dans ce cas nous obtiendrons

$$\frac{\partial \delta u_x}{\partial x} = \frac{\sigma^0_{xx}}{4\pi\mu}\,\frac{\lambda + 2\mu}{\lambda + \mu}\sum_{i=1}^{N} \epsilon_i \ln \frac{(x_i - x)^2 + z^2}{(x_{i+1} - x)^2 + z^2}. \qquad (10)$$

Cette expression montre que les corrections aux déformations de marées sont maximales dans les régions où la fonction $\epsilon_x(x)$ subit une fracture.

En conclusion il faut remarquer que les valeurs σ^0_{xx} entrant en (8 à 10) représentent la valeur totale des tensions de marées incluant aussi bien les valeurs de ces grandeurs dues à la présence des forces de marées de volume aussi bien que les valeurs liées à la charge de surface provenant de la marée océanique.
Dans les régions voisines de l'océan, les déformations provenant des charges de surface peuvent avoir le même ordre de grandeur

que les déformations provenant des forces de volume et c'est pourquoi on doit en tenir compte dans les formules (8 à 10).

BIBLIOGRAPHIE

1. H a r r i s o n Y.C. Cavity and topographic effects in tilt and strain measurements.Journ. Geophys. Res., v.81, 1976, p.319-328.

2. L a t y n i n a L.A., K a r m a l e e v a R.M. T i k h o m i r o v A.V., K h a s i l e v L.E. Secular and tidal deformations recorded in the fault zone of the Northeastern Tien Shan. J.Terrestrial and space techniques in Earthquake prediction research. Proceedings of the international workshop on Monitoring Crustal Dynamics in Earthquake Zones Strasbourg, Aug.29 - Sept.5, 1978, p.407-420.

Traduction

Détermination des variations des amplitudes des ondes de marées

T. CHOJNICKI

Prévision des tremblements de Terre, n° 8, pp. 126-135 - 1988

Des études des variations d'amplitude des ondes de marées terrestres sont entreprises déjà depuis longtemps. Les chercheurs obtiennent souvent différents résultats des valeurs des facteurs d'amplitudes de ces ondes au cours de différentes périodes de temps dans une même station à l'aide du même instrument. Il existe deux types principaux de variations des amplitudes des ondes de marées :

1) effectives c'est à dire provoquées par des variations physiques de l'écorce terrestre ou de l'intérieur de la Terre;

2) fictives c'est à dire provoquées par des erreurs d'étalonnage, par des erreurs de processus de calcul dans la méthode d'analyse des observations ou par des variations d'effets purement locaux en relation avec le socle sur lequel est placé l'instrument.

Dans la théorie des marées terrestres l'existence de variations du premier type est problématique. En revanche l'existence de variations du second type est un fait incontestable. Nous examinons ici l'approche du problème uniquement du point de vue calcul et nous essayons de déterminer les variations possibles des amplitudes sans explication des causes ce qui est évidemment impossible lors de l'application des méthodes de calcul seule.

La découverte de l'existence des variations des amplitudes des ondes de marées a une grande importance pour l'étude de la validité de l'étalonnage, de l'existence de bruits locaux etc... mais elle peut également avoir une grande importance pour la recherche de précurseurs des tremblements de Terre. Nous proposons ici un procédé de détermination de ces variations que l'on étudie à présent au Laboratoire de géodésie physique du Centre de Recherhes Cosmiques à Varsovie.

Si nous analysons une série quelconque d'observations de marées terrestres par la méthode des moindres carrés nous pouvons déterminer la valeur des erreurs d'observations en fonction de temps. Dans le résidu il y aura également des informations sur l'allure des variations temporelles des amplitudes des ondes.

Pour simplifier le problème nous poserons que les amplitudes de toutes les ondes varient pareillement c'est à dire que l'étude de ces variations sera incluse dans la détermination de la variation du facteur moyen d'amplitude. Les variations de ce facteur apparaissent dans le résidu par l'apparition d'une courbe semblable à la marée mais avec une amplitude très faible et inhabituelle dans les différentes périodes, dépendant de la différence du facteur dans ces périodes par rapport à sa valeur moyenne. Entre les périodes séparées la phase de cette courbe variera par rapport à la courbe de marée même de 180° en raison du fait que le facteur d'amplitude instantané est plus grand ou plus petit que le facteur moyen dans une période donnée de temps.

Prenant cela en considération on constate qu'il sera difficile de découvrir cette courbe par un procédé analogique ou numérique. De même l'analyse de Fourier du résidu ne donne pas d'information concrète sur la courbe des variations puisqu'elle sera donnée en forme de somme de sinusoïdes qui masque la dépendance de cette courbe en fonction de la courbe de marée.

Il existe cependant un procédé simple de représentation imagée de la courbe des variations. En désignant par P_t la valeur de la marée observée à l'instant t et par K_t la relation de l'amplitude instantanée à l'amplitude moyenne, on peut écrire :

$$K_t = \frac{P_t + V_t}{P_t} \qquad (1)$$

et alors la fonction temporelle

$$F(t) = K_t \qquad (2)$$

représentera de façon concrète les variations des amplitudes des ondes de marées.

Du point de vue pratique la valeur obtenue par la formule (1) serait insuffisamment précise. Il vaut mieux la calculer comme valeur moyenne des valeurs obtenues dans la période de t_{-n} à t_n de sorte que cette valeur représentera l'époque t_0. On peut également introduire un poids égal $/P_t/$ et outre cela éliminer des valeurs par lesquelles nous calculons la moyenne K, valeurs où le poids $/P_t/$ est plus petit qu'une certaine limite prise précédemment $/P_t/$ min.

Finalement nous pouvons écrire

$$K_t = \frac{1}{m} \sum_{i = t_{-n}}^{t_n} \frac{P_i + V_i}{P_i} ; \qquad (3)$$

ou

$$K_t = \frac{1}{\sum |P_i|} \cdot \sum_{i=t_{-n}}^{t_n} \frac{P_i \cdot V_i}{P_i} \cdot |P_i| \qquad (4)$$

$$|P_i| \geq |P_i|\, min$$

où : M est le nombre des époques dans la période allant de t_{-n} à t_n pour lesquelles

$$|P_i| \geq |P_i|\, min\ ;$$

Dans nos applications initiales de la méthode décrite on a pris les déterminations suivantes et leurs variantes :

- les époques de représentation de t_o des valeurs calculées K_t ont été initialisées au début des jours c'est à dire que les accroissements de la fonction (2) sont égaux aux jours;
- les périodes d'après lesquelles on a calculé les valeurs moyennes de K_t c'est à dire les périodes de t_{-n} à t_n ont été examinées avec des variantes de 1 à 9 jours;
- les limitations $|P_i|$ min sont égales à 20 µgal ou 2 millisec en fonction du type de composante de la marée analysée.

Dans le cadre de notre collaboration on a fait l'analyse d'une série de 3 mois d'observations de la composante horizontale EW dans la station de Polouchniko. Cette série a déjà été étudiée précédemment par les méthodes généralement adoptées d'analyse harmonique, après quoi on l'a soumise à l'analyse des variations des amplitudes d'après le schéma décrit plus haut. La figure 1 présente la courbe de la variation des amplitudes pour la période t = 1 jour.

Cette méthode nous donne des résultats identiques à ceux obtenus dans le cas où on fait des analyses séparées pour différentes périodes sur lesquelles nous espérons partager la série étudiée des observations. Mais notre procédé est incomparablement moins cher du point de vue de l'efficacité de l'utilisation de l'ordinateur puisqu'elle se réalise sans dépense dans la marche de chaque analyse harmonique.

D'autre part le problème présenté ici est un procédé d'utilisation de la courbe obtenue pour l'amélioration des résultats de l'analyse. Ce problème exige une étude plus précise. Comme illustration nous donnerons les résultats obtenus à l'aide du premier procédé de l'utilisation de la courbe calculée c'est à dire la multiplication des valeurs observées par les valeurs correspondantes de la fonction (2) et du nivellement réitéré de ces observations.

La figure 2 représente la fonction (2) dont on a fait l'approximation par quatre variantes : à l'aide de 5, 13, 28 et 75 points. La variation de la fonction dans l'intervalle entre les points est considérée comme linéaire. La table 1 donne les résultats de l'analyse des observations non corrigées. Dans la table 2 nous donnons les résultats de l'analyse des observations améliorées sur la courbe pointillée 5. Dans la table 3 nous trouvons les résultats tels que le facteur d'amplitude de l'onde M_2 soit partout le même et égal à la valeur obtenue par l'analyse des observations non corrigées c'est à dire 0,72607.

Des tables 4 et 5 il découle que la divergence des valeurs des paramètres de marées des ondes séparées obtenues par l'analyse des observations non corrigées est beaucoup plus grande que ce qui résulte de la théorie. Cela montre que ces observations sont perturbées pour une certaine raison. Dans les tables 4 et 5 on a calculé les moyennes des paramètres des grandes ondes et les erreurs de ces moyennes. Les valeurs de ces erreurs montrent que dans les observations améliorées la divergence des paramètres diminue nettement et les valeurs des mêmes paramètres moyens s'approchent de la valeur théorique $\gamma = 0,72$.

Cela établit la conséquence positive de la correction des variations d'amplitude provoquées dans ce cas probablement par des erreurs d'étalonnage.

Il faut souligner encore une fois que le problème de l'utilisation de la courbe des variations des amplitudes est ouvert et est un problème très délicat. L'un des facteurs les plus importants est le problème lié aux époques auxquelles la fonction (2) prend la valeur K = 1. Cela signifie que à ces époques les observations sont sans erreurs. Ainsi par exemple si la cause des variations de la courbe sont les erreurs d'étalonnage il faut admettre que l'étalonnage dans ces périodes est correct. Il peut cependant ne pas en être ainsi. Les périodes de coupure de la courbe des variations avec la droite dont les ordonnées sont égales à l'unité découlent des règles de l'effet de la méthode des moindres carrés et apparaitront toujours plus ou moins au milieu de la courbe des variations entre leurs valeurs minimales et maximales. Il s'ensuit que, pour utiliser cette courbe avec plus de précision pour la correction des observations, il faut avoir encore un point ou des points pour lesquels nous sommes absolument sûrs de la valeur de nos observations et déplacer ainsi la courbe des variations par rapport au système de coordonnées afin que la ligne des ordonnées égales à l'unité coupe la courbe des variations aux époques de ces observations sûres.

Le problème du procédé de l'utilisation de la courbe des variations est lié à la nature de ces variations. Si on considère cette variation comme réelle, existant physiquement dans la nature des phénomènes de marées, alors on peut considérer la courbe des variations comme une information objective obtenue dans le processus de l'analyse et la terminant. D'autre part, si on considère les variations comme fictives, découlant de l'imperfection des instruments, du processus ou de l'analyse des observations alors il faut examiner des méthodes sûres d'utilisation de l'information obtenue sur les variations pour améliorer les mauvaises observations.

TABLE 1

WYROWNANIE KONCOWE - FINAL ADJUSTMENT CHOJNICKI METHOD

KONCOWE WYNIKI OBLICZEN - OCENA DOKLADNOSCI NA PODSTAWIE RESIDUUM
FINAL RESULTS OF COMPUTATIONS - ESTIMATION OF ACCURACY BASED ON RESIDUAL

0.0 1978 415 0 0 00101110 55.5800 -36.5500 200 -90.0000 981.515729

STATION POLUSHKINO EAST-WEST COMPONENT USSR

55 35 N 36 33 E H 200 M
INSTITUTE OF GEOPHYSICS IN MOSCOW - K.M.ANOCHINA IRK=0/RMAX=0
VERTICAL PENDULUM ASKANIA IN THE HOLE NO 1

LEAST SQUARE ANALYSIS IN CLASSICAL MANNER (CHOJNICKI)
FILTRATION OF OBSERVATIONS / FILTER 51/ 833
POTENTIAL CARTWRIGHT-EDDEN-(DOODSON) / COMPLETE EXPANSION
COMPUTATION - ZAKLAD GEODEZJI PLANETARNEJ CBK PAN - WARSZAWA - COMPUTER IBM/370

78 3 8 1-78 5 2022

TOTAL NUMBER OF DAYS 73 1774 READINGS

WAVE GROUP ARGUMENT	N	SYMBOL	ESTIM.AMPL. VALUE	ESTIM.AMPL. R.M.S.	AMPLITUDE FACTOR VALUE	AMPLITUDE FACTOR R.M.S.	PHASE DIFFERENCE VALUE	PHASE DIFFERENCE R.M.S.	RESIDUALS AMPL.	RESIDUALS PHASE
105.-139.	65	Q1	0.21	0.19	0.31541	0.27650	88.865	50.188	0.51	155.1
143.-149.	26	O1	2.92	0.19	0.67200	0.04431	-8.227	3.776	0.43	-102.7
152.-158.	22	M1	0.33	0.18	1.02382	0.57283	20.307	32.039	0.14	52.4
161.-168.	33	PSK1	4.14	0.18	0.75443	0.03247	-3.178	2.464	0.43	-32.2
172.-177.	22	J1	0.28	0.18	1.03847	0.67475	8.361	37.262	0.10	23.9
181.-1E3.	37	OO1	0.12	0.17	1.13804	1.60073	-155.485	80.624	0.19	-164.7
207.-23X.	41	2N2	0.07	0.07	0.61664	0.67744	-60.532	62.873	0.07	-125.5
243.-248.	24	N2	1.16	0.12	0.82814	0.08402	-14.178	5.816	0.33	-60.1
252.-258.	26	M2	6.75	0.12	0.72607	0.01310	-11.404	1.032	1.35	-80.0
262.-267.	17	L2	0.32	0.12	0.94645	0.34973	13.029	21.153	0.11	42.2
271.-2X5.	47	S2K2	3.60	0.12	0.77132	0.02628	-6.966	1.952	0.57	-49.7
327.-375.	17	M3	0.08	0.02	0.91951	0.26362	-46.270	16.427	0.06	-104.3

R.M.S.ERROR M-ZERO 1.5850 MILISEC

R.M.S.ERROR FOR BANDS D 5.5765 SD 3.5789 TD 0.9063

O1/K1 0.8907 1-O1/1-K1 1.3357 M2/O1 1.0805

REFERENCE EPOCH 1978 4 15 0.0 JULIAN DATE 2443613.50

TABLE 2

WYROWNANIE KONCOWE - FINAL ADJUSTMENT CHOJNICKI METHOD

KONCOWE WYNIKI OBLICZEN - OCENA DOKLADNOSCI NA PODSTAWIE RESIDUUM
FINAL RESULTS OF COMPUTATIONS - ESTIMATION OF ACCORACY BASED ON RESIDUAL

0.0 1978 415 0 0 00101110 55.5800 -36.5500 200 -90.0000 981.515729

STATION POLUSHKINO EAST-WEST COMPONENT USSR

55 35 N 36 33 E H 200 M
INSTITUTE OF GEOPHYSICS IN MOSCOW - K.M.ANOCHINA PD5 . IRK= 5/RMAX=0
VERTICAL PENDULUM ASKANIA IN THE HOLE NO 1

LEAST SQUARE ANALYSIS IN CLASSICAL MANNER (CHOJNICKI)
FILTRATION OF OBSERVATIONS / FILTER 51/ 833
POTENTIAL CARTWRIGHT-EDDEN-(DOODSON) / COMPLETE EXPANSION
COMPUTATION - ZAKLAD GEODEZJI PLANETARNEJ CBK PAN - WARSZAWA - COMPUTER IBM/370

78 3 8 1-78 5 2022

TOTAL NUMBER OF DAYS 73 1774 READINGS

WAVE GROUP ARGUMENT	N	SYMBOL	ESTIM.AMPL. VALUE	ESTIM.AMPL. R.M.S.	AMPLITUDE FACTOR VALUE	AMPLITUDE FACTOR R.M.S.	PHASE DIFFERENCE VALUE	PHASE DIFFERENCE R.M.S.	RESIDUALS AMPL.	RESIDUALS PHASE
105.-139.	65	Q1	0.25	0.17	0.37256	0.25488	103.729	39.171	0.58	155.0
143.-149.	26	O1	2.82	0.18	0.64913	0.04083	-7.772	3.601	0.43	-116.4
152.-158.	22	M1	0.29	0.17	0.89455	0.52782	24.352	33.791	0.13	70.8
161.-168.	33	PSK1	4.00	0.16	0.72929	0.02991	-3.626	2.349	0.34	-48.4
172.-177.	22	J1	0.22	0.17	0.82499	0.62175	5.651	43.221	0.04	31.1
181.-1E3.	37	OO1	0.11	0.16	1.05871	1.47519	-145.300	79.853	0.18	-158.8
207.-23X.	41	2N2	0.03	0.04	0.25366	0.33708	-33.730	76.202	0.05	-163.5
243.-248.	24	N2	1.08	0.06	0.77134	0.04185	-10.520	3.110	0.22	-62.9
252.-258.	26	M2	6.60	0.06	0.70981	0.00652	-11.212	0.526	1.29	-85.9
262.-267.	17	L2	0.36	0.06	1.07697	0.17418	18.105	9.260	0.16	44.8
271.-2X5.	47	S2K2	3.52	0.06	0.75330	0.01309	-7.701	0.996	0.55	-59.2
327.-375.	17	M3	0.08	0.02	0.93900	0.25328	-40.018	15.455	0.05	-98.1

