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Working Group on High Precision Tidal Data Processing

Bonn Meeting, october 4-6, 1988

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Preface

During the 10th International Symposium on Earth Tides in Madrid, September 1985, the new working group was formed with the name

'Working Group on High Precision Tidal Data Processing'

According to the resolution no. 8 the tasks of this working group were to examine the introduction of new advances in computing technology and instrumentation into tidal data processing and to report back to the next meeting of the Permanent Commission on Earth Tides.

Therefore, the group met already in October 1986. The results of this meeting are published in BIM 99, 1987, and a report was prepared for the meeting of the PCET during the IUGG General Assembly in Vancouver, August 1987.

Now, a second meeting of the working group took place at the Institute for Theoretic Geodesy, University of Bonn, October 4 - 6, 1988. Again Prof. M. Bonatz was our host and provided good conditions for the meeting of the members of the group and other interested scientists. In all more than 28 scientists from 11 countries were present.

The content of this volume covers the topics of the meeting. The conclusions drawn are printed at first place. They allow an overview over the following papers.

As the chairman of the working group I wish to thank all participants for their contributions and Prof. Bonatz for his hospitality. We all acknowledge the support from the 'Deutsche Forschungsgemeinschaft'. I thank Prof. Melchior for publishing all the material in the Bulletin of the International Center of Earth Tides.

Gerhard Jentzsch

Conclusions

drawn at the meeting of the working group on

HIGH PRECISION TIDAL DATA PROCESSING

Bonn, October 4 - 6, 1988

1. Instrumentation (general)

Since the feedback system for LaCoste-Romberg (LCR) G and D meters is now available it should be compared to the ET meters as well as to other feedback electronics.

First encouraging results have been achieved with the feedback of ASKANIA gravimeters by the group in Madrid. This efforts should be continued and supported by a subgroup consisting of our colleagues G. Montes, M.v. Ruymbeke and Z. Simon.

The influence of humidity changes inside gravimeters should be observed.

The group realizes the progress made in Brussels regarding the calibration of gravimeters applying a lift. The development of this method should be continued with regard to the application in the observation site and it should be compared with other methods.

The range of the feedback of LCR gravimeters should be increased.

Sensors of environmental conditions should be compared to standards frequently.

2. Superconducting Gravimeter

We acknowledge the recommendation of SSG.5 regarding the calibration. Up to now absolute gravimeters are only useful for the check of the long-term drift.

Data acquisition should be carried out with at least 5.5 digits (better 6.5 digits) averaging over 10 seconds providing one sample every 10 seconds.

For pressure recordings at the 0.1 millibar accuracy a mercury standard should be considered.

The timing accuracy should be better than 0.1 second.

The local groups are responsible for the control of environmental conditions like temperature, humidity, ground water in at least hourly values.

The tilt of the pillar should be recorded in order to check tilt compensation and stability.

3. Tidal development

We realize the progress made since the last meeting. The activities of the subgroup should be continued including B. Ducarme. The comparison of the developments of Q.W. Xi and Y. Tamura should be repeated on the basis of more precise ephemeris.

4. Tidal recording and analysis

We realize the importance of long-term recordings with one gravimeter (like in Potsdam). These measurements should be continued since they are very valuable for studies of time variations of body tides and loading tides.

5. Tidal residuals / ocean loading

The development of local shelf models should be continued. These models should be made available to the ICET data bank.

The calculations of loading effects should be based upon Schwiderski's maps refined by local models and applying a Green's function derived from the PREM earth model including a local crustal structure.

6. Space techniques

We realize the future possibilities of space techniques in tidal research.

The results of space techniques should be compared with local ground techniques for the separation of different effects of ocean and earth tides.

7. Data bank

We realize the contributions of colleagues to this data bank. Still further contributions are needed, esp. with regard to load vectors.

Local cotidal maps should be provided.

8. Next meeting

The group will meet again during the Eleventh International Symposium on Earth Tides, Helsinki, July 31 - August 5, 1989.

Calibration and Stability of the Gravimeter LCR ET 18

by

G. Jentzsch and J. Melzer*

1. Introduction

The gravimeter LaCoste & Romberg ET 18 was delivered with mechanical feedback in 1974; in 1983 it was transformed to electrostatic feedback.

The instrument was installed in Germany, England and Fennoscandia ; since it was used for high precision tidal measurements the calibration and stability of this instrument are of special interest.

In the following some of the results are given and discussed.

2. Short installation history

In 1974 the gravimeter - LaCoste & Romberg ET 18 - was delivered to the University of Clausthal, Geophysical Institute. After tests it recorded for dynamical calibration in Bidston/England (June to December 1975) and in Hannover (June 1976 to May 1978). Both records and an additional tilt-table calibration resulted in a calibration factor which is 1.28% larger than that given by the manufacturer (0.07881 $\mu\text{gal}/\text{cu}$ instead of 0.07781 $\mu\text{gal}/\text{cu}$).

From May 1978 the gravimeter was recording at the Tidal Observatory in Berlin, and from April 1980 until April 1983 it was used in the 'Blue Road Project' in Norway and Sweden (Asch et al., 1987).