R.M.S.ERROR M-ZERO 1.4115 MILISEC

R.M.S.ERROR FOR BANDS D 5.1381 SD 1.7826 TD 0.8708

O1/K1 0.8901 1-O1/1-K1 1.2961 M2/O1 1.0935

REFERENCE EPOCH 1978 4 15 0.0 JULIAN DATE 2443613.50

TABLE 3

WYROWNANIE KONCOWE - FINAL ADJUSTMENT CHOJNICKI METHOD

KONCOWE WYNIKI OBLICZEN - OCENA DOKLADNOSCI NA PODSTAWIE RESIDUUM
FINAL RESULTS OF COMPUTATIONS - ESTIMATION OF ACCURACY BASED ON RESIDUAL

0.0 1978 415 0 0 00101110 55.5800 -36.5500 200 -90.0000 981.515729

STATION POLUSHKINO EAST-WEST COMPONENT USSR

55 35 N 36 33 E H 200 M
INSTITUTE OF GEOPHYSICS IN MOSCOW - K.M.ANOCHINA PD5 IRK= 5/RMAX=0
VERTICAL PENDULUM ASKANIA IN THE HOLE NO 1

LEAST SQUARE ANALYSIS IN CLASSICAL MANNER (CHOJNICKI)
FILTRATION OF OBSERVATIONS / FILTER 51/ 833
POTENTIAL CARTWRIGHT-EDDEN-(DOODSON) / COMPLETE EXPANSION
COMPUTATION - ZAKLAD GEODEZJI PLANETARNEJ CBK PAN - WARSZAWA - COMPUTER IBM/370

78 3 8 1-78 5 2022

TOTAL NUMBER OF DAYS 73 1774 READINGS

WAVE GROUP			ESTIM.AMPL.		AMPLITUDE FACTOR		PHASE DIFFERENCE		RESIDUALS	
ARGUMENT	N	SYMBOL	VALUE	R.M.S.	VALUE	R.M.S.	VALUE	R.M.S.	AMPL.	PHASE
105.-139.	65	Q1	0.26	0.17	0.38110	0.25488	103.729	39.171	0.58	154.5
143.-149.	26	O1	2.88	0.18	0.66401	0.04083	-7.772	3.601	0.41	-107.8
152.-158.	22	M1	0.30	0.17	0.91505	0.52782	24.352	33.791	0.13	68.7
161.-168.	33	PSK1	4.10	0.16	0.74600	0.02991	-3.626	2.349	0.41	-39.3
172.-177.	22	J1	0.23	0.17	0.84389	0.62175	5.651	43.221	0.05	28.5
181.-1E3.	37	OO1	0.12	0.16	1.08296	1.47519	-145.300	79.853	0.18	-158.6
207.-23X.	41	2N2	0.03	0.04	0.25947	0.33708	-33.730	76.202	0.05	-163.0
243.-248.	24	N2	1.11	0.06	0.78901	0.04185	-10.520	3.110	0.24	-58.2
252.-258.	26	M2	6.75	0.06	0.72607	0.00652	-11.212	0.526	1.33	-79.6
262.-267.	17	L2	0.37	0.06	1.10164	0.17418	18.105	9.260	0.17	43.5
271.-2X5.	47	S2K2	3.60	0.06	0.77056	0.01309	-7.701	0.996	0.60	-53.2
327.-375.	17	M3	0.08	0.02	0.96051	0.25328	-40.018	15.455	0.05	-96.4

AMPLITUDE FACTORS MULTIPLICATED BY 1.022911

R.M.S.ERROR M-ZERO 1.4115 MILISEC

R.M.S.ERROR FOR BANDS D 5.1381 SD 1.7826 TD 0.8708

O1/K1 0.8901 1-O1/1-K1 1.3228 M2/O1 1.0935

REFERENCE EPOCH 1978 4 15 0.0 JULIAN DATE 2443613.50

TABLE 4

	P0	P5	P13	P28	P75	P75
1	0,315	0,373	0,499	0,546	0,576	0,643
01	0,672	0,649	0,638	0,628	0,627	0,597
1	0,754	0,729	0,718	0,716	0,716	0,687
2	0,828	0,771	0,786	0,798	0,780	0,751
2	0,726	0,710	0,706	0,706	0,708	0,686
2K2	0,771	0,753	0,761	0,760	0,760	0,745
cp.	0,678	0,664	0,685	0,692	0,695	0,685
	076	061	043	037	032	024
	1,59	1,41	1,37	1,33	1,34	1,35
	5,58	5,14	4,89	4,70	4,64	4,45
	3,58	1,78	1,61	1,59	1,53	1,53

TABLE 5

	P0	P5	P13	P28	P75	P75
1	0,315	0,381	0,513	0,562	0,591	0,680
01	0,672	0,664	0,656	0,646	0,643	0,631
P K1	0,754	0,746	0,738	0,736	0,735	0,726
2	0,828	0,789	0,808	0,821	0,801	0,794
M2	0,726	0,726	0,726	0,726	0,726	0,726
2K2	0,771	0,771	0,782	0,782	0,780	0,788
cp.	0,678	0,680	0,704	0,712	0,713	0,724
	076	062	044	038	033	026

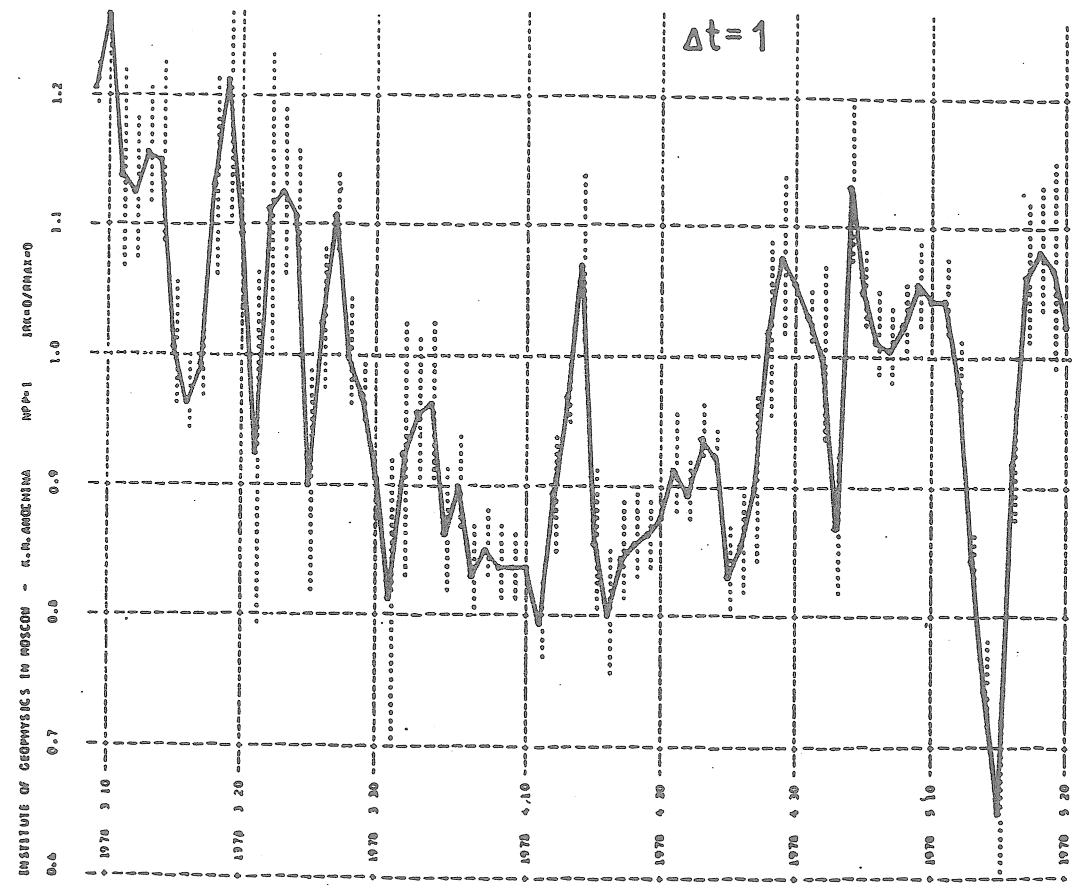


FIG.1

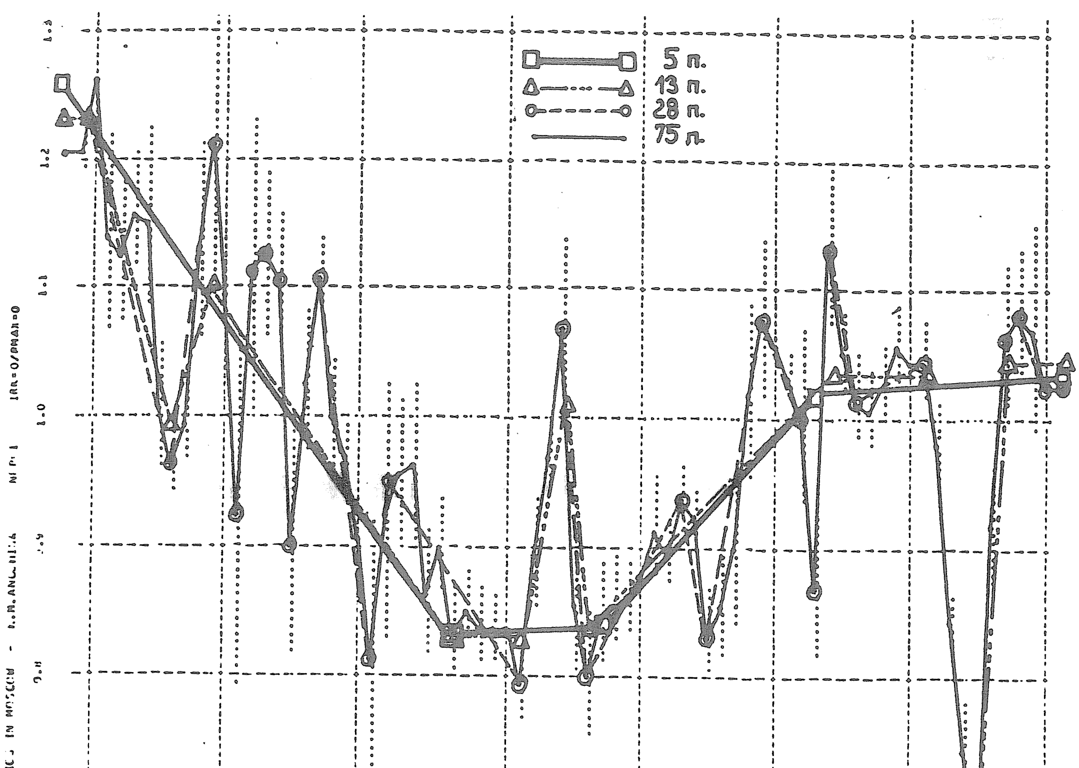


FIG.2

Traduction

Résultats préliminaires des observations de marées à Irkoutsk

N.N. Pariiski, D.G. Gridniev,
You.K. Saritcheva, V. You. Timofeyev

(Institut de Physique de la Terre,
Institut de Géologie et Géophysique
de l'Académie des Sciences d'U.R.S.S.)

Prévision des tremblements de Terre N°8 pp 187-190 - 1988.

Des observations des variations de marées de la pesanteur sont organisées à Irkoutsk depuis l'automne 1979 conformément à un programme du profil de marée transsibérien. Ces observations se font à l'aide de quatre gravimètres dont deux (GS-12 N°180 et SKG [1]) appartiennent à l'Institut de Physique de la Terre de l'Académie des Sciences d'URSS, un (GS-12 N°186) à l'Institut de géologie et de géophysique de l'Académie des Sciences d'URSS et un (GS-11 N°159) à l'Observatoire Gravimétrique de Poltava de l'Institut de Géophysique de l'Académie des Sciences d'Ukraine.

Les observations sont faites dans le bâtiment principal de la Filiale Sibérienne de l'Institut de l'Union d'étude scientifique des mesures physico-techniques et radiotechniques situé sur le sommet d'une colline de cote 520m, à 400m du bord du réservoir d'eau d'Irkoutsk. La colline est d'âge jurassique. Le gisement des couches est horizontal. Sur le bord du réservoir d'eau on observe des phénomènes de glissement de terrain.

Les gravimètres sont placés dans une salle thermostatisée dans le soubassement du bâtiment, sur un socle en béton qui constitue aussi le sol de la salle. La température de thermostatisation de la salle a été maintenue au niveau de 24°C avec une précision de 0,1°.

En même temps que les observations gravimétriques on fait des observations des variations de la densité de l'air [2, 3], des inclinaisons du socle avec des clinomètres à quartz HK-1 [4, 6], des variations de la température dans la salle avec un thermographe à résistance [7]. L'enregistrement des variations de la densité de l'air, des inclinaisons du socle et de la température visent à étudier leur influence sur les indications des gravimètres. L'estimation de l'apport des inclinaisons du socle dans les résultats des observations a montré que dans certains cas cet apport peut atteindre 2 à 3% de la valeur du facteur gravimétrique [8 - 10]

L'enregistrement de tous les appareils se fait sur deux enregistreurs à 4 canaux. Les marques horaires sont assurées par le service de l'heure.

L'alimentation des thermostats des gravimètres et des lampes des photoéléments est assurée par des stabilisateurs de tension, fabriqués à l'Institut de Physique de la Terre de l'Académie des Sciences d'URSS. Le contrôle de la tension sur les lampes des photoéléments se fait à l'aide d'un volmètre M-106 (classe de précision 0,5).

La détermination de l'échelle d'enregistrement des gravimètres se fait par des déplacements couplés. En outre, l'échelle d'enregistrement se détermine également par des séries de trois déplacements avec une sensibilité diminuée du galvanomètre [11,12].

La sensibilité des gravimètres est de 2,5 à 3,0 μgal/mm, pour les clinomètres, 30 mm/s d'arc, pour la densité de l'air - 10^{-4} kg/m^3 mm, du thermographe de 100 mm/degré.

Les observations faites de septembre 1979 à février 1988 avec les gravimètres 180 et 186 sont soumises à l'analyse harmonique par la méthode de Vénédikov. Les valeurs du facteur gravimétrique δ sont données dans la table qui donne également les valeurs de δ obtenues à l'aide des mêmes gravimètres à Novossibirsk pour la période d'observations de 1977 à 1979 [19].

Comme le montre la table, les valeurs de δ des ondes M_2, S_2, K_1, O_1 à Irkoutsk correspondent dans les limites des erreurs avec les valeurs obtenues à Novossibirsk avec ces mêmes gravimètres.

La concordance interne du gravimètre 180 est plus élevée que celle du gravimètre 186. Cependant aussi bien à Novossibirsk qu'à Irkoutsk les valeurs provenant du gravimètre 186 étaient inférieures en moyenne de 2% à celles du gravimètre 180.

Les valeurs moyennes de δ (M_2, S_2, K_1, O_1) obtenues à Novossibirsk par les observations des gravimètres SKG et 159 et sont égales à 1,1627 ± 0,0043 et 1,1746 ± 0,0040 [13]. Le gravimètre 186 donne donc des valeurs de δ inférieures en comparaison avec les trois gravimètres fonctionnant simultanément avec lui à Novossibirsk. Il parait évident que le constructeur a fait une erreur lors de l'étalonnage du gravimètre 186 sur son polygone et que les données de départ pour la détermination de la valeur de division du gravimètre par le procédé des billes ne sont pas sûres. Cependant la vérification par étalonnage sur le polygone de Alma-Alta et par la méthode d'inclinaison à Irkoutsk a montré, dans les limites des erreurs, l'exactitude de la détermination de la valeur de division par le constructeur. La raison des valeurs inférieures de δ obtenues par le gravimètre 186 reste peu claire. A présent, à Irkoutsk, la détermination de l'échelle d'enregistrement du gravimètre 186 se fait par inclinaisons [8]. Ce procédé n'exige pas l'application d'un équipement de calcul et peut aider à déceler la cause des valeurs inférieures de δ.

La table montre que les erreurs de détermination des valeurs de δ à Irkoutsk sont sensiblement plus grandes qu'à Novossibirsk. Les résultats donnés ici pour les observations à Irkoutsk doivent être considérés comme préliminaires. Pour augmenter la précision de la détermination des valeurs de δ à Irkoutsk il faut analyser l'influence des inclinaisons du socle, des variations de la densité de l'air et de la température sur les indications des appareils et effectuer également des observations à la station de marée de l'Institut de l'Ecorce Terrestre S.O de l'Académie des Sciences d'URSS située assez loin du réservoir d'eau d'Irkoutsk. L'étude de l'action des inclinaisons, de la variation de la densité de l'air et de la température sur les indications des appareils exige une longue et minutieuse réduction des données.