In summer 1983 the gravimeter was converted to electrostatic feedback and supplied with electronic levels by J. Larson (Maryland Instrumentation). Another record was carried out in Berlin to check and recalibrate the instrument (February 1984 to April 1985).

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At the end of May 1985 the first statical calibration was carried out at the vertical base-line Hannover, just before the gravimeter was installed in the limestone mine Lohja in Southern Finland where the long water-tube tiltmeters of Kääriäinen (1979) are recording.

After a second calibration at the Hannover base-line in November 1986 the ET 18 was used in the tidal gravity project in Denmark (Jahr, 1989) until February 1987 to serve as a reference for two ASKANIA gravimeters.

In March 1987 the third run at the Hannover base-line was carried out and since that time the instrument is recording in Metsähovi/Finland (close to Lohja) together with a borehole-tiltmeter of the ASKANIA type to form a 3-component-station.

3. Data acquisition

The mechanical feedback includes the recording of the turns of the spindulum applying an angular encoder. A range of about 1.6 mgal covering about 20% of the total range (8 mgal) of the measuring-screw was digitized with a 14 bit recorder thus providing a resolution of about 0.1 μ gal/bit. According to the frequency characteristics of the feedback 1 minute samples were recorded without additional filtering.

After the conversion to electrostatic feedback the recorder had to be changed, too: An analog filter/amplifier replaced the counting unit. We chose the cut-off-period of the active 4-pole filter to about 60 seconds and recorded 30 seconds samples. The free oscillation channel was separated by an analog band-pass with a range from about 2 minutes to 2 hours and an amplification factor of 60.

In Metsähovi we use now a high-resolution computer controlled data acquisition system with an A/D-converter of 6 1/2 digits equivalent to 22 bit. Thus, by integrating over 10 seconds and by numerical filtering we record 20 seconds samples without any further analog data treatment. Tides are resolved with about 18 bit, and seismological signals and free oscillations of the Earth still cover about 12 bit.

4. Calibration

Before the conversion to electrostatic feedback the calibration factor of 0.07781 $\mu\text{gal}/\text{cu}$ given by the manufacturer was redetermined to 0.07881 $\mu\text{gal}/\text{cu}$ (+ 1.28%) by parallel recording in Bidston/England and by a tilt-table investigation in Hannover by Wenzel (Rosenbach, 1978); no phase shift was observed.

The corrected calibration factor was used for the analysis of the record taken at the Tidal Observatory in Berlin (1978 - 1980; Asch et al., 1987). Results were compared with the tidal parameters obtained in Potsdam (Asch et al., 1986a) where the gravimeter GS 15-222 was used and independently calibrated. It turned out that the tidal parameters there were nearly 1% smaller than those obtained with the ET 18 in Berlin.

After the conversion to electrostatic feedback the ET 18 recorded again in Berlin for dynamical calibration (April 1984 to May 1985). The calibration factor was determined by averaging over the amplitude ratios of the four wavegroups O1, P1S1K1, M2, S2K2 weighted by their respective signal-to-noise ratios. In addition the instrument was calibrated three times at the vertical base-line Hannover.

The feedback voltage (maximum operation range is ± 10 V) comprises about 20% of the spindulum range equivalent to about 20,000 counter units. The total range of the spindulum (99,000 cu) covers a little less than 8 milligals. Thus, the applicable gravity difference used is 6.1403 mgal between points 190 and 250 of the staircase at the vertical base-line Hannover.

Each measurement was taken 5 minutes after completing the setup. The electronic levels were used to repeat identical setups. The resolution is 50 times that of the liquid bubbles (± 0.3 arcsec compared to ± 15 arcsec).

Table 1 gives the results of the three calibration runs at Hannover. The error intervals of all three determined calibration factors overlap. The errors decrease with increasing numbers of measured gravity differences, and taking into account these numbers the weighted mean of 0.07932 $\mu\text{gal}/\text{mV}$ has an error of $\pm 0.3\%$.

Table 1: Results of the calibration campaigns at the vertical base-line Hannover

date of calibra- tion	number of measured changes	sensi- tivity	scale- factor	calibration- factor
		mV / cu	10^{-5} mgal / cu	10^{-5} mgal / mV
May 1985	7	.9883	7.8002 $\pm 0.7\%$	7.893 $\pm 0.7\%$
Nov 1986	12	.9786	7.7798 $\pm 0.4\%$	7.950 $\pm 0.4\%$
Mar 1987	18	.9825	7.7985 $\pm 0.3\%$	7.937 $\pm 0.3\%$

Table 2: Scale-factors determined during the calibration campaigns in Hannover (electrostatic feedback) and their percental differences compared to Wenzel and the manufacturer (mechanical feedback)

scale-factors 10^{-5} mgal / cu			
date	Wenzel (7.881)	calibration- line 'Hannover'	LaCoste & Romberg (7.781)
May 1985	-1.03%	7.800 $\pm 0.7\%$	+0.24%
Nov 1986	-1.28%	7.780 $\pm 0.4\%$	-0.01%
Mar 1987	-1.04%	7.799 $\pm 0.3\%$	+0.23%