Les observations ont montré que les inclinaisons diurnes du socle atteignent 0,5 à 1,0 seconde d'arc, les variations saisonnières quelques dizaines de seconde d'arc. On observe une bonne corrélation entre les inclinaisons et les variations de la densité de l'air, et également entre les inclinaisons et la dynamique du réservoir d'eau. Ainsi lors d'une forte baisse d'eau (le niveau du réservoir d'eau baissant de 80 cm en 3 heures) les clinomètres enregistraient une variation de l'inclinaison de 12 secondes d'arc vers le réservoir. Cela peut s'expliquer par la diminution du niveau des eaux souterraines. Dans une série de cas, avant les tremblements de Terre se produisant dans la zone Baïkale, on observe des inclinaisons anormales dont la valeur atteint 5 à 8 secondes d'arc.

Valeurs de δ

Station	N° des gravimètres	Nombre de jours	δ M_2	S_2	K_1	O_1
Irkoutsk	180	406	1,1634	1,1615	1,1480	1,1504
			±0,0103	±0,0215	±0,0121	±0,0201
	186	618	1,1511	1,1062	1,1257	1,1304
			±0,0195	±0,0429	±0,0160	±0,0263
	180	406		$(M_2, S_2, K_1, O_1)_\rho = 1/\varepsilon^2$	= 1,1561 ±0,0035	
	186	618		$(M_2, S_2, K_1, O_1)_\rho = 1/\varepsilon^2$	= 1,1317 ±0,0073	
Novossibirsk	180	646	1,1605	1,1589	1,1513	1,1662
			±0,0011	±0,0029	±0,0021	±0,0042
	186	552	1,1368	1,1231	1,1195	1,1462
			±0,0015	±0,0056	±0,0069	±0,0011
	180	646		$(M_2, S_2, K_1, O_1)_\rho = 1/\varepsilon^2$	= 1,1586 ±0,0023	
	186	552		$(M_2, S_2, K_1, O_1)_\rho = 1/\varepsilon^2$	= 1,1387 ±0,0044	

BIBLIOGRAPHIE

1. Гриднев Д.Г., Осинская С.В., Певзнер Б.Н. Стационарный кварцевый гравиметр. – В сб.: Приливные деформации Земли. – Тр. ИФЗ АН СССР. М.: Наука, 1975, с.113–123.

2. Гриднев Д.Г., Прохоровский Г.С. Макет фотоэлектрического кварцевого барографа. – В сб.: Приливные деформации Земли. – Тр. ИФЗ АН СССР. М.: Наука, 1975, с.167–172.

3. А.С. № 569885 (СССР). Микробарограф. / Д.Г.Гриднев. Опубл. в Б.И. 1977, № 31.

4. Буланже Ю.Д., Гриднев Д.Г., Давыдов В.И. и др. Кварцевый наклономер НК–1. – В сб.: Приливные деформации Земли.– Тр. ИФЗ АН СССР. М.: Наука, 1975, с.149–158.

5. А.С. № 433341 (СССР). Способ эталонирования наклономеров./ Д.Г.Гриднев, Ю.Д.Буланже. Опубл. в Б.И. 1974, № 23.

6. А.С. № 828154 (СССР). Наклономер./ Ю.Д.Буланже, Д.Г.Гриднев, В.И.Давыдов и др. Опубл. в Б.И. 1981, № 17.

7. Гриднев Д.Г. Исследования стационарного кварцевого гравиметра с фотоэлектрической записью показаний. – В сб.: Изучение земных приливов. – Тр. ИФЗ АН СССР. М.: Наука, 1964, с.98–110.

8. Гриднев Д.Г. Эталонирование гравиметров фирмы "Аскания" и определение масштаба записи приливных изменений силы тяжести наклоном. Температурные наклоны постаментов. – В сб.: Вращение и приливные деформации Земли, вып.6. Киев: Наукова думка, 1974, с.42–54.

9. Гриднев Д.Г. Температурные наклоны земной поверхности и вызываемые ими искажения в показаниях приливных гравиметров. – В сб.: Приливные деформации Земли. – Тр. ИФЗ АН СССР. М.: Наука, 1975, с.142–149.

10. Гриднев Д.Г., Сарычева Ю.К., Тимофеев В.Ю. Наклоны земной поверхности и изменения плотности воздуха перед некоторыми землетрясениями (тезисы доклада). – В сб.: Современные движения земной коры. Кишинев, 1982, с.44–45.

11. Парийский Н.Н., Перцев Б.П., Гриднев Д.Г. и др. Приливные изменения силы тяжести в Алма-Ате. – В сб.: Гравиметрические исследования, № 1, серия Результаты МГГ. М., Изд-во АН СССР, 1960, с.27–33.

12. Гриднев Д.Г., Сарычева Ю.К., Тимофеев В.Ю. Об определении масштаба записи приливорегистрирующего гравиметра GS – 12 № 186. – В жур.: Геология и геофизика, № 3. Новосибирск: Наука, 1982, с.96–101.

13. Баленко В.Г., Барсенков С.Н., Булацен В.Г. и др. Приливные изменения силы тяжести в Новосибирске. – В сб.: Медленные деформации Земли. – Тр. ИФЗ АН СССР. М.: Наука, 1984.

cf. B.I.M. n°95, p 6458, Table 9 1985

Traduction

Comparaison des résultats des mesures communes des variations de marées de pesanteur effectuées d'après le programme du K A P G.

Z. Simon, V.A. Volkov, G. You Dittfeld, P. Varga, A. Venedikov.

(Institut de Physique de la Terre de l'Académie des Sciences d'U.R.S.S. Moscou).

Prévision des tremblements de Terre N° 8 pp 25-53 - 1988

Introduction

Nous avons publié dans le travail [1] les résultats des observations des marées de la pesanteur réalisées d'après le plan de coopération internationale dans le cadre du K A P G à Poulkovo, Obninsk (URSS), Potsdam (Allemagne), Pecny (Tchécoslovaquie) Tihany (Hongrie) et Sofia (Bulgarie). On a réduit 20 séries de mesures avec un nombre total de 100872 ordonnées. On a utilisé les gravimètres GS 11 N° 201, GS 15 N° 220, 221, 222, 224, 228 et BN07. Dans chaque station les mesures ont été faites au minimum par deux gravimètres.

La réduction des mesures a été faite par la méthode M74 de Venedikov [2, 3] en quelques variantes. Ensuite on a réalisé une analyse commune [4] de toutes les séries dans chaque station. Enfin on a analysé les 20 séries ensemble. Cette analyse a été faite en deux fois. Une première fois sans les corrections géophysiques; la seconde fois on a introduit des corrections aux effets indirects et inertiels pour les ondes O_1, K_1, M_2 et S_2. En outre on a effectué d'autres variantes de l'analyse globale.

On peut faire une interprétation des résultats selon différents buts. Dans ce travail nous comparons les valeurs obtenues pour les paramètres de marées δ (facteur d'amplitude) et κ (retard de phase) pour les ondes principales O_1, K_1, M_2 et S_2 présentées en [1], et on a analysé leur précision en tenant compte de la possibilité d'erreurs systématiques des séries séparées de mesures. Cette approche a été faite pour que l'analyse à l'aide du critère de Fisher, qui se fait à chaque analyse globale, témoigne dans la plupart des cas de l'absence de paramètres communs des ondes de marées dans toutes les séries.

On propose les paramètres communs les plus approchants pour le calcul des corrections des mesures gravimétriques sur le territoire de l'Europe de l'Est.

Estimation des résultats

La table 1 donne les résultats des analyses des séries séparées de mesures. Si, dans une station, on a réalisé avec un gravimètre quelques séries de mesures, on indique le numéro de la série après le numéro du gravimètre. Pour chaque station on a fait également les valeurs moyennes des paramètres et le résultat de l'analyse globale. Si pour une station on a fait plusieurs variantes de l'analyse avec un groupement différent des ondes, on donne dans la table celle pour laquelle le groupement est le moins détaillé. Lors de l'analyse globale on n'a utilisé que le le poids à l'intérieur des séries [1, 4]. Toutes les séries interviennent dans l'analyse globale avec le même poids dont la valeur ne dépendait que de leur longueur. C'est pourquoi les valeurs moyennes des paramètres coïncident bien avec le résultat de l'analyse globale. Nous donnons à la fin de la table les valeurs moyennes des résultats des analyses globales des stations séparées et le résultat de l'analyse globale des 20 séries (sans les corrections géophysiques). Les erreurs quadratiques moyennes $m\delta$ et $m\varkappa$ données dans la table 1 sont obtenues au cours du processus d'analyse et ne caractérisent que la précision interne des séries.

Les erreurs quadratiques moyennes $m\delta$ et $m\varkappa$ données également dans la table 1 sont obtenues dans le processus d'analyse et caractérisent uniquement la précision interne des séries.

Les figures 1 à 6 présentent les graphiques des valeurs des paramètres de marées de la table 1. En outre, pour la station de Pecny, sont inclus les résultats des mesures simultanées avec le gravimètre GS 11 n° 131 pour autant qu'ils ne soient pas pris dans l'analyse globale. Les graphiques montrent des différences constantes importantes entre les valeurs de δ des différentes séries dans les stations de Obninsk et Tihany. Cela indique des erreurs systématiques dans la calibration des enregistrements. Les valeurs de sont plus affectées d'erreurs accidentelles mais on observe ici aussi des différences systématiques par exemple dans les stations de Poulkovo, Potsdam, Pecny. La cause peut provenir d'erreurs dans les corrections du retard instrumental mais la cause la plus probable peut être l'application systématiquement différente de l'échelle d'ordonnées sur le papier enregistreur.

Nous essayerons d'évaluer la valeur de l'erreur systématique d'une série et de l'erreur quadratique moyenne réelle accidentelle. Dans ce but nous utiliserons les valeurs des facteurs d'amplitude δ et le retard en temps $\tau = \varkappa / \omega$ (sec) pour les ondes O_1, K_1 et M_2 qui ont été obtenues par des séries séparées. Les retards en temps τ sont utilisés au lieu des retards de phase parce que la même erreur systématique dans le temps doit apparaître dans les phases pour différentes valeurs en fonction de la vitesse angulaire de l'onde ω (°/sec). Les valeurs des paramètres δ et τ obtenues par une seule série sont considérées comme indépendantes du point de vue des erreurs accidentelles pourvu qu'elles soient obtenues par une équation commune des mêmes données. Cette hypothèse est possible parce que les coefficients de corrélation entre elles sont, comme nous le savons, très faibles. La détermination précise des erreurs de départ doit être faite en même temps que l'analyse globale.

L'analyse des erreurs est faite séparément pour les facteurs d'amplitude δ (table 2) et séparément pour les retards en temps τ (table 3). On suppose ainsi que les valeurs δ et τ sont affectées, d'une part de l'erreur accidentelle variant d'une série à l'autre et d'une onde à l'autre, dont la valeur quadratique moyenne est σ; et d'autre part de l'erreur systématique variant d'une série à l'autre mais constante pour toutes les ondes d'une seule série, dont la valeur quadratique moyenne est S. Le calcul de ces erreurs a été fait dans les deux variantes.

Dans la première variante toutes les valeurs de δ et τ sont considérées comme également précises du point de vue des erreurs accidentelles. Nous donnons dans les tables 2 et 3 les valeurs observées de δ et τ pour les trois ondes de marées examinées et leur moyenne arithmétique de chaque série δ, τ. Les valeurs moyennes δ et τ doivent être constantes pour chaque station si on ne considère pas les erreurs de mesures.

On a calculé pour chaque onde, dans chaque station, la moyenne arithmétique des valeurs δ et τ d'après toutes les séries et l'erreur quadratique moyenne d'une mesure de μ_0. Par les valeurs μ_0 pour les ondes O_1 K_1 et M_2 on a calculé ensuite la dispersion moyenne $\mu_1^2 = [\mu_0^2] / 3$. Les erreurs aussi bien accidentelles que systématiques influencent habituellement cette dispersion, ainsi

$$\mu_1^2 = \sigma^2 + S^2$$

Dans la dispersion μ_2^2 calculée par les valeurs de δ et τ l'erreur accidentelle est neutralisée de sorte que

$$\mu_2^2 = \sigma^2/_3 + S^2 \qquad (2)$$

Sur la base des deux dispersions il est impossible de déterminer pour chaque station les erreurs quadratiques moyennes de σ et S.

Les erreurs σ et S calculées pour les stations séparées et données dans les tables 2 et 3 sont insuffisamment sûres puisqu'elles sont obtenues sur la base d'un faible nombre d'observations. Leur estimation plus sûre est possible sur la base des dispersions M_1^2 et M_2^2 calculées au moyen de toutes les séries dans toutes les stations d'après les formules

$$M_1^2 = \frac{\sum_i \left[\mu_{1i}^2 (n_i - 1)\right]}{\sum_i (n_i - 1)}, \qquad (3)$$

$$M_2^2 = \frac{\sum_i \left[\mu_{2i}^2 (n_i - 1)\right]}{\sum_i (n_i - 1)},$$

où l'indice i désigne la station et n le nombre de séries dans la station. On a obtenu les valeurs suivantes :

$\sigma_\delta = \pm 0.0043$ $\sigma_\tau = \pm 77$ s

$S_\delta = \pm 0.0064$ $S_\tau = \pm 26$ s

Dans le cas des valeurs δ l'erreur systématique est plus forte que l'erreur accidentelle. Si on prend $\delta = 1{,}16$ cela correspond à une erreur quadratique moyenne relative de la calibration de l'enregistrement égale à ± 0,55 %. Dans le cas des valeurs τ, l'erreur systématique est moins forte que l'erreur accidentelle. Si la vitesse de mouvement de la bande d'enregistrement de 30 mm/heure l'erreur systématique correspondante est de ± 0.2 mm. L'hypothèse d'une même précision des valeurs τ pour les différentes ondes peut être incorrecte. On peut espérer bientôt une précision égale des valeurs de $\mathcal{H}$. Si on désigne l'erreur quadratique moyenne de ces valeurs par $\sigma_{\mathcal{H}}$ les erreurs quadratiques moyennes des valeurs de τ seront différentes pour les différentes ondes c'est à dire $\sigma_{\mathcal{H}}/\omega$. Les dispersions de μ_0 auront également des valeurs différentes :

$$\mu_0^2 = G_{\mathcal{H}}^2 / \omega^2 + S^2$$

et la dispersion moyenne

$$\mu_1^2 = [\mu_0^2]/3 = \frac{G_{\mathcal{H}}^2}{3} \left[\frac{1}{\omega^2}\right] + S^2. \quad (4)$$

La dispersion de μ_2^2 calculée d'après les valeurs de τ sera

$$\mu_2^2 = \frac{G_{\mathcal{H}}^2}{9} \left[\frac{1}{\omega^2}\right] + S^2. \quad (5)$$

En comparant les équations (4), (5) et (1), (2) nous obtiendrons:

$$G_{\mathcal{H}}^2 = G_t^2 \frac{3}{\left[\frac{1}{\omega^2}\right]} .$$

Pour les valeurs ω des ondes O_1, K_1 et M_2 nous avons :

$\sigma_{\mathcal{H}} = 0{,}00464\ \sigma_\tau$

Les valeurs résultantes des erreurs accidentelles et systématiques des retards de phase seront dans le cas suivant :

$\sigma_{\mathcal{H}} = \pm 0.36°$

$S_{\mathcal{H}} = \pm 26\ \omega$

Les erreurs quadratiques totales complètes des paramètres de marées δ et obtenues par une seule série de mesures, seront

$M_\delta^2 = \sigma_\delta^2 + S_\delta^2 = 0{,}0077^2$

$$M^2_{\varkappa} = \sigma^2_{\varkappa} + S^2_{\varkappa} = (0{,}36)^2 + (26\omega)^2$$

Dans la seconde variante de l'analyse des erreurs, les valeurs δ et τ sont considérées inégalement précises du point de vue des erreurs accidentelles et c'est pourquoi on utilise les poids calculés sur la base des erreurs m_δ, $m_\varkappa$ obtenues par l'analyse.