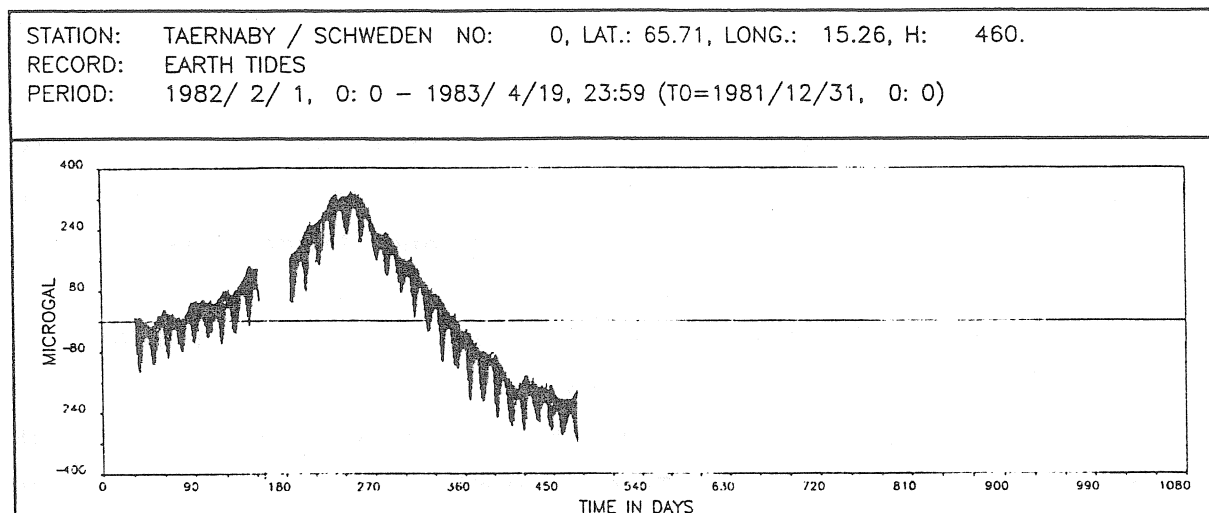


Figure 1: Annual wave in the record of Tärnaby (1981 to 1983)

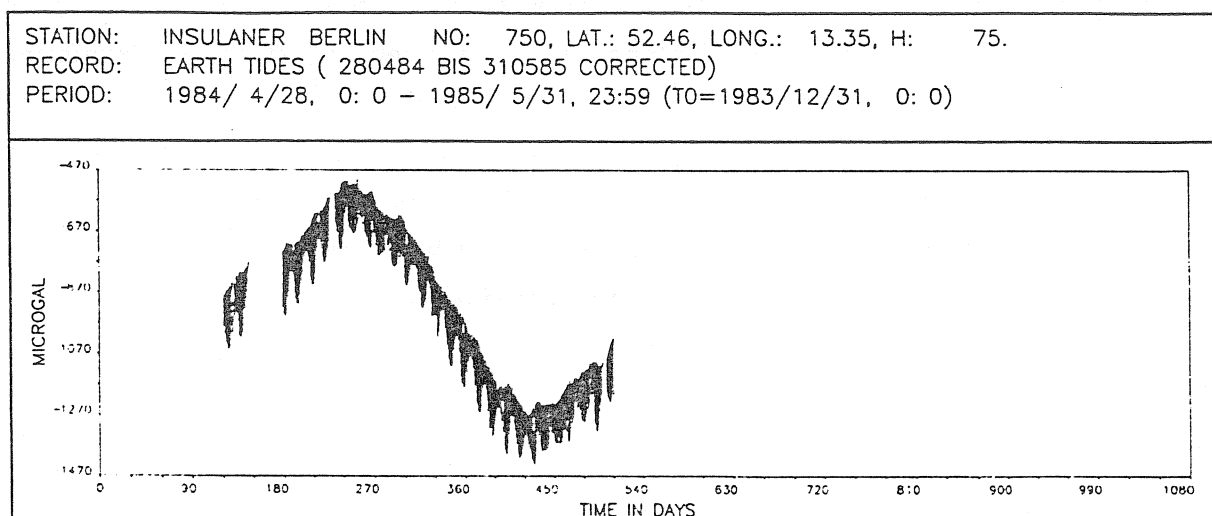


Figure 2: Annual wave in the record of Berlin (April 1984 to May 1985)

In table 2 the comparison of the determined scale factors to the previous calibrations is given. The result is astonishing: The new values are close to those given by the manufacturer. Provided that the previous calibrations are correct this means that our gravimeter has changed by about 1% after the conversion.

5. Stability

The strong annual wave of about ± 350 μgal amplitude was also observed by other authors. Surprisingly, there was a very strong correlation between the annual wave at Berlin and Potsdam (Asch et al., 1986b).

The gravimeter GS 15-222 lost this signal after stabilizing the atmospheric humidity in the measuring room.

The annual wave occurred also at other recording places of the ET 18 showing always the same phase: Tärnaby/Sweden (1981 - 1983, see figure 1), Berlin (1984 - 1985, see figure 2), and Metsähovi/Finland (1987 until now). Even the short records from Lohja/Finland (August 1985 - April 1986) and Copenhagen/Denmark (November 1986 - February 1987) fit into the pattern of that annual wave.