Pour cela les erreurs quadratiques moyennes de l'unité de poids sont prises égales à : $m_{o\delta} = \pm\ 0{,}0050$ et $m_{o\tau} = \pm\ 50$ sec. de sorte que

$$p_\delta = (0{,}0050/m_\delta)^2, \qquad p_\tau = (50\omega/m_\varkappa)^2.$$

Les poids des valeurs moyennes de δ et τ sont déterminés d'après la relation

$$\bar{p} = q/\left[\frac{1}{p}\right].$$

Dans les tables 2 et 3 on a calculé pour chaque onde de marée dans chaque station la moyenne pondérée et l'erreur quadratique moyenne de l'unité de poids μ_o. L'erreur accidentelle effective de l'unité de poids de σ_o et l'erreur systématique S entrant dans la valeur μ_o. Selon [5] nous avons

$$\mu_o'^2 = \sigma_o^2 + p S^2,$$

où

$$p = \frac{[p] - [p^2]}{[p](n-1)}.$$

Si nous calculons à nouveau dans chaque station, sur la base des valeurs μ_o', la dispersion moyenne $\mu_1'^2 = [\mu_o'^2]/3$ on aura une première équation pour la détermination de σ_o et S :

$$\mu_1'^2 = S_o^2 + \frac{1}{3}[p] \cdot S^2. \qquad (6)$$

Nous obtiendrons une seconde équation sur la base de la dispersion μ_2 calculée en fonction des valeurs de δ (τ) :

$$\mu_2'^2 = \sigma_o^2 + \bar{p}\, S^2, \qquad (7)$$

où

$$\bar{p} = \frac{[\bar{p}]^2 - [\bar{p}^2]}{[\bar{p}](n-1)}.$$

Les valeurs de σ_o et S calculées pour les stations séparées sont données dans les tables 2 et 3. Des 20 séries on a obtenu sur la base des dispersions de M_1^2 et M_2^2 calculées de façon analogue à M_1 et M_2 les valeurs suivantes

$\sigma_{o\delta} = \pm\ 0{,}0091$ $\qquad$ $\sigma_{o\tau} = \pm\ 63$ sec.

$S_\delta = \pm\ 0{,}0067$ $\qquad$ $S_\tau = \pm\ 29$ sec.

Pour les erreurs systématiques on a obtenu des valeurs pratiquement les mêmes que dans le cas de l'analyse sans pondération. Pour les erreurs quadratiques moyennes accidentelles d'unité de poids on a obtenu des valeurs plus grandes que celles qui ont été prises pour la détermination des poids. Par conséquent, les valeurs effectives des erreurs quadratiques moyennes accidentelles sont plus grandes que celles qui sont obtenues par l'analyse de la marée.

Les erreurs quadratiques moyennes totales des paramètres provenant d'une seule série de mesures

$$M_\delta'^2 = \frac{G_{o,\delta}^2}{p_\delta} + S_\delta^2 = \frac{G_{o,\delta}^2}{m_{o,\delta}^2} \; m_\delta^2 + S_\delta^2 ,$$

$$M_\varkappa'^2 = \omega^2\left(\frac{G_{o,\tau}}{p_\tau} + S_\tau^2\right) = \frac{G_{o,\tau}^2}{m_{o,\tau}^2} \; m_\varkappa^2 + 10^2 \, S_\tau^2$$

Avec les résultats que nous avons obtenus nous avons :

$$M_\delta'^2 = (1{,}82 \, m_\delta)^2 + 0{,}0067^2 ,$$

$$M_\varkappa'^2 = (1{,}26 \, m_\varkappa)^2 + (29 \, \omega)^2 .$$

Les deux variantes de l'analyse des erreurs effectuées présentent deux approches opposées. Dans le premier cas, sans l'utilisation des poids, nous négligeons les erreurs quadratiques moyennes internes déterminées par l'analyse et nous considérons comme importantes les sources extérieures des erreurs qui ont les périodes de marées et qui, pour l'onde donnée, sont constantes dans une seule série. Dans le second cas nous supposons que les erreurs quadratiques moyennes obtenues par l'analyse de la marée donnent la caractéristique complète de la précision des paramètres de marées. La réalité est entre ces deux hypothèses. Mais la première variante de l'analyse donne des résultats plus réels. Dans la seconde variante on obtient, dans certaines stations, des valeurs fictives de l'erreur quadratique moyenne σ_0 et notamment dans le cas du retard de temps τ, ce n'est pas seulement parce qu'on a utilisé un petit nombre de données pour leur détermination mais avant tout à cause de la relation irréelle des poids calculés sur la base des erreurs quadratiques moyennes m_δ et $m_\varkappa$.

L'augmentation de précision des valeurs des paramètres de marées pour chaque station est possible d'une part par l'allongement des séries des mesures, d'autre part par l'augmentation du nombre de séries. Dans le premier cas on ne diminue que la composante accidentelle de l'erreur quadratique moyenne totale. C'est pourquoi il vaut mieux augmenter le nombre de séries de mesures dans la station. L'influence des deux composantes sur l'erreur quadratique moyenne totale du résultat diminuera proportionnellement à la racine carrée du nombre des séries. Cela a avant tout une valeur pour les facteurs d'amplitude δ pour lesquels la composante systématique est importante. Enfin cela n'est correct que dans le cas où l'erreur systématique est variable d'une série à l'autre. Les séries de mesures avec différents gravimètres satisfont à cette condition.

Cependant on a montré précédemment que pour des séries avec un même gravimètre on peut avoir différentes erreurs systématiques dans les valeurs de δ. Ce fait peut être expliqué par les résultats du gravimètre G_s 15 N° 228 [6]. Pour cet appareil on a constaté que lors de la calibration on obtient différentes valeurs de l'échelle d'enregistrement dans les limites de 3% en fonction de quoi chaque partie de l'échelle du micromètre a été utilisée. Il est possible que des propriétés analogues des micromètres puissent se retrouver pour les autres gravimètres.

Le procédé le plus efficace pour augmenter la précision de la valeur de δ serait l'élimination de l'erreur systématique de calibration de l'enregistrement. Pour le gravimètre G_s 15 N°228 on a élaboré une méthode d'étalonnage par la vis de mesure sans utilisation du micromètre publiée dans le travail [6].

Interprétation des résultats

Nous donnons dans la table 4 les valeurs observées de δ pour les ondes O_1, K_1, M_2 et S_2 d'après les résultats de l'analyse globale dans des stations séparées. Nous donnons également dans la table les valeurs des corrections de l'influence des océans, des effets d'inertie, utilisés en [1] et les valeurs corrigées des paramètres des ondes. En outre, pour chaque station, nous donnons dans la table les différences de δ (O_1 - K_1) caractérisant la résonance avec la nutation diurne dont la valeur moyenne est égale à δ (O_1 - K_1) = 0,0147.

Il convient de noter que dans la plupart des stations on a fait des mesures ultérieures. Les valeurs de δ et que nous donnons ici ne représentent que le résultat des mesures faites dans le cadre de la collaboration internationale. Les valeurs précisées des paramètres de marées en tenant compte des mesures ultérieures seront publiées plus tard après la réduction des données.

Les valeurs mesurées et corrigées des facteurs d'amplitudes δ pour les stations séparées sont données sur la figure 7. Si on ne tient pas compte des résultats pour l'onde S_2 qui sont soumis dans un degré important aux influences extérieures, des différences systématiques atteignant plus de 1% sont évidentes entre les stations. L'introduction de corrections géophysiques n'élimine pas ces différences.

Il ne faut pas proposer les mêmes valeurs des paramètres pour les différentes stations. Mais puisqu'on a démontré plus haut l'existence des erreurs systématiques dans les séries séparées, on peut faire l'hypothèse que les différences dans les valeurs de δ pour les différentes stations s'expliquent nettement par les erreurs dans les calibrations de l'enregistrement, bien plus que par des causes géophysiques.

Les valeurs des retards de phases sont représentées sur la figure 8. Même si on ne prend pas en considération les résultats pour l'onde S_2 la divergence est ici nettement plus grande que dans le cas des valeurs de δ. Ainsi les différences systématiques constantes entre les stations sont également nettement sensibles.

De ce qui a été dit il s'ensuit qu'il faut perfectionner la méthode d'observation des marées gravimétriques avec le fait qu'on a éliminé ou au moins diminué les erreurs systématiques dans les valeurs δ et χ. La bonne solution de ce problème doit amener à une augmentation sensible de la précision réelle des résultats.

En conclusion nous ferons l'essai d'une interprétation géodésique des résultats des observations de marées gravimétriques communes c'est à dire que nous déterminerons les paramètres des ondes de marées pour calculer les corrections aux mesures gravimétriques précises dans le domaine donné. Nous partirons ainsi de l'hypothèse que la cause la plus importante des différences entre les résultats des stations sont les erreurs systématiques des séries séparées des mesures et que les variations régionales des paramètres de marées sont plus petites. Ces erreurs sont le mieux neutralisées dans l'analyse globale des 20 séries de mesures pour toutes les stations. L'analyse globale a été faite dans les deux variantes : avec l'introduction des corrections géophysiques et sans ces corrections. L'utilisation des corrections géophysiques n'a apporté pratiquement aucune augmentation de la précision des paramètres des ondes de marées [1]. Cela signifie que les erreurs systématiques sont de beaucoup les plus importantes.

C'est pourquoi nous considérons à présent comme le plus souhaitable d'utiliser pour le calcul des marées dans les mesures gravimétriques en Europe de l'Est des valeurs unifiées des paramètres de δ et obtenues par l'analyse globale des 20 séries sans corrections géophysiques. Les erreurs systématiques comprenant des variations régionales sont moyennées. Il est ainsi tout à fait suffisant de partager les ondes en 6 diurnes, 5 semi diurnes et 1 terdiurne. L'analyse avec une séparation plus détaillée des ondes n'a pas donné de résultats plus précis. Pour les ondes à longue période il faut jusqu'à présent utiliser les valeurs théoriques $\delta = 1{,}160$, $= 0^{\circ}$ conformément au modèle 1 de Molodenski. Ces valeurs seraient utilement précisées par l'introduction des corrections de l'influence des océans.

L'onde de marée constante [7] doit être également incluse dans les corrections de marées selon la résolution N° 15 (Canberra). Un point de vue opposé est proposé en [8]. Nous considérons cette question jusqu'à présent comme ouverte puisque l'observation de cette résolution amènerait à de grandes difficultés pratiques. Les valeurs des paramètres des ondes de marées recommandées ici, destinées au calcul des corrections dans les mesures gravimétriques sur le territoire de l'Europe de l'Est, sont données dans la table 5.

Pour estimer la précision avec laquelle ces paramètres sont représentatifs pour ce territoire, nous ferons l'analyse des erreurs pour les ondes O_1, K_1 et M_2. Nous donnons dans la table 6 les valeurs de δ et obtenues par l'analyse commune dans des stations séparées. Nous trouverons l'erreur quadratique moyenne accidentelle σ qui contient également les écarts des valeurs réelles des paramètres dans les stations en fonction des moyennes pour tout le territoire. Ainsi nous proposerons que ces écarts soient variables en signe et en valeur. Ensuite nous trouverons l'erreur systématique S constante de tous les résultats dans une seule station. L'analyse a été faite comme précédemment et sans pondération. Les poids calculés sur la base de la précision interne n'ont ici évidement pas de valeur. A la suite de ces calculs nous obtenons les résultats suivants :

$\sigma_\delta = \pm\ 0.0027$ $\qquad\qquad$ $\sigma_\varkappa = \pm\ 0.32°$

$S_\delta = \pm\ 0.0057$ $\qquad\qquad$ $S_\varkappa = +\ 59$

Les erreurs quadratiques moyennes complètes des paramètres δ et $\varkappa$ d'une seule station sont égales à

$$M_\delta^2 = \sigma_\delta^2 + S_\delta^2 = 0.0063^2$$

$$M_\varkappa^2 = \sigma_\varkappa^2 + S_\varkappa^2 = (0.32°)^2 + (59\omega)^2$$

Si on suppose que la composante systématique ne provient que des erreurs des observations nous obtenons les erreurs suivantes de la représentativité des paramètres proposés obtenus pour 6 stations :

$$\eta_\delta^2 = \sigma_\delta^2 + S_\delta^2/6 = 0{,}0036^2$$

$$\eta_x^2 = \sigma_\varkappa^2 + S_\varkappa^2/6 = (0{,}32°)^2 + (24\omega)^2$$

Si nous calculons ces erreurs pour les trois ondes données et comparons avec la table 6 en [9] nous obtiendrons l'erreur des corrections de marées égale à ± 30 nm s^{-2} (3 μgal). C'est la limite inférieure pour l'erreur des corrections. Nous obtiendrons la limite supérieure si nous considérons que les différences systématiques entre les stations s'expliquent par des causes géophysiques. Dans ce cas il faut utiliser les erreurs quadratiques moyennes complètes des paramètres. Pour les erreurs des corrections de marées nous obtiendrons dans ce cas : ± 50 nm.s^{-2} (5 μgal). On n'a pas inclu dans l'estimation proposé plus haut de la précision des corrections l'influence des erreurs dans les valeurs adoptées pour les paramètres des ondes à longue période.

BIBLIOGRAPHIE

1. D i t t f e l d H.-J., S i m o n Z., V a r g a P. and ext. Global Analysis of Earth Tide Observations of KAPG in Obninsk, Potsdam, Pechy, Tihany and Sofia. Study of Earth Tides, KAPG, W.G. 3.3, Bull. N°4, Hungary, Geoph. Inst. "R. Eötvos", Budapest, 1981.

2. В е н е д и к о в А.П. Анализ земноприливных регистраций. KAPG, W.G. 3.3 - Study of the Earth Tides, Bull. N°1, Hungary, Geoph. Inst. "R.Eötvös", Budapest, 1978.

3. V e n e d i k o v A.P. Analysis of Earth Tidal Data. 8 th Intern. Symp. Earth Tides, Bonn, 1977.

4. V e n e d i k o v A.P. Simultaneous Analysis of Different Earth Tidal Records. BIM N°81, 1979, 5020 p.

5. B ö h m J. Vyrovnavaci pocet. SNTL, Praha, 1964.

6. B r o z J., H o l u b S., S i m o n Z. Calibration of GS-15 N°228 Earth Tide Gravimeter Record. Study of the Eart Tide, KAPG, W.G.3.3, Bull. N°5, Hung. Geoph. Inst. "R.Eötvös", in print.

7. Bulletin Géodesique 54, N°3, 1980.

8. S i m o n Z. A Comment on Honkassalo's Correction. Studia geoph. et geod. 24, 1980, 92 p.

9. S i m o n Z. Earth Tide Corrections for Gravimetric Measurements. Study of the Earth Tides, KAPG, W.G.3.3, Bull. N°2. Hung. Geoph.Inst. "R.Eötvös", Budapest, 1979.

TABLE 1

Résultats des mesures.

Série	Station gravimètre	O₁ δ	O₁ k°	K₁ δ	K₁ k°	M₂ δ	M₂ k°	S₂ δ	S₂ k°
1	Poulkovo G_S 201	1,1626	-0,34	1,1455	-0,94	1,1754	-0,40	1,1673	-0,50
		± 68	0,33	± 41	±0,22	± 43	±0,21	± 89	±0,45
2	BN 07	1,1488	-0,59	1,1481	-1,57	1,1873	-1,20	1,1668	-0,46
		± 50	0,25	± 29	±0,15	± 27	±0,13	± 58	±0,29
3	G_S 220	1,1574	-0,12	1,1427	-0,60	1,1844	-0,31	1,1752	1,79
		± 40	0,20	± 21	±0,12	± 20	±0,10	± 44	±0,21
	Moyenne arithm.	1,1563	-0,35	1,1454	-1,04	1,1824	-0,64	1,1698	0,28
	Analyse globale	1,1563	-0,28	1,1449	-0,99	1,1830	-0,60	1,1701	0,55
		± 30	±0,15	± 16	±0,09	± 17	±0,08	± 36	±0,18
4	Obninsk BN 07	1,1531	-0,16	1,1480	0,18	1,1995	-0,31	1,1915	0,02
		± 61	±0,30	± 40	±0,20	± 20	±0,10	± 44	±0,20
5	G_S 220.1	1,1477	0,08	1,1331	-0,05	1,1825	-0,32	1,1528	1,24
		± 42	±0,21	± 22	±0,12	± 21	±0,10	± 44	±0,21
6	G_S 220.2	1,1505	-0,03	1,1348	-0,01	1,1775	-0,09	1,1704	1,43
		± 29	±0,14	± 18	±0,09	± 11	±0,06	± 26	±0,12
7	G_S 221.1	1,1713	-0,20	1,1553	-1,11	1,1991	-0,13	1,2183	-1,92
		± 76	±0,37	± 49	±0,24	± 42	±0,20	± 89	±0,40
8	G_S 221.2	1,1510	0,20	1,1410	0,10	1,1821	-0,26	1,1593	0,19
		± 25	±0,12	± 17	±0,08	± 12	±0,06	± 27	±0,12
9	G_S 224	1,1724	0,18	1,1585	0,25	1,2047	0,00	1,1989	-0,39
		± 20	±0,10	± 13	±0,07	± 8	±0,04	± 20	±0,09
	Moyenne arithm.	1,1577	0,01	1,1451	-0,11	1,1909	-0,18	1,1819	0,10
	Analyse globale	1,1584	0,07	1,1455	-0,12	1,1902	-0,17	1,1807	0,08
		± 20	±0,10	± 11	±0,06	± 10	±0,05	± 20	±0,09