Up to now the reason for this annual wave is still unknown. Some authors have found correlations with atmospheric humidity (compare Bastien & Goodacre, 1989), but also air temperature seems a possible source: During the installation in Metsähovi/Finland we observed a strong correlation between temperature changes of the air and gravity. The records of the levels and the temperatures of the inner and outer heaters showed correlations as well. This gives rise to the idea that thermal gradients produced deformations of the gravimeter box which may have changed the geometry. These findings are still under consideration.

6. Conclusions

Regarding the calibration of our LCR Earth tide meter no. 18 at the vertical base-line in Hannover we can conclude that the accuracy obtained was improved with increasing numbers of readings taken especially during the night. The application of electronic levels was crucial.

Regarding the long-term drift the observed strong annual wave shows a strong coherence at all places, were this gravimeter recorded. This is astonishing because the environmental conditions were different: In some stations we supplied an additional temperature stabilization, but this was not always possible. Now we also record humidity to investigate this question.

7. Acknowledgements

We wish to thank our friends from Denmark (Niels Andersen, Danish Geodetic Institute), Sweden (Kent Hallin, Lantmäteriet) and Finland (Hannu Ruotsalainen, Finnish Geodetic Institute) for the maintenance of the gravimeter and their support. Our former co-worker Günter Asch was responsible for the installation. We acknowledge the funding of all the work by the German Research Soc. (Deutsche Forschungsgemeinschaft).

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Improvements of the vertical oscillating platform
for the calibration of gravimeters

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An inertial force is induced by the oscillating vertical displacement $A \sin(\omega t)$ of a platform supporting a feedback LaCoste Romberg gravimeter (Van Ruymbeke, 1986). The resulting acceleration dg is proportional to the square of the frequency. The effects of the vertical gradient and of the residual tilt are eliminated by operating at different frequencies.

The gravimeter LCR 336 placed on the vertical oscillating platform recorded the gravity tides during one year.

During the first six months we performed only three calibrations to ensure the best stability to the instruments. The variation of the sensitivity found with this method is confirmed by the Nakai procedure. The value obtained for the amplitude factor $\delta(O_1)$ corresponds to a diminution of 1.4 % of the accepted value of the Brussels system. The error is of the order of 0.5 %.

During the second period we performed simultaneous calibrations every ten days. The standard deviation of the repeated calibrations is only 0.2 %. However we noticed that calibration produced jumps of the sensitivity. This section of the records is thus not suitable for the determination of precise tidal factors.

It is clear that the precision of the gravimeter is the main limitation and it was thus decided to improve the quality of the tidal records.

The new feedback system using the Maximum Voltage Retroaction (MVR) principle (Van Ruymbeke, 1989) was implemented on the gravimeter LCR3 placed on the platform.

A new data acquisition system with a one second sampling was used. Minute averages are directly computed and hourly values are obtained by smoothing and interpolation using spline functions.

Results from a two months series show a decrease of 0.8 % for the $\delta(O_1)$ value and of 1.1 % for the $\delta(M_2)$ value.

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Bull. Inf. Marées Terrestres, 103,

New Developments of Feedback Electronics

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1. Introduction

To install a feedback electronics on a gravimeter it is necessary to apply a variable force in order to keep the moving mass at a fixed position.

Two methods have been experimented :

- 1 - the force is produced by the action of a variable magnetic field on a permanent magnet (Orejana & Vieira, 1983).
- 2 - a differential electrostatic force is applied on the plates of the capacitive transducer (Harrison & Sato, 1984; Röder & alii, 1987)

In this case the force is proportional to the square of the modulated voltage difference. It is also possible to replace this force modulation by the action of a constant force applied discontinuously (Vaillant & alii, 1986). This principle is called Pulse Width Modulation (P.W.M.).

Our developments on the LaCoste-Romberg gravity meters showed that a modulated electrostatic excitation allows to measure directly the capacities of the bridge. The built in CPI card is thus no more required.

As shown on figure 1 the voltage drop across the resistor R in series with capacitor C is a function of the capacitor when a modulated signal V is applied.

The use of a "OR exclusive" is a very simple way to obtain a signal directly proportional to the RC constant of the circuit (figure2).

2. The VRL 86 system

A CPI card based on this principle gave very good results (figure 3).

The main advantage of this system is to allow to connect the mass of the power supply to any part of the gravimeter except to the two fixed plates of the capacitive transducer.

To implement an electrostatic feedback system it is enough to replace the primary oscillator CMOS 4060 by a Pulse Width Modulation circuit (figure 4).

The PWM circuit consist in a single CMOS circuit and a transistor. It can be directly installed on the print shown in figure 3.

With a 300 Hz frequency it is possible to measure directly digitally the period of inversion by counting a 10 Mhz oscillator. (figure 5)

These circuits have been tested and a tidal analysis performed on the records of a transformed gravimeter confirming the quality of the system.

However the excursion of the modulation should be limited to $\pm 20\%$ in order to avoid that any asymmetry of the excitation voltage V (figure 4) could perturb the output signal Y producing a non linearity. With the selected components the voltage was limited to 18 volts and the feedback range to about 2 milligal.