10	Potsdam *BN* 07	1,1558	0,06	1,1443	0,15	1,1914	1,36	1,2082	0,58
		± 28	±0,14	± 18	±0,09	± 11	±0,05	± 22	±0,12
11	G_8 222	1,1517	-0,09	1,1420	0,06	1,1894	1,17	1,1955	0,28
		± 12	±0,06	± 8	±0,04	± 3	±0,02	± 7	±0,04
	Moyenne arithm.	1,1538	-0,02	1,1432	0,10	1,1904	1,26	1,2018	0,43
	Analyse globale	1,1536	0,04	1,1426	0,11	1,1898	1,26	1,2024	0,40
		± 15	±0,08	± 10	±0,05	± 6	±0,03	± 12	±0,06
12	Pecny *BN* 07	1,1546	-0,53	1,1334	1,72	1,1832	0,75	1,1988	0,49
		± 59	±0,29	± 40	±0,21	± 21	±0,10	± 37	±0,20
13	G_6 220	1,1517	-0,03	1,1392	0,19	1,1859	0,88	1,1918	0,25
		± 24	±0,12	± 15	±0,09	± 11	±0,05	± 21	±0,11
14	G_8 222	1,1523	-0,13	1,1360	0,37	1,1871	1,01	1,1984	0,32
		± 31	±0,16	± 20	±0,12	± 14	±0,07	± 23	±0,12
15	G_6 228	1,1489	-0,12	1,1392	-0,03	1,1836	0,22	1,1921	-0,52
		± 26	±0,13	± 16	±0,09	± 17	±0,08	± 30	±0,16
	Moyenne arithm.	1,1519	-0,20	1,1370	0,56	1,1850	0,72	1,1953	0,14
	Analyse globale	1,1522	-0,19	1,1373	0,58	1,1848	0,73	1,1958	0,17
		± 22	±0,11	± 14	±0,08	± 9	±0,04	± 16	± 0,08
16	Tihany *BN* 07.1	1,1618	-0,31	1,1670	-0,58	1,1922	0,05	1,2021	-3,28
		± 92	±0,46	± 52	±0,28	± 32	±0,16	± 67	±0,35
17	*BN* 07.2	1,1520	0,04	1,1414	-0,76	1,1858	0,64	1,1789	-1,72
		± 105	±0,52	± 48	±0,29	± 38	±0,18	± 83	±0,43
18	G_8 220	1,1563	0,05	1,1464	-0,12	1,1900	0,29	1,1888	0,32
		± 23	±0,11	± 13	±0,08	± 12	±0,06	± 22	±0,11
	Moyenne arithm.	1,1567	-0,07	1,1516	-0,49	1,1893	0,33	1,1899	-1,56
	Analyse globale	1,1567	-0,03	1,1493	-0,38	1,1889	0,33	1,1895	-0,83
		± 30	±0,15	± 17	±0,10	± 14	±0,07	± 28	±0,15
19	Sofia G_8 220	1,1444	-0,07	1,1273	0,29	1,1751	0,45	1,1784	0,73
		± 17	±0,09	± 11	±0,06	± 5	±0,03	± 12	±0,06
20	G_6 221	1,1452	0,01	1,1288	-0,01	1,1746	0,31	1,1653	0,29
		± 13	±0,07	± 8	±0,04	± 5	±0,03	± 12	±0,06
	Moyenne arithm.	1,1448	-0,03	1,1280	0,14	1,1748	0,38	1,1718	0,51
	Analyse globale	1,1448	-0,04	1,1280	0,15	1,1749	0,39	1,1723	0,53
		± 11	±0,06	± 7	±0,04	± 4	±0,02	± 9	±0,04
	Moyenne arithm. toutes stations	1,1537	-0,07	1,1413	-0,11	1,1853	0,32	1,1851	0,15
	Analyse globale toutes stations	1,1538	-0,03	1,1424	-0,11	1,1842	0,39	1,1848	0,16
		± 9	±0,05	± 5	±0,03	± 4	±0,02	± 9	±0,04

TABLE 2

Analyse des erreurs des facteurs δ

Station gravimètre	O_1		K_1		M_2		Moyenne arithm.		$\mu_1^2\cdot10^8$ / $\mu_2^2\cdot10^8$	$G\cdot10^4$ / $s\cdot10^4$
	δ	p	δ	p	δ	p	$\bar{\delta}$	$\bar{p}$		
1	2	3	4	5	6	7	8	9	10	11
Poulkovo G_s 201	1,1626	0,5	1,1455	1,5	1,1754	1,4	1,1612	2,8		
BN 07	1,1488	1,0	1,1481	3,0	1,1873	3,4	1,1614	5,5		
G_s 220	1,1574	1,6	1,1427	5,7	1,1844	6,2	1,1615	9,2		
Moyenne arithm.	1,1563		1,1454		1,1824		1,1614		3158	± 69
μ_o, μ_a	± 70		± 27		± 62		± 2		4	im
Moyenne pondérée	1,1555		1,1447		1,1842		1,1614		4550	± 91
μ'_o, μ'_a	± 62		± 54		± 83		± 3		9	im
$p, \bar{p}$		0,95		2,93		3,13		5,24		
Obninsk BN 07	1,1531	0,7	1,1480	1,6	1,1995	6,2	1,1669	3,9		
G_s 220.1	1,1477	1,4	1,1331	5,2	1,1825	5,7	1,1544	8,4		
G_s 220.2	1,1505	3,0	1,1348	7,7	1,1775	20,7	1,1543	17,5		
G_s 221.1	1,1713	0,4	1,1553	1,0	1,1991	1,4	1,1752	2,2		
G_s 221.2	1,1510	4,0	1,1410	8,6	1,1821	17,4	1,1580	21,3		
G_s 224	1,1724	6,2	1,1585	14,8	1,2047	39,1	1,1785	35,5		
Moyenne arithm.	1,1577		1,1451		1,1909		1,1646		12261	± 39
μ_o, μ_a	± 111		± 106		± 115		± 106		11236	± 104
Moyenne pondérée	1,1597		1,1460		1,1923		1,1660		130426	± 185
μ'_o, μ'_a	± 193		± 298		± 515		± 461		212521	± 117
$p, \bar{p}$		2,30		5,82		12,94		13,02		
Potsdam BN 07	1,1558	3,2	1,1443	7,7	1,1914	20,7	1,1638	18,3		
G_s 222	1,1517	17,4	1,1420	39,1	1,1894	277,8	1,1610	103,7		
Moyenne arithm.	1,1538		1,1432		1,1904		1,1624		431	± 7
μ_o, μ_a	± 29		± 16		± 14		± 20		400	± 20
Moyenne pondérée	1,1523		1,1424		1,1895		1,1614		5199	im.
μ'_o, μ'_a	± 67		± 58		± 88		± 110		12100	± 24
$p, \bar{p}$		5,39		12,89		38,46		31,12		

Pecny BN 07	1,1546	0,7	1,1334	1,6	1,1832	5,7	1,1571	4,1		
G_s 220	1,1517	4,3	1,1392	11,1	1,1859	20,7	1,1589	24,4		
G_s 222	1,1523	2,6	1,1360	6,2	1,1871	12,8	1,1585	14,4		
G_s 228	1,1489	3,7	1,1392	9,8	1,1836	8,6	1,1572	18,4		
Moyenne arithm.	1,1519		1,1370		1,1850		1,1579		558	± 27
μ_o, μ_a	± 23		± 28		± 19		± 9		81	im
Moyenne pondérée	1,1511		1,1382		1,1855		1,1582		2418	± 59
μ'_o, μ'_a	± 33		± 54		± 57		± 35		1225	im
$p, \bar{p}$		2,62		6,54		11,05		14,15		
Tihany. BN 07.1	1,1618	0,3	1,1670	0,9	1,1922	2,4	1,1737	1,9		
BN 07.2	1,1520	0,2	1,1414	1,1	1,1858	1,7	1,1597	1,5		
G_s 220	1,1563	4,7	1,1464	14,8	1,1900	17,4	1,1642	26,7		
Moyenne arithm.	1,1567		1,1516		1,1893		1,1659		7329	±59
μ_o, μ_a	± 49		± 136		± 33		± 71		5041	±62
Moyenne pondérée	1,1564		1,1472		1,1899		1,1646		7747	±65
μ'_o, μ'_a	± 26		± 143		± 46		± 99		9801	±42
$p, \bar{p}$		0,49		1,83		3,56		3,11		
Sofia G_s 220	1,1444	8,6	1,1273	20,7	1,1751	100,0	1,1489	51,7		
G_s 221	1,1452	14,8	1,1288	39,1	1,1746	100,0	1,1495	87,2		
Moyenne arithm.	1,1448		1,1280		1,1748		1,1492		58	±8
μ_o, μ_a	± 6		± 11		± 4		± 4		16	im
Moyenne pondérée	1,1449		1,1283		1,1748		1,1493		1537	±50
μ'_o, μ'_a / $p, \bar{p}$										im
	± 19		± 55		± 35		± 34		1156	
		10,92		27,03		100,00		64,93		
					De toutes les séries sans poids				6032	± 43
									4781	± 64
					De toutes les séries avec poids				49337	± 91
									78511	± 67

TABLE 3

Analyse des erreurs des valeurs τ

Station gravimètre	O_1 τ	O_1 p	K_1 τ	K_1 p	M_2 τ	M_2 p	Moyenne arithm. τ	Moyenne arithm. $\bar{p}$	μ_0^2 / μ_s^2	σ / s
1	2	3	4	5	6	7	8	9	10	11
Poulkovo G_8 201	- 88	0,3	-225	0,9	- 50	3,7	-121	2,1		
BN 07	-152	0,6	-376	1,9	-149	9,6	-226	3,9		
G_8 220	- 31	0,9	-144	3,0	- 39	16,2	- 71	6,2		
Moyenne arithm.	-90		-248		- 79		-139		7122	±36
μ_0, μ_s	±61		±118		± 61		± 79		6241	±76
Moyenne pondérée	-80		-233		- 76		-130		24255	±im
μ_0', μ_s'	±52		±179		±195		±170		28900	±205
$p, \bar{p}$		0,58		1,76		8,49		3,72		
Obninsk BN 07	-41	0,4	43	1,1	-39	16,2	- 12	2,7		
G_8 220.1	21	0,8	- 12	3,0	-40	16,2	- 10	5,7		
G_8 220.2	- 8	1,9	- 2	5,4	-11	45,0	- 7	12,3		
G_8 221.1	-52	0,3	-266	0,8	-16	4,0	-111	1,7		
G_8 221.2	52	2,6	24	6,8	-32	45,0	15	16,3		
G_8 224	46	3,8	60	8,9	0	101,2	35	23,2		
Moyenne arithm.	3		- 26		-23		-15		5611	±68
μ_0, μ_s	±44		±121		±16		±50		2500	±31
Moyenne pondérée	28		19		-14		11		9880	±90
μ_0', μ_s'	±42		±129		±106		±97		9409	±12
$p, \bar{p}$		1,44		3,92		32,50		9,15		
Potsdam BN 07	15	1,9	36	5,4	169	64,8	73	12,4		
G_8 222	-23	10,4	14	27,3	145	405,0	45	66,6		
Moyenne arithm.	- 4		25		157		59		425	±6
μ_0, μ_s	±27		±16		±17		±20		400	±20

1	2	3	4	5	6	7	8	9	10	11
Moyenne pondérée	-17		18		148		49		12185	± 65
μ_o, μ_s	±48		± 47		±179		±91		8281	± 14
$p, \bar{p}$		3,23		9,00		111,72		20,93		
Pecny BN 07	-137	0,4	412	1,0	93	16,2	123	2,7		
G_s 220	- 8	2,6	45	5,4	109	64,8	50	15,4		
G_s 222	- 34	1,5	89	3,0	125	33,1	60	8,6		
G_s 228	- 31	2,2	- 7	5,4	27	25,3	- 4	13,3		
Moyenne arithm.	- 52		135		89		57		13645	±128
μ_o, μ_s	± 58		± 189		± 43		± 52		2704	*im*
Moyenne pondérée	- 30		60		96		39		35218	*im*
μ_o, μ_s	± 46		±224		±231		±129		16641	± 79
$p, \bar{p}$		1,55		3,40		31,64		9,22		
Tihany BN 07.1	-80	0,2	-139	0,6	6	6,3	-71	1,2		
BN 07.2	10	0,1	-182	0,5	80	5,0	-31	1,0		
G_s 220	13	3,1	- 29	6,8	36	45,0	7	18,3		
Moyenne arithm.	-19		-117		41		-32		3473	±54
μ_o, μ_s	±53		± 79		± 37		±39		1521	± 23
Moyenne pondérée	8		- 47		37		1		5585	±45
μ_o, μ_s	±27		± 91		± 88		±63		3969	±31
$p, \bar{p}$		0,30		0,97		9,61		2,00		
Sofia G_s 220	-18	4,6	69	12,1	56	180,0	36	29,6		
G_s 221	3	7,6	- 2	27,3	39	180,0	13	52,0		
Moyenne arithm	- 8		34		48		24		956	±32
μ_o, μ_s	±15		±50		± 12		± 16		256	*im*
Moyenne pondérée	- 5		20		48		21		23218	*im*
μ_o, μ_s	± 36		±206		±161		±100		10000	±21
$p, \bar{p}$		5,77		16,78		180,01		37,74		
					Из всех серий без весов				6540	±77
									2628	±26
					Из всех серий с весами				17867	±63
									12928	±29

TABLE 4

Introduction des corrections géophysiques

Station	Onde	Измеренные значения		Океанические поправки		Инерциальные поправки	Исправленные значения		$\Delta\delta(O_1-K_1)$
		δ	k	$\Delta\delta$	$\Delta k'$	$\Delta\delta'$	δ	k	
1	2	3	4	5	6	7	8	9	10
Poulkovo	O_1	1,1563	-0,28	0,0046	-0,14	0,0021	1,1630	-0,42	
	K_1	1,1449	-0,99	0,0015	-0,06	0,0022	1,1486	-1,05	0,0144
φ =59,77°	M_2	1,1830	-0,60	-0,0180	-0,45	-0,0033	1,1617	-1,05	
λ =30,32°	S_2	1,1701	0,55	-0,0083	0,89	-0,0035	1,1583	1,44	
Obninsk	O_1	1,1584	0,07	0,0039	-0,12	0,0020	1,1643	-0,05	
	K_1	1,1455	-0,12	0,0026	-0,08	0,0021	1,1502	-0,20	0,0141
φ =55,17°	M_2	1,1902	-0,17	-0,0165	-0,30	-0,0033	1,1704	-0,47	
λ =36,45°	S_2	1,1807	0,08	-0,0056	0,46	-0,0035	1,1716	0,54	
Potsdam	O_1	1,1536	0,04	0,0061	-0,04	0,0020	1,1617	0,00	
	K_1	1,1426	0,11	0,0020	-0,13	0,0021	1,1467	-0,02	0,0150
φ =52,38°	M_2	1,1898	1,26	-0,0273	-0,78	-0,0034	1,1591	0,48	
λ =13,07°	S_2	1,2024	0,40	-0,0254	0,34	-0,0036	1,1734	0,74	
Pecny	O_1	1,1522	-0,19	0,0058	-0,03	0,0020	1,1600	-0,22	
	K_1	1,1373	0,58	0,0023	-0,12	0,0021	1,1417	0,46	0,0183
φ =49,92°	M_2	1,1848	0,73	-0,0271	-0,88	-0,0034	1,1543	-0,15	
λ =14,78°	S_2	1,1958	0,17	-0,0225	0,40	-0,0036	1,1697	0,57	
Tihany	O_1	1,1567	-0,03	0,0052	-0,03	0,0019	1,1638	-0,06	
	K_1	1,1493	-0,38	0,0031	-0,11	0,0021	1,1545	-0,49	0,0093
φ =46,90°	M_2	1,1889	0,33	-0,0225	-0,66	-0,0034	1,1630	-0,33	
λ =17,89°	S_2	1,1895	-0,83	-0,0174	0,24	-0,0036	1,1685	-0,59	
Sofia	O_1	1,1448	-0,04	0,0049	-0,04	0,0019	1,1516	-0,08	
	K_1	1,1280	0,15	0,0044	-0,12	0,0020	1,1344	0,03	0,0172
φ =42,68°	M_2	1,1749	0,39	-0,0186	-0,55	-0,0035	1,1528	-0,16	
λ =23,33°	S_2	1,1723	0,53	-0,0127	0,15	-0,0037	1,1559	0,68	
							Среднее		0,0147

TABLE 5

Valeurs géophysiques des paramètres de marées pour l'Europe de l'Est

Onde	δ	k	Onde	δ	k
longue période	1,160	0,0	longue période	1,160	0,0
Q_1	1,155	0,2	$2N_2$	1,175	0,9
O_1	1,154	0,0	N_2	1,186	1,0
M_1	1,151	0,5	M_2	1,184	0,4
K_1	1,142	-0,1	L_2	1,140	-0,1
J_1	1,165	-0,8	S_2	1,185	0,2
OO_1	1,183	2,9	M_3	1,029	-0,8

TABLE 6

Estimation de la précision

Station	O_1		K_1		M_2		Moyenne arithm.		$\mu_0^2\cdot10^6$ / $\mu_2^2\cdot10^6$	μ_0^2 / μ_2^2	$G\cdot10^6$ / $S\cdot10^6$	G / S
	δ	τ	δ	τ	δ	τ	$\bar{\delta}$	$\bar{\tau}$	δ	τ	δ	τ
Poulkovo	1,1563	-72	1,1449	-237	1,1830	-75	1,1614	-128				
Obninsk	1,1584	18	1,1455	- 29	1,1902	-21	1,1647	- 11				
Potsdam	1,1536	10	1,1426	26	1,1898	157	1,1620	64				
Pecni	1,1522	-49	1,1373	139	1,1848	91	1,1581	60				
Tihany	1,1567	- 8	1,1493	- 91	1,1889	41	1,1650	- 19				
Sofia	1,1448	-10	1,1280	36	1,1749	48	1,1492	25				
Moyenne arithm	1,1537	-18	1,1413	- 26	1,1853	40	1,1601	- 2	3847	8111	23	68
μ_0, μ_2	± 49	± 35	± 76	± 128	± 58	± 82	± 59	± 71	3481	5041	57	59

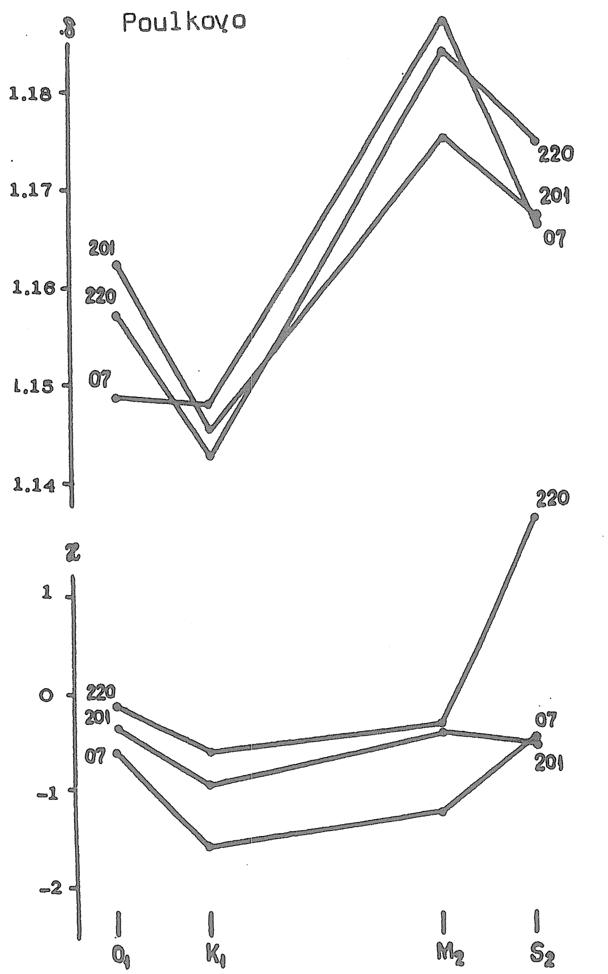


Fig. 1

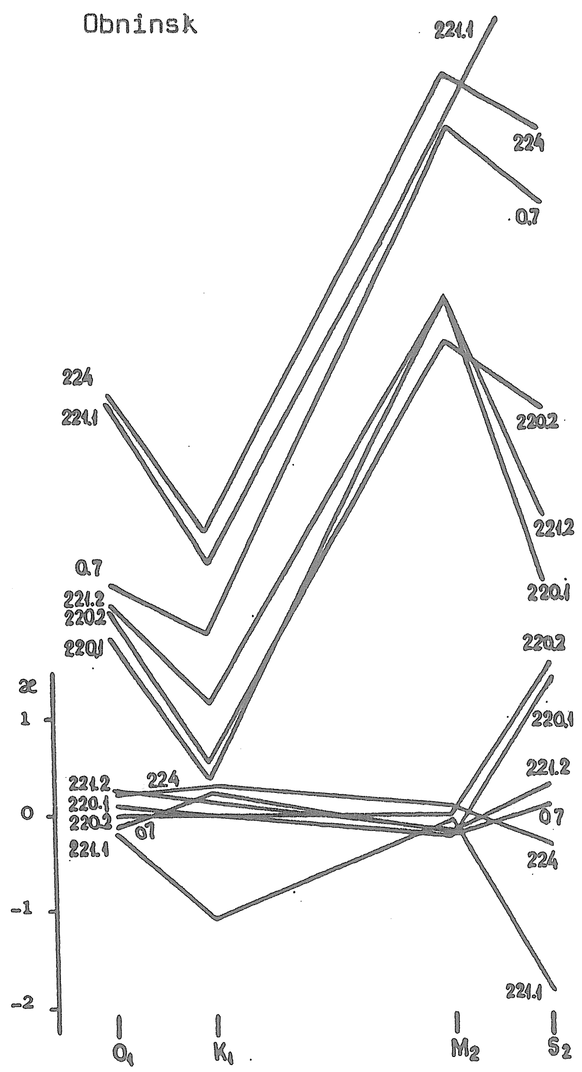


Fig. 2

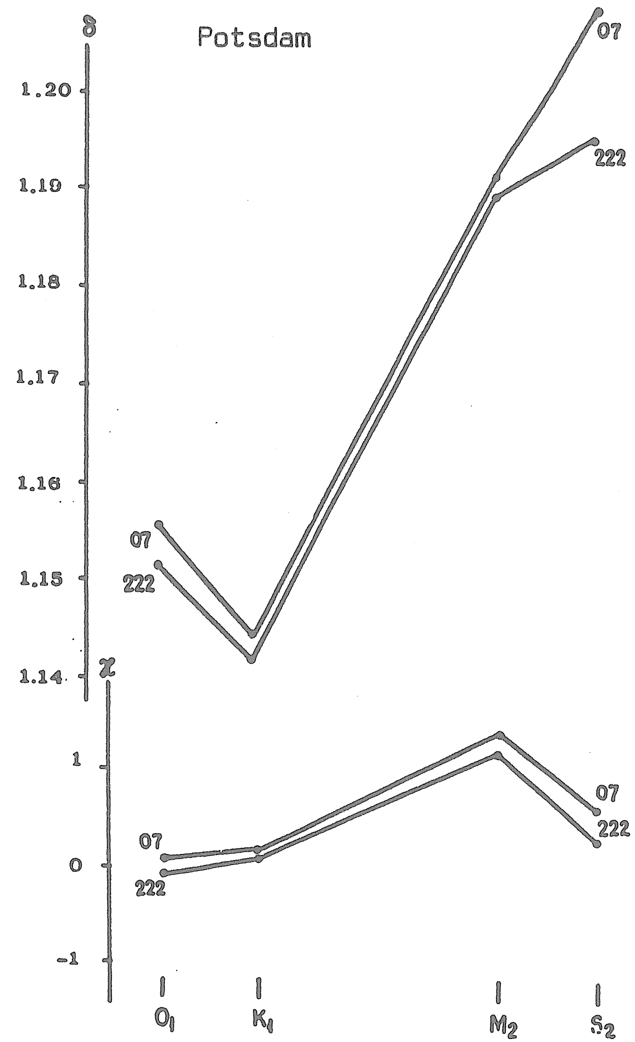


Fig. 3

Graphiques des paramètres de marées de δ et $\varkappa$

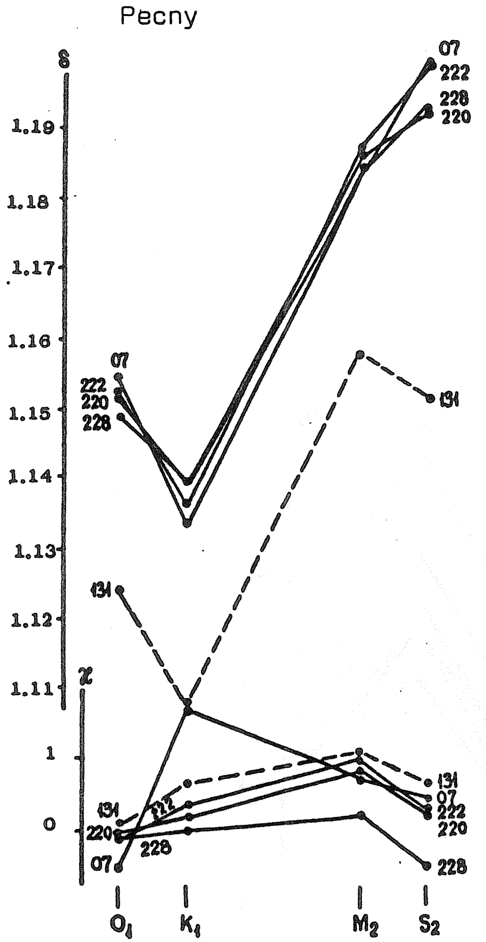


Fig. 4

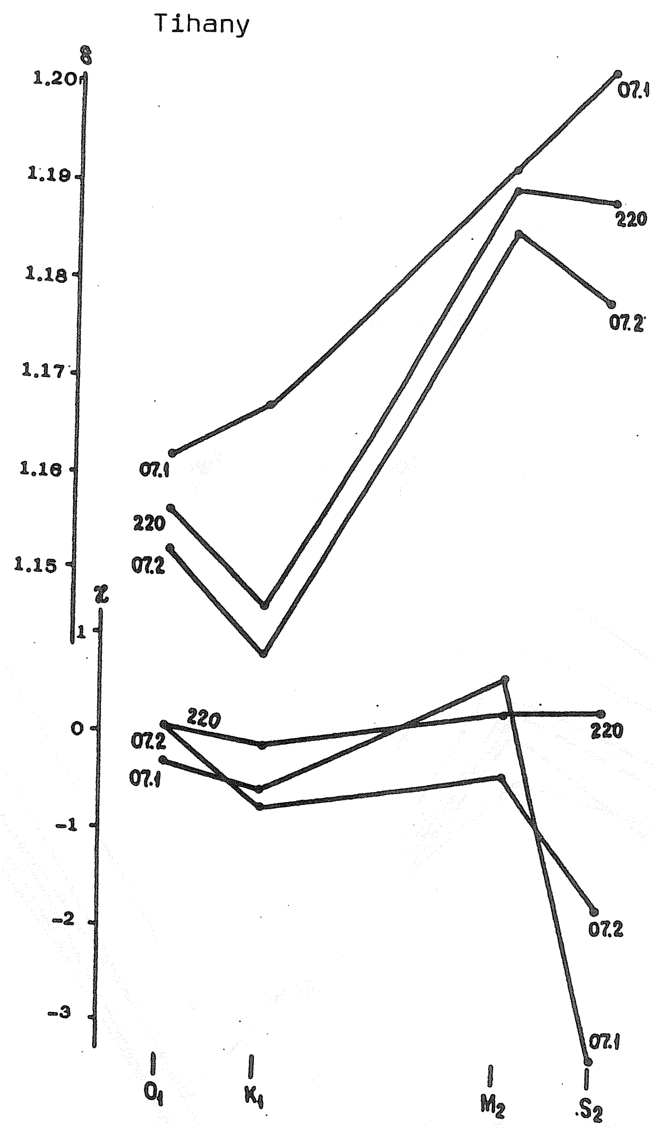


Fig. 5

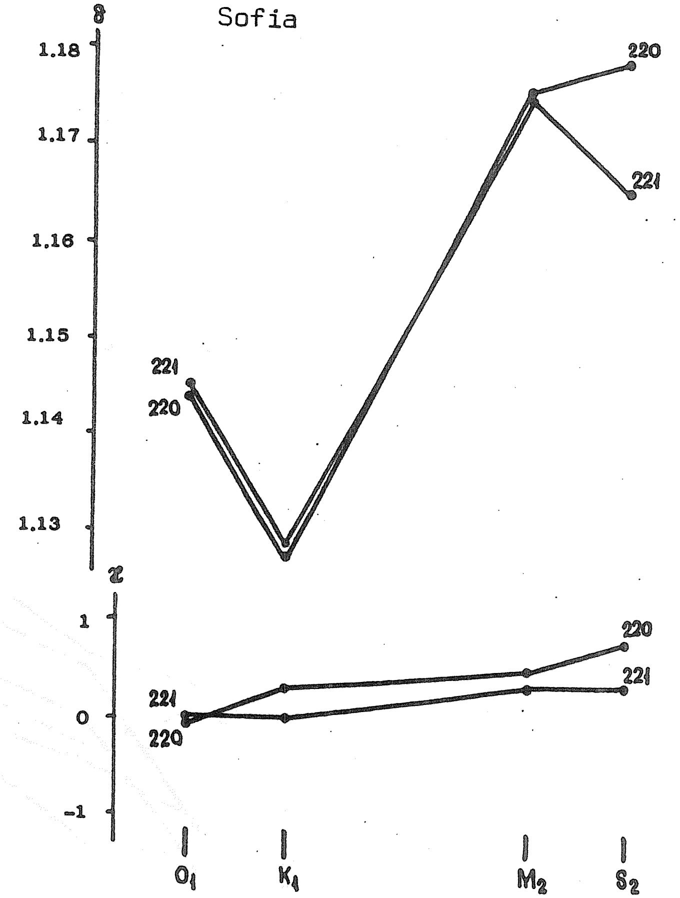


Fig. 6

Graphiques des paramètres de marées de δ et $\varkappa$

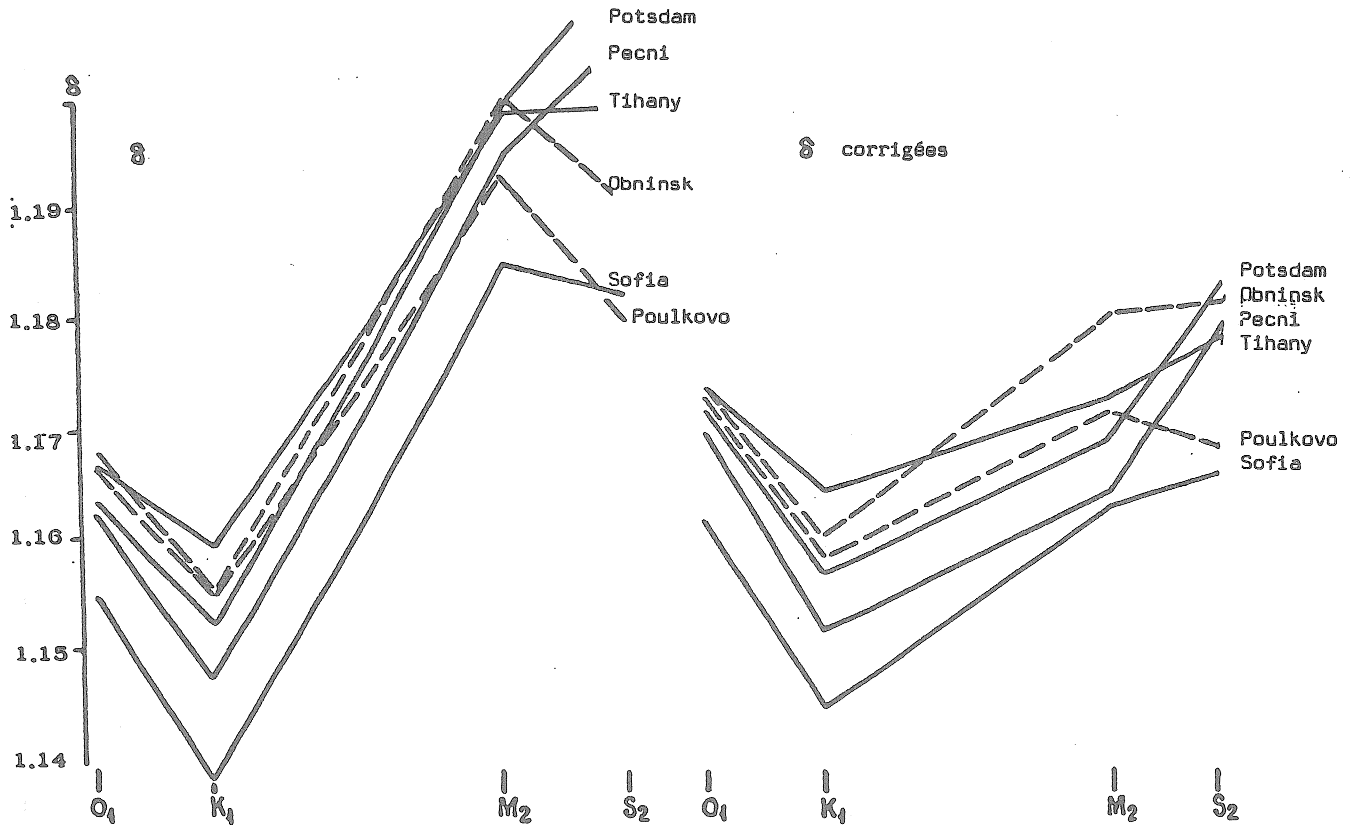


Figure 7 Valeurs du facteur δ avant et après les corrections géophysiques

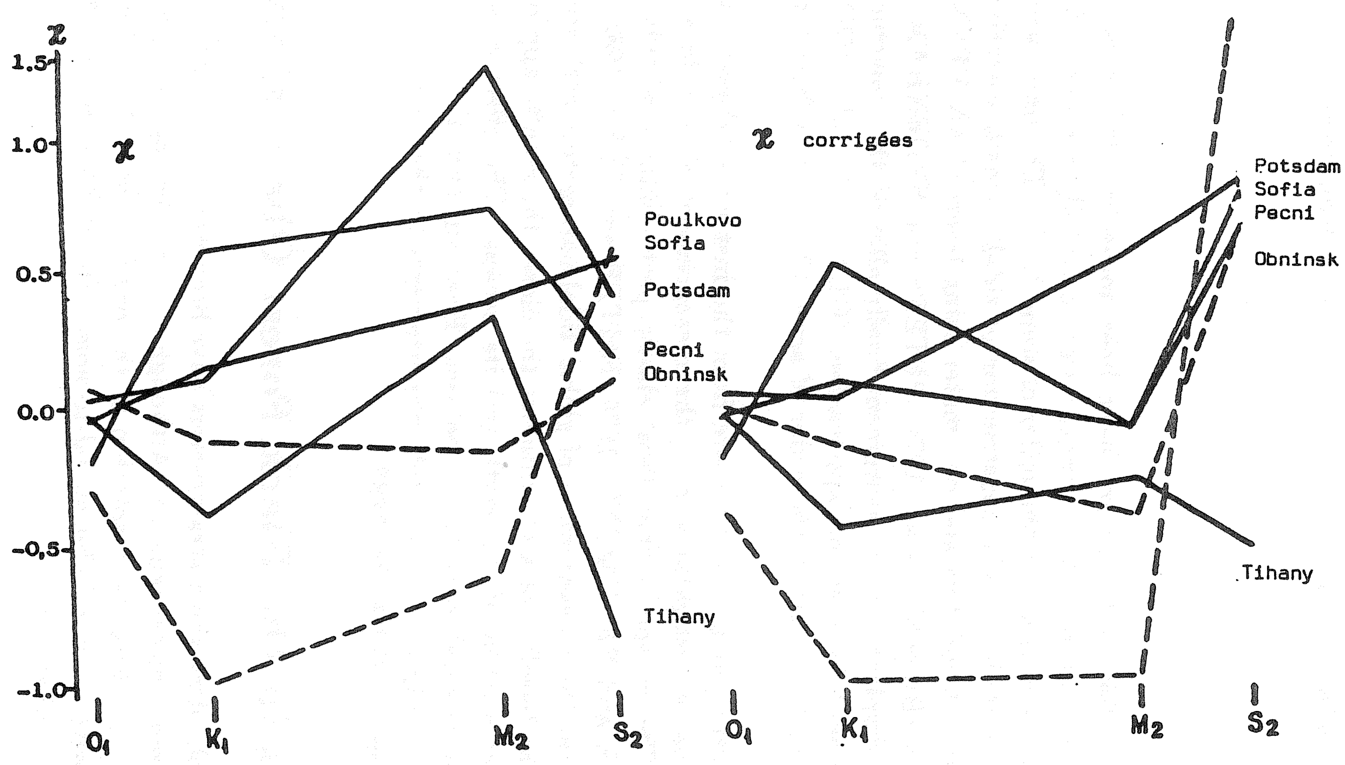


Figure 8 Valeurs du déphasage $\varkappa$ avant et après les corrections géophysiques.

Ultra-short strainmeters : Tides are in the smallest cracks

Walter Zürn, Dieter Emter and Heinz Otto
Black Forest Observatory Schiltach (BFO), Heubach 206, D-7620 Wolfach, Federal Republic of Germany.

Abstract

Tidal strains have been measured in the BFO with strainmeters having base lengths of some centimeters. This was possible because of very local strain enhancement due to inhomogeneities.

1 Introduction

Since the time of BENIOFF (1959) tidal strains have been measured with strainmeters having lengths of some tens of meters. These lengths were originally necessary for adequate resolution of tidal strains which are in the order of some 10^{-8} . The improvement of methods to measure displacements led to the construction of tidally sensitive strainmeters with base lengths between some decimeters and one meter using lasers (VAN VEEN et.al., 1966) or inductive transducers (BILHAM et al., 1974; PETERS and SYDENHAM, 1974). The application of capacitive displacement measurement finally allowed the construction of strainmeters that are short enough for borehole applications (GLADWIN, 1984; MENGLIN and MINGJIN, 1986); they resolve strain in the order of some 10^{-11} over a base length of some centimeters.
However, ultra-short strainmeters, as nice as they are from the instrumental point of view, should only be used for special investigations, e.g. the measurement of very local deformations or displacements. The reason is that local elastic perturbations of tidal strain (HARRISON, 1976; BERGER and BEAUMONT, 1976; EMTER and ZÜRN, 1985; SATO and HARRISON, 1990) prevent the measurement of global tidal parameters with strainmeters. In general, the longer the baseline of a strainmeter the closer the results approximate predicted values using present models of the earth and its oceans. The effect of a larger baseline is to average over severe small scale perturbations. However, even then one should not try to derive global tidal parameters from such measurements at the accuracy level of 1% , which is needed at present (WILHELM, 1982).

2 Measurements in the Black Forest Observatory

In recent years two very simple ultra-short strainmeters have been used in the Schiltach Observatory for certain displacement measurements. Although their measuring base was about a decimeter or less and their instrumental resolution in the order of some 10^{-8}, both showed surprisingly clear tides with a remarkable signal-to-noise ratio in their records because obviously the tides are locally enhanced.

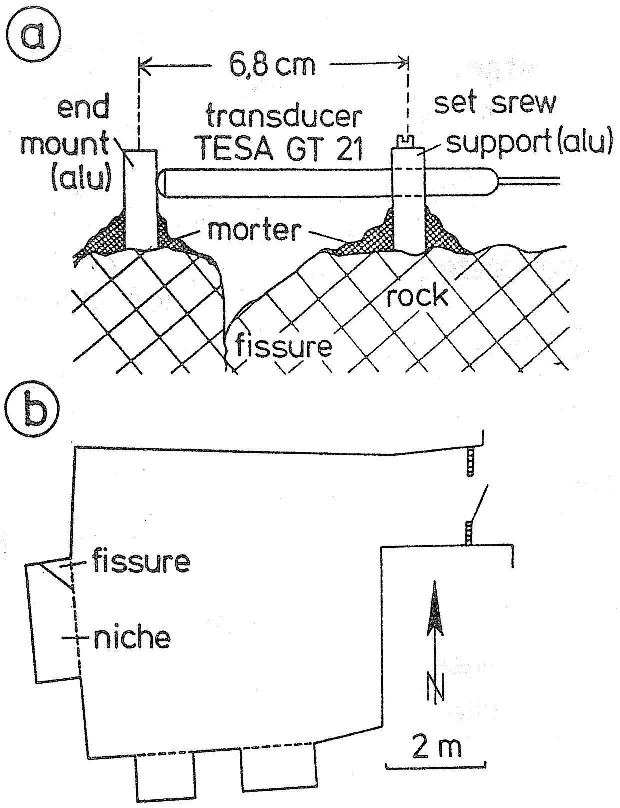


Figure 1: a.) Sketch of the 6.8 cm-strainmeter for the experiment in the tiltmeter niche (not scaled). b.) Position within the tiltmeter-vault.