On all sides of the range the feedback signal remains constant and the non compensated motion of the mass is detected at the analog output. It makes easier the use of this system for field measurements.

3. The Maximum Voltage Retroaction System VRL 88

During this systematic investigation of the feedback systems many other principles have been tested.

One of them is specially interesting as it allows to increase the feedback voltages up to several hundred volt.

It becomes thus possible to increase very much the feedback range of the gravity meters and eventually to suppress the use of the micrometer for field measurements.

The Maximum Voltage Retroaction (M.V.R.) is a special case of the previous design. If one increases indefinitely the gain of the circuit loops in figure 4, the signal will induce a maximum force on one capacitor and no force on the other or inversely. Let us adapt an electronics witch (figure 6) able to determine during the charge of the capacitors which one has the smallest capacity (The charge of the circuit with the smallest RC value will be quicker). It corresponds to the largest distance between the moving plate and the fixed plates of the capacitive transducer.

The signal will be applied on its terminals until the mass, drawn back by the electrostatic force, reaches and finally overshoots its equilibrium position.

Then the system reverses and the full polarization voltage is applied to the other capacitor to draw back the mass again.

At each inversion of V the application of the full feedback force is governed by the choice of the smallest capacity.

This system is auto stabilized and can be connected with a direct or inverse polarity. It has an infinite gain by definition.

An electronics working with an 18 Volt supply is experimented on the calibrating platform installed at the Royal Observatory of Belgium (Van Ruymbeke, 1989).

The sensitivity is perfectly stable and the precision is at least equal to the electronics precedly developed (Van Ruymbeke, 1985).

A CPI card with a 60 Volt supply allowed to compensate an offset of more than 40 milligal produced by the rotation of the micrometer.

Although a permanent oscillation of the beam is visible in the microscope eyepiece, the recording curve remains normal.

The applied voltages seem to be limited by a technical problem. The connector to the capacitive bridge is very small and the capacitance existing between the two conductors connected to it produces inductions. A dead band is introduced in the decision making process concerning the signals W_1 and W_2 (figure 6). The slight hysteresis so generated is producing a residual oscillation of the acting force. This reaction can be minimized by adjusting some parameters of the VRL 88 CPI card.

A very simple counter (figure 7) can be developed similarly to the circuit of figure 4.

The principle of the maximum voltage retroaction can also be applied to vertical or horizontal pendulums, to high precision balances, to water tubes or to seismometers.

Acknowledgements

This research was supported by the Royal Observatory of Belgium (O.R.B.) and the "Ecole Catholique des Arts et Métiers" at Brussels (E.C.A.M.). The author is indebted to B.Ducarme and R.Laurent for their help. The analysis of the tidal records have been performed at the International Center for Earth Tides (I.C.E.T.).

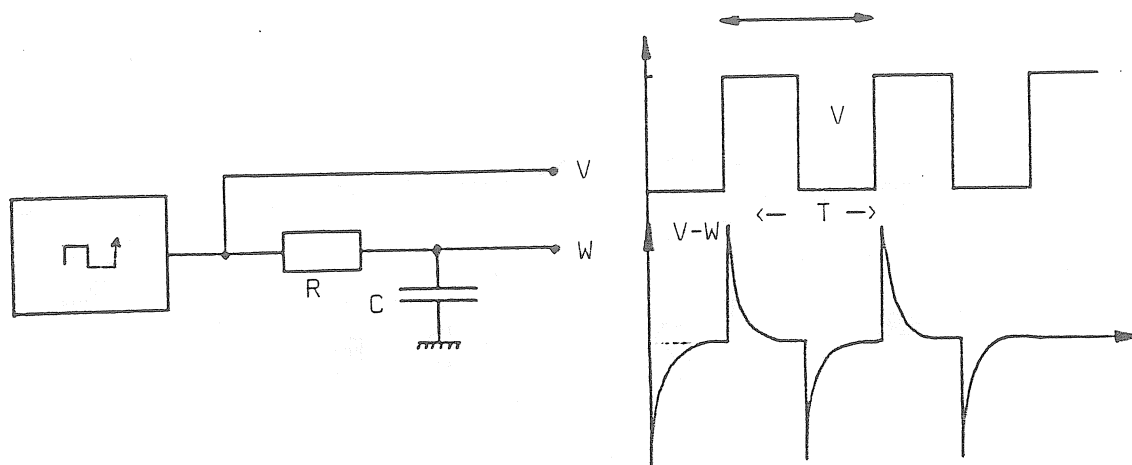


Figure 1 : Voltage drop across a resistor R for a signal V applied on a RC circuit.

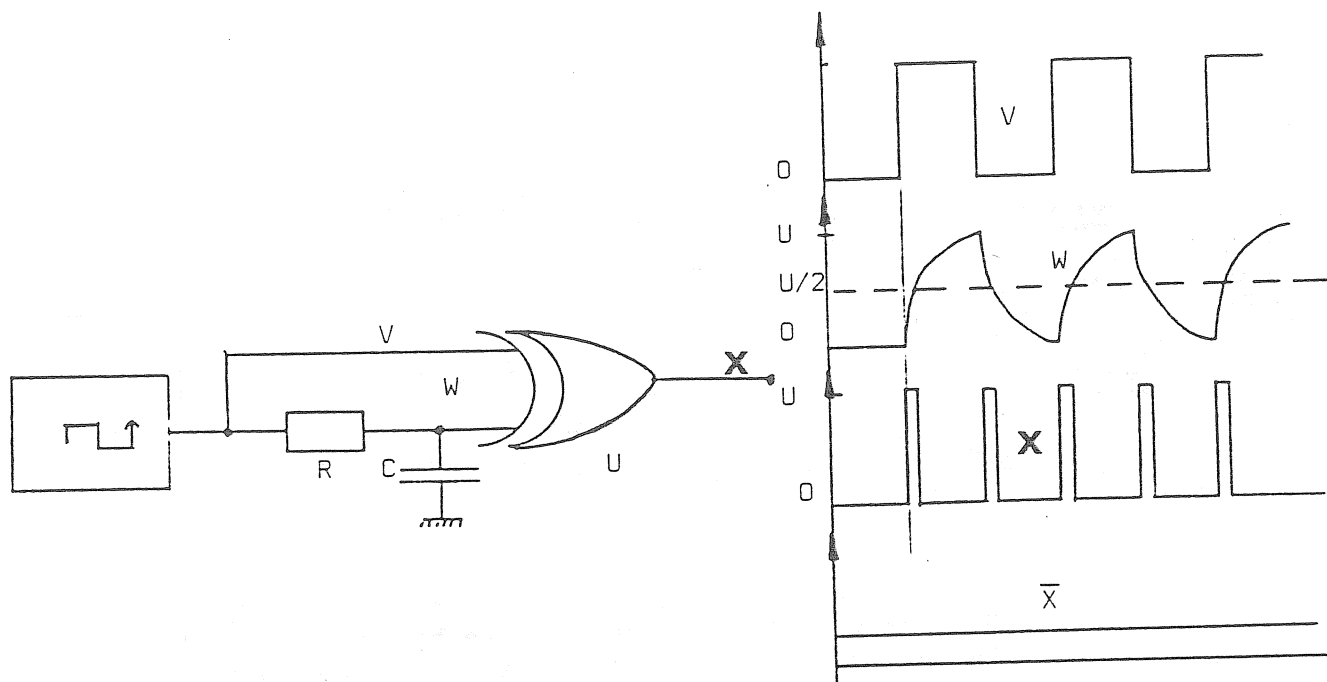


Figure 2 : The output signal X from an OR exclusive is high when V and W are not equal.
Its mean value \bar{X} is thus proportional to the RC value.

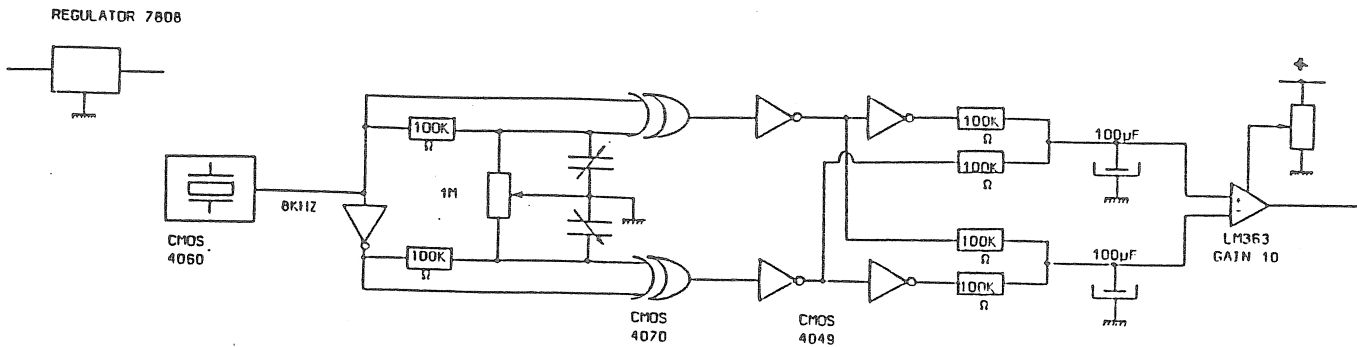


Figure 3 : The VRL 86 CPI card has been installed on LaCoste Romberg model G 487.
The circuit consists only of 5 integrated circuits mounted on a print of 3 x 3 cm².
The 1 MΩ potentiometer allows to adjust the reading line.