The first instrument (Fig.1a) was placed perpendicular to a rock fissure at the bottom of one of the niches in the Schiltach tiltmeter vault (see MÄLZER et al., 1979 and Fig.1b) in an azimuth of N 43^0 E. The aim of this installation was to check whether the strong drift of one of the horizontal pendulums installed in that niche was caused by movements of that fissure. The instrument simply consisted of an industrial displacement transducer of the type TESA GT21 fixed on one side of the fissure and pushed with a small force (0.6 N) against the end mount on the other side. Inductive displacement transducers of this type had already been used for strainmeters (SYDENHAM, 1969; BILHAM et al., 1974). Their resolution is limited by the electronic noise level to around 1 nm. The transducers are calibrated by the manufacturer to better than 1% ; this was confirmed by calibration with a HEWLETT PACKARD laser interferometer.

The second instrument was (Fig.2a) very similar to the first one, only the end mounts were not cemented but bolted to the base and the displacement transducer was of the type TESA GT41. It was placed in EW-direction across a 5 cm wide gap between two recently constructed concrete piers in the seismic vault of the Schiltach observatory in order to measure relative horizontal displacements of these piers during the aging of the concrete. These new seismometer piers are about 25 cm high and are cemented directly on the granit

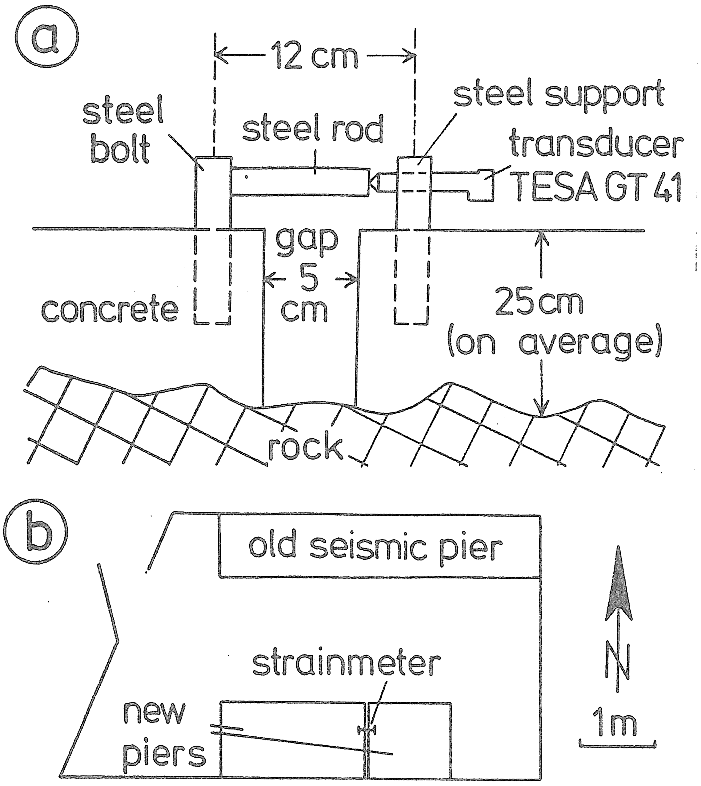


Figure 2: a.) Sketch of the 12 cm-strainmeter between the seismic piers (not exactly scaled). b.) Position within the seismic vault.

floor of the Schiltach seismic vault (Fig 2b). The ends of the strainmeter were each bolted to the concrete at a distance of 3.5 cm from the edges of the piers, so that the distance between them amounted to 12 cm.

3 Tidal results

3.1 General

As already mentioned the records of both instruments showed besides a certain drift clear signals with tidal periodicities (Fig.3). The temperature in the Schiltach mine has a stability at the level of some mK (see MÄLZER et al., 1979). Therefore we exclude temperature effects to be a major error source.

3.2 Data Analysis.

Data from both instruments were recorded on analog chart recorders for several weeks, and then a few days of the best data were digitized by hand with a sampling interval of 1 hr together with the local air pressure. The directly measured quantity is the relative displacement of the two end mounts projected onto the axis of the transducer. Converting this quantity into linear strain is not a simple matter for such short instruments because azimuth and baselength of the 'strainmeter' are not clearly defined. In general, the apparent

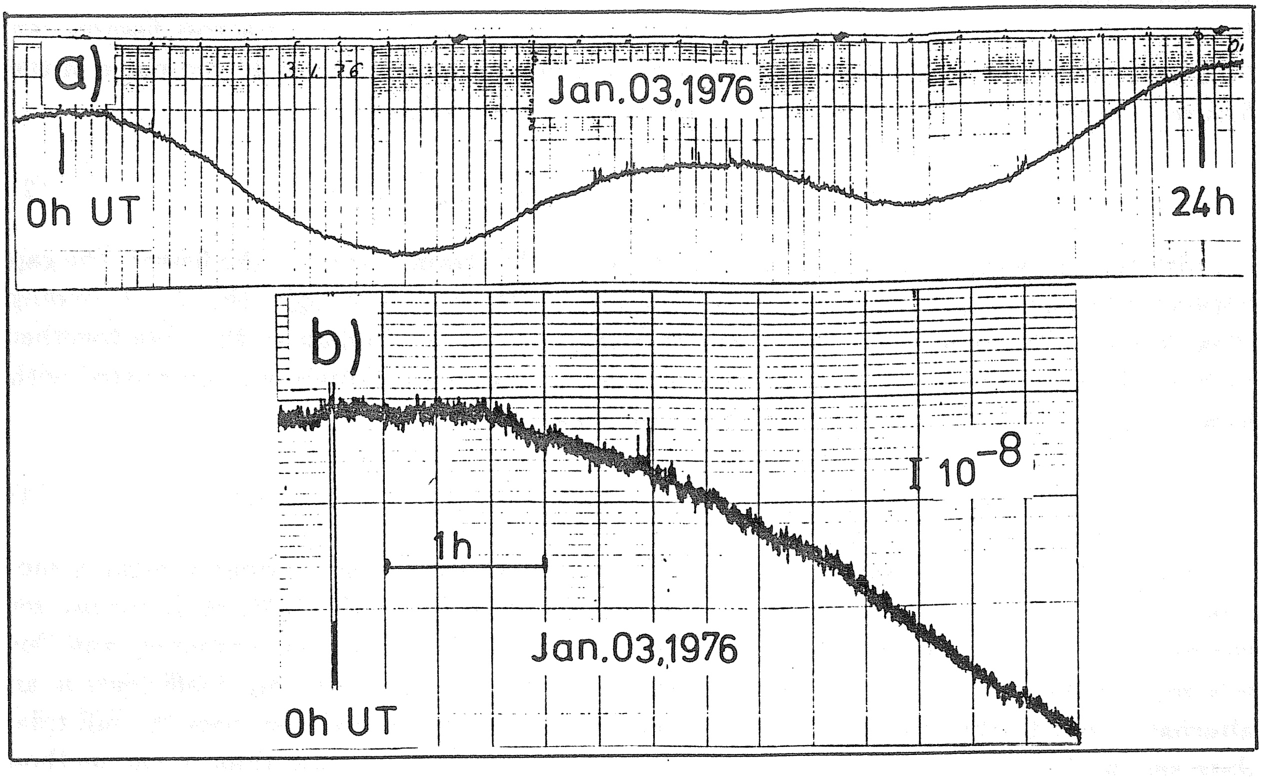


Figure 3: Examples of the analog record of the niche-strainmeter: a.) One full day with one tidal cycle. b.) Enlarged section with scale.

strains obtained by dividing the measured displacements by some characteristic distance l_0 of the installation are not necessarily elastic strains related by Hooke's law to stress components in the material. This is especially true for measurements across gaps as in the case of our seismic piers or the case of a large cleft as reported by BEAVAN et al.(1979). We choose the distance between the bolts as the baselength l_0 of the two instruments. Nevertheless, these displacements are caused by the tidal stresses and since there is no evidence for a nonlinear tidal response of the earth (AGNEW, 1981) they must be a linear function of the tidal stresses at distance (KING et al., 1976; BERGER and BEAUMONT, 1976; AGNEW, 1986). Therefore the tidal displacements measured can be expressed as a linear combination of the only three independent tidal strain components $\epsilon_{ij}(t)$ that exist at distance under near-surface-conditions:

$$\frac{\Delta l}{l_0}(t) = A_{\theta\theta} \cdot \epsilon_{\theta\theta}(t) + A_{\lambda\lambda} \cdot \epsilon_{\lambda\lambda}(t) + A_{\theta\lambda} \cdot \epsilon_{\theta\lambda}(t) \qquad (1)$$

where the A_{ij} are the so-called strain-strain coupling coefficients. They provide a mapping of the tidal strains of a simple earth model (with or without ocean tides) onto the measured local deformation (AGNEW, 1986) and depend strongly on the installation. Basically the $\epsilon_{ij}(t)$ could also be taken to represent the strain components in the vicinity of the heterogeneity under investigation. But then assumptions of free-surface condition and only three independent components of the strain are violated and these components would have to be modelled from the strains at distance and the larger scale inhomogeneities.