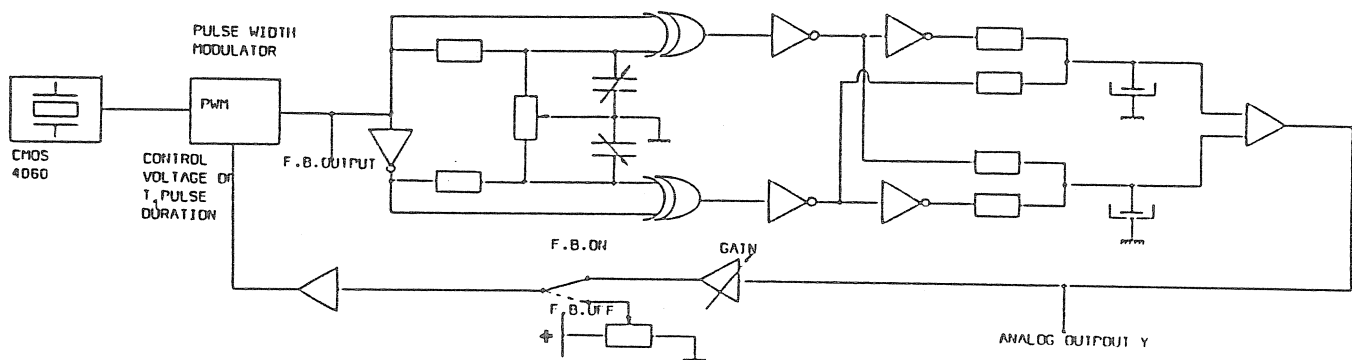


Figure 4 : This CPI card allows to work in normal conditions (FB OFF) as well as to operate in feedback mode (FB ON).
The analog output provides a voltage proportional to the beam offset.
After filtering the F.B. output signal V allows to monitor the true feedback force.

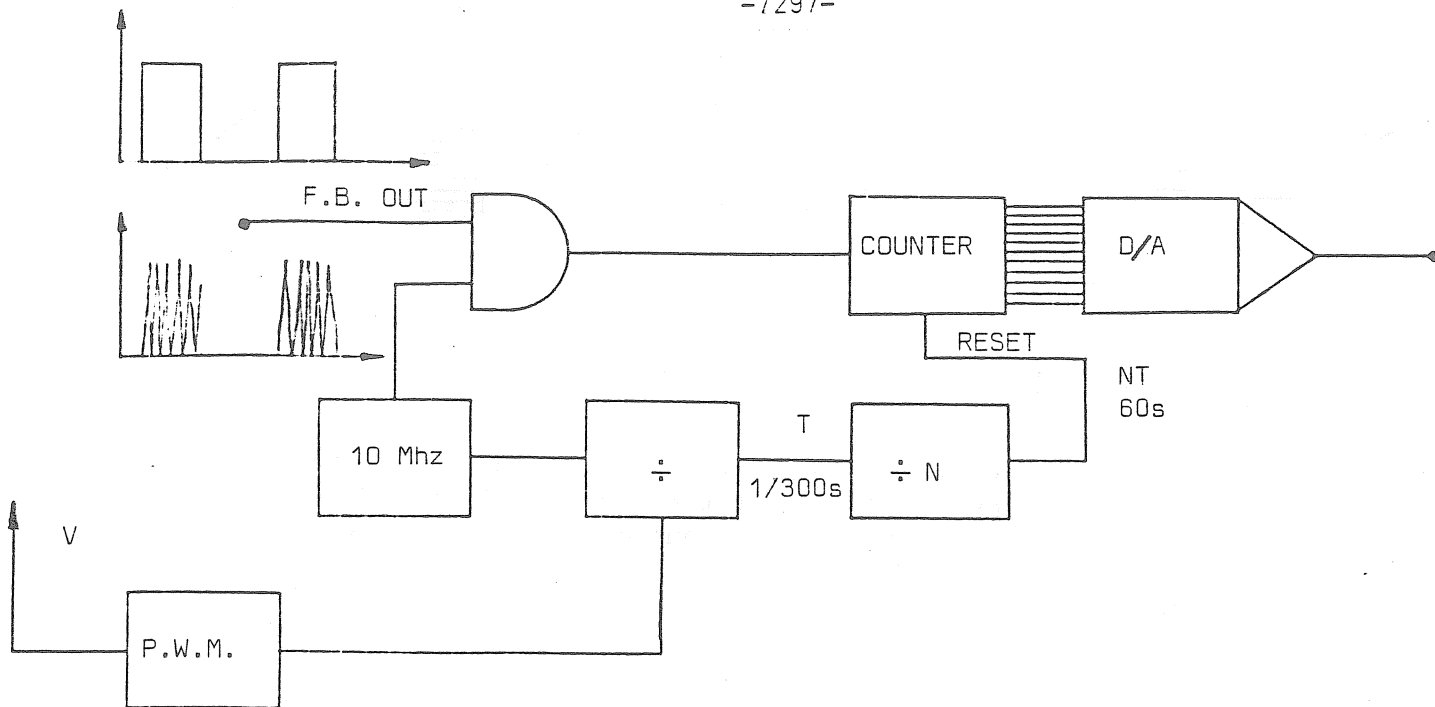


Figure 5 : T is the triggering period of the PWM circuit and NT defines the integration period of the signal (typically 1 minute).