The $\epsilon_{ij}(t)$ ideally would contain the effect of the ocean tides but since we only want to get an idea of this coupling, strains computed for an earth model without oceans were used which is a shortcoming of this method. Previous investigations about strain enhancement across fissures (BEAVAN et al., 1979) have shown that displacements occuring across fissures can best be modelled by strain components in a coordinate system referred to the plane of the fissure. Therefore the measured displacements will be modelled by:

$$\frac{\Delta l}{l_0}(t) = A_{pp} \cdot \epsilon_{pp}(t) + A_{ll} \cdot \epsilon_{ll}(t) + A_{lp} \cdot \epsilon_{lp}(t) \tag{2}$$

with the following tidal strain components: $\epsilon_{pp}(t)$ the strain normal to the fissure (the gap between the piers), $\epsilon_{ll}(t)$ the strain along the fissure (the gap) and $\epsilon_{lp}(t)$ the corresponding shear strain. This model was least squares fitted in the time domain to the data together with a drift polynomial. For both cases it was found that the residuals were correlated with local air pressure p(t); therefore our final model was:

$$\frac{\Delta l}{l_0}(t) = A_{pp} \cdot \epsilon_{pp}(t) + A_{ll} \cdot \epsilon_{ll}(t) + A_{lp} \cdot \epsilon_{lp}(t) + A_p \cdot p(t) + a_0 + a_1 t + a_2 t^2. \tag{3}$$

The tidal strain components on an elastic, spherical earth were computed using a modified version of LONGMAN's (1959) algorithm,which was thought sufficiently precise for the purpose here. The strains caused by the ocean tides were neglected completely and this is a more serious shortcoming. This method of determining the coupling coefficients is an alternative one to that proposed by KING et al.(1976). The method here uses the full tidal data set (all harmonics), which results in a better signal-to-noise ratio than in the method of KING et al.(1976) which uses two tidal constituents only. However, in the latter method, ocean load effects can more easily be taken into account. For records as short as ours here the latter method cannot be used because at least a month of data is required for adequate separation of the major constituents in a tidal analysis.

3.3 Results

3.3.1 Fissure in the tiltmeter niche

For the instrument across the fissure the baselength was chosen to be 6.8 cm, the distance between the centers of the end mounts which were cemented to the rock (see Fig.1). A short, typical (i.e.not the worst) record section of this instrument on the analog chart is shown in Fig.3. The results of the least squares analysis using the model of Eq.3 are listed in Table 1 along with the formal uncertainties.

The time series shown in Fig.4 from top to bottom are: $\epsilon_{lp}, \epsilon_{pp}, \epsilon_{ll}, \Delta l/l_0(t)$ with linear and parabolic drift subtracted, air pressure p and finally the post-fit residuals r (units are $n\epsilon$ except for barometric pressure (0.1 hPa)). The table shows that only the linear drift coefficient is less than the standard deviation, so it cannot be claimed to be different from zero. For this strainmeter, the drift was resonably small compared to the tidal signal, it amounts to about $0.6 \cdot 10^{-6}$ /a. A priori we expected the coupling coefficient A_{pp} for the strain parallel to the strainmeter, i.e. perpendicular to the fissure to be significantly larger than the others, but this is not found in the results. The reason for this expectation was a simple one: normal strain on the crack should lead to a larger displacement of the opposite sides of the crack than is expected in any other direction (see BEAVAN et al., 1979). This

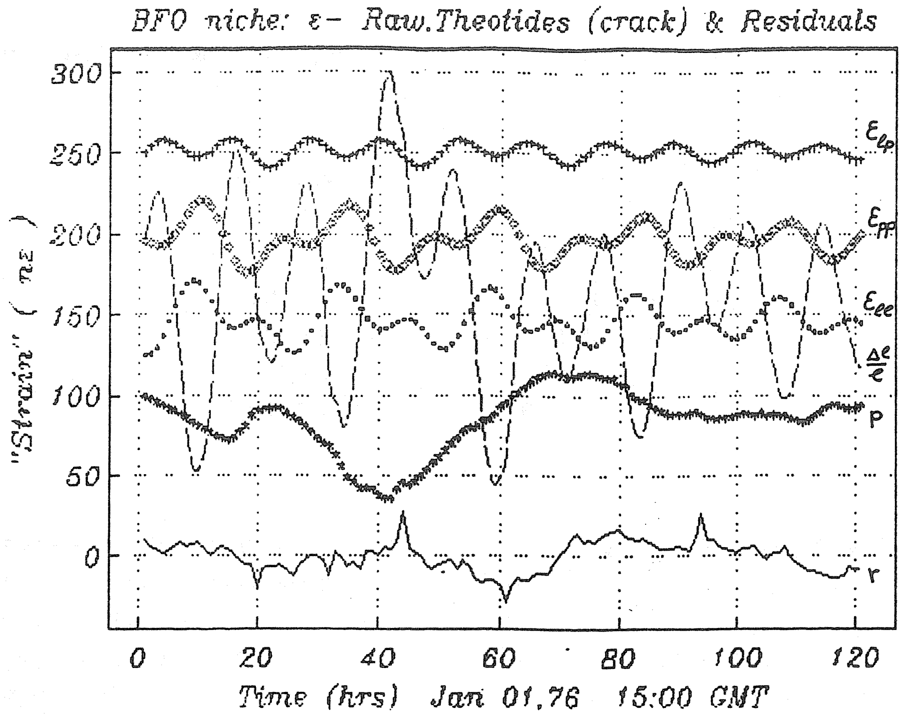


Figure 4: See text.

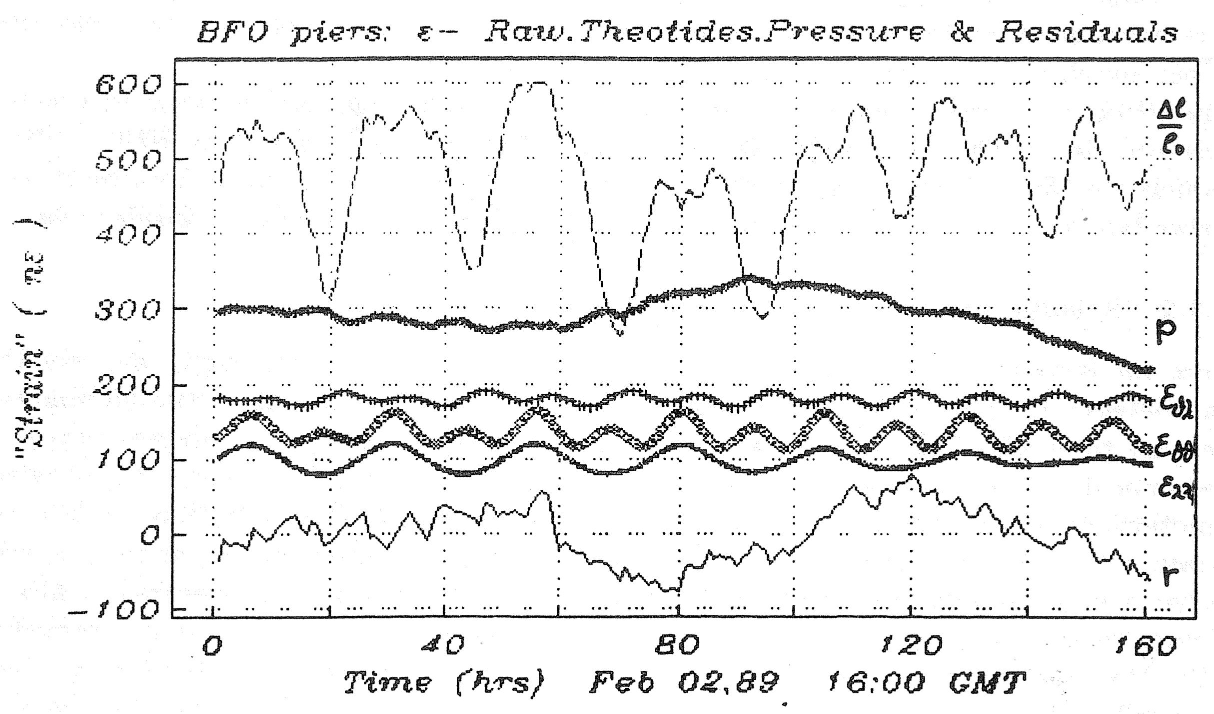


Figure 5: See text.

Table 1: Results of least squares analysis of the strainmeter across the fissure in the niche. Time series of 121 hrs from Jan. 01, 1976, 15:00 GMT ($\Delta t = 1$ hr).

coefficient	units	stand.dev.
A_{pp} = -2.05	-	±0.11
A_{ll} = -2.39	-	±0.11
A_{lp} = +4.77	-	±0.20
A_p = -5.56	$n\epsilon/hPa$	±0.25
a_1 = -0.073	$n\epsilon/hr$	±0.10
a_2 = -0.0096	$n\epsilon/hr^2$	±0.0008

idea was supported by an additional experiment. The same strainmeter was set-up along the same azimuth on compact rock next to the fissure and tidal signals did not show up above the noise. However, with this simplistic conjecture we did not take into account, that the local strain field in the vicinity of the crack is already modified by other inhomogeneities (see EMTER and ZÜRN, 1985). There are severe effects at successively smaller scales distorting the strain field used as the input to the coupling matrix: Geology (Rhinegraben), severe topography in EW-direction, the cavity and finally the niche. To show, as in BEAVAN et al.(1979) that the strain across the fissure is enhanced, one would first have to model the effects of the other heterogeneities. This, however, would be a major task with many uncertainties. The mapping by the coefficients A_{ij} refers to tidal strains on a spherically symmetric earth model (strains at distance) and therefore cannot simply have the properties of our simplistic conjecture; i.e. A_{pp} being the largest coefficient.
The effect of barometric pressure is negative, i.e. we observe compression when barometric pressure rises. This is probably caused by an effect studied in detail by HEIL (1985), namely that for a certain period range the interior of the mine due to the air lock reacts like a standard barometer and is deformed by the difference between outside and inside pressure.

3.3.2 Seismic piers

Here the distance of 12 cm between the bolts was chosen as the baselength and not the gap between the piers which is 5 cm wide. The geometry in this case is a little bit simpler, because the walls of the seismic vault, the piers and the strainmeter are aligned nearly in the same direction, i.e. with the principal geographical directions. Fig.5 shows the following functions of time for the analyzed time series of this instrument (top to bottom): Observed strain with drift subtracted, barometric pressure p, the tidal strains $\epsilon_{\theta\lambda}, \epsilon_{\theta\theta}$ and $\epsilon_{\lambda\lambda}$ as well as the postfit residuals r (all units are strains in $n\epsilon$ except for barometric pressure (0.1 hPa)). Comparison of the observed 'strain' with the theoretical strain components shows immediatelly, that the observed tidal curve had no resemblance with any of the theoretical ones, especially not with the EW-strain which was expected to be the major signal. As a matter of fact, the curve shows the typical modulation pattern of tidal gravity in these latitudes. Therefore it was clear from the beginning, that probably all theoretical strain components couple significantly into the observations. This result can be seen in Table 2. $A_{pp}(A_{\lambda\lambda})$ is still the dominant coefficient as expected from the simplistic conjecture above, but only about a factor of two larger compared to the other strain components. For an explanation of the other significant contributions the arguments of 3.3.1 hold here also.

Table 2: Results of least squares analysis of the strainmeter between the seismic piers. Time series of 161 hrs from Feb. 02, 1989, 16 GMT ($\Delta t = 1$ hr).

coefficient			units	stand.dev.
$A_{ll} = A_{\theta\theta}$	=	-3.71	-	±0.34
$A_{pp} = A_{\lambda\lambda}$	=	+8.75	-	±0.41
$A_{lp} = A_{\theta\lambda}$	=	-5.13	-	±0.63
A_p	=	-8.29	$n\epsilon/hPa$	±1.79
a_1	=	-64.0	$n\epsilon/hr$	±0.38
a_2	=	-0.0055	$n\epsilon/hr^2$	±0.0025

The pressure coefficient has the same sign and the same order of magnitude as in the previous case. The drift in this case was 2000 times larger and showed increasing compression. The concrete piers were built only four weeks before these data were obtained. Under the conditions of 100% relative humidity in the mine concrete is expected to expand (AGNEW, 1986), qualitatively one therefore expects a shortening of the distance between the piers.

4 Conclusions

The results of these investigations have demonstrated that tidal strain can be amplified by small scale inhomogeneities such that it can easily be measured with low-resolution short strainmeters. These observations demonstrate again the omnipresence of tidal deformations, elastic strains or merely displacements. The deviations of the sign and magnitude of the measured coupling coefficients from unity are large for both cases. We think that this is typical rather than the exception. At small scales it is very hard to model elastic effects that may result in enhancements of tidal strain or in very local tilts. Spatial variability is very high at small range but probably existing at all scales, because there is no apparent break in the spectrum of lateral inhomogeneities.

In contrast to the results here, BEAVAN et al.(1979) found for a huge vertical cleft in the BFO that the strain component normal to the cleft had the strongest influence. Using observed strains in the neighbourhood this could be modelled theoretically. This is not possible for the results presented here as the situation is much more complicated. The tidal strain field is not only distorted by large scale effects of geology and topography (see EMTER and ZÜRN, 1985) as in the case of the cleft problem but furthermore heavily distorted by the cavity effects in the vaults. In the case of the fissure in the niche we have the additional influence of the niche itself so that all the elastic effects cannot be modelled with a reasonable effort and with confidence. The fact that the strain components normal and parallel to the fissure have an unexpectedly smaller influence on the measured displacement between the two sides of the fissure than for instance shear strain shows that the situation is complicated indeed.

The seismic piers are cemented directly on the floor of the seismic vault (Fig.2). The tidal strain at their base should correpond to that at the floor of the vault which cannot be modelled for the same reasons as above but is not expected to be largely enhanced (HARRISON, 1976). The enhancement of tidal strain perpendicular to the gap at the top of the piers ($A_{\lambda\lambda}$) could be caused partly by tilting of the piers due to the tidal strain at their base. The fact that other components of the strain tensor contribute in nearly the same way to measured displacement shows, however, that such a model is too simplistic.

Acknowledgements

We thank H.Mälzer and W.Großmann for continuous support and help. P.Rydelek and H.-G.Wenzel critically read the text. H.-G.Wenzel helped finishing the manuscript. All this is gratefully acknowledged.

References

AGNEW, D.C., 1981: Nonlinearity in rock: Evidence from earth tides. J. Geophys. Res., **86**, 3969-3978.

AGNEW, D.C., 1986: Strainmeters and tiltmeters. Rev.of Geophysics, **24**, 579-624.

BEAVAN, J.; BILHAM, R.; EMTER, D.; KING, G.C.P., 1979: Observation of strain enhancement across a fissure. Deutsche Geod.Komm., **B 231**, 47-58.

BENIOFF, H., 1959: Fused-quartz extensometers for secular,tidal and seismic strains. Bull. Geol. Soc. Am., **70**, 1019-1032.

BERGER, J.; BEAUMONT, C., 1976: An analysis of tidal strain observations from the United States of America: II.The laterally inhomogeneous tide. Bull. Seism. Soc. Am., **66**, 1821-1846.

BILHAM, R.G.; KING, G.C.P.; EMTER, D., 1974: Kilometre and centimetre earth strainmeters. Nature, **249**, 25-26.

EMTER, D.; ZÜRN, W., 1985: Observations of local elastic effects on earth tide tilts and strains. In 'Earth Tides' (Ed.J.C.Harrison), Benchmark Papers in Geology Series, Van Nostrand Reinhold, New York, 309-327.

GLADWIN, M.T., 1984: High-precision multicomponent borehole deformation monitoring. Rev. Sci. Instr., **55**, 2011-2016.

HARRISON, J.C., 1976: Cavity and topographic effects in tilt and strain measurements. J. Geophys. Res., **81**, 319-328.

HEIL, E., 1985: Untersuchungen von Vertikal-Extensometermessungen im Observatorium Schiltach. Dissertation Univ.Karlsruhe, 141p..

KING, G.C.P.; ZÜRN, W.; EVANS, R.; EMTER, D., 1976: Site correction for long period seismometers, tiltmeters and strainmeters. Geophys. J. R. astr. Soc., **44**, 405-411.

LONGMAN, I.M., 1959: Formulas for computing the tidal accelerations due to the moon and the sun. J. Geophys. Res., **64**, 2351-2355.

MÄLZER, H.; KIESEL, H.; ZÜRN, W.; EMTER, D.; SCHLEMMER, H.; OTTO, H., 1979: Gezeitenneigungsmessungen im Geowissenschaftlichen Gemeinschaftsobservatorium Schiltach. Deutsche Geod.Komm., **B 245**, 41p..

MENGLIN, C.; MINGJIN, L., 1986: Observation of linear strain of earth tide by capacitive-type borehole earth strainmeter at Xiangshan and preliminary results of harmonic analysis. Proc.10th Int. Symp. Earth Tides, Madrid 1985 (Ed.R.Vieira), Cons. Sup. Invest. Cient., Madrid, 281-286.

PETERS, J.A.; SYDENHAM, P.H., 1974: Solid tides recorded with 1 m interval mechanical strainmeter. Nature, **250**, 43-44.

SATO, T.; HARRISON, J.C., 1990 : Local effects on tidal strain measurements at Esashi, Japan. Geophys. J. Int., **102**, 513-526.

SYDENHAM, P.H., 1969: A tensioned wire strain seismometer. J. Sci. Instr., **2**, 1095-1097.

VAN VEEN, H.J.; SAVINO, J.; ALSOP, L.E., 1966: An optical Maser strainmeter. J. Geophys.Res., **71**, 5478-5479.

WILHELM, H., 1982: Dispersion effect in earth tide parameters. J. Geophysics, **51**, 147-149.