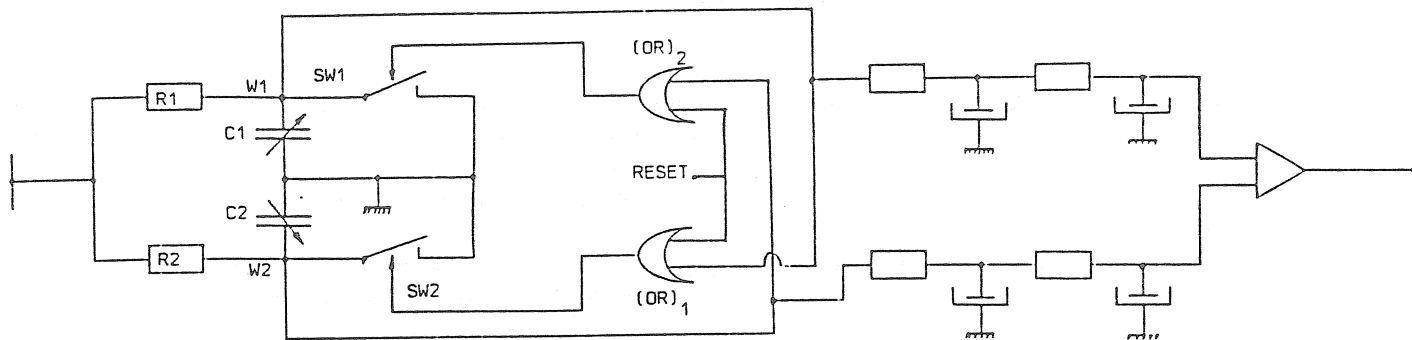


Figure 6 : Description of the Maximum Voltage Retroaction principle.

When the RESET signal is high both SW₁ and SW₂ switches are closed and no signal is present at the terminals of capacitors C₁ and C₂.

Putting RESET back to zero will enable the switches to open and to charge the capacitors through R₁ and R₂.

Let us suppose that the slew rate of W₁ is shorter i.e. R₁C₁ is smaller than R₂C₂. It follows that the (OR)₁ will activate SW₂ before (OR)₂ can activate SW₁. When SW₂ is closed, W₂ drops to zero and will never activate SW₁. From now on the voltages W₁ and W₂ will remain constant until a new RESET pulse closes SW₂ and puts W₁ back to zero. W₁ and W₂ being identical again a new cycle will start. A maximum electrostatic force may thus be successively applied on each branch of the capacitive bridge.

A measure of the ratio of active cycles for C₁ and C₂ will provide a real value of the feedback force.

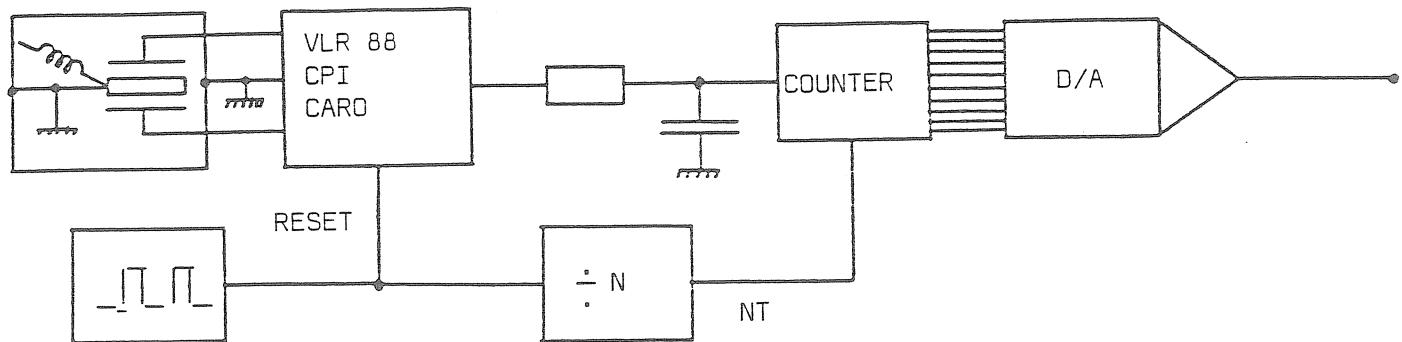


Figure 7 : A digital device used to count how many times during N cycles the SW₁ switch was activated by W₂. This result is transformed into an analog signal directly proportional to the feedback force.

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Resolution and dynamics of tidal recordings in Norway and Finland

by

Günter Asch*

extended abstract:

Geophysical sensors like earth tide gravity meters and borehole tiltmeters belong to the group of most sensitive mechanical devices, being available today. Their main features include wide dynamic range and extreme broadband resolution. These aspects are not always taken into consideration when dealing with analogue recording systems.

Therefore, this work introduces a digital recording system for the above mentioned sensors which includes the modern concept of data acquisition and data processing.

Within the scope of two projects being carried out in Scandinavia, three Askania borehole tiltmeters and one LaCoste&Romberg earth tide gravity meter have been put into operation. In Norway two tiltmeters are recording the loading signal of a reservoir. In Finland, a three component station is recording the whole spectrum of crustal dynamics, ranging from free mode signals of the earth to active crustal deformation. After the mechanical construction of the tiltmeter has been discussed and the expected signal- and frequency range evaluated, model calculations are presented in order to determine the true transfer property of the pendulum around its natural resonance frequency. The introduction of an improved preamplifier stage for the borehole tiltmeter then terminates the part on tiltmeters.

In the following the principle of operation of the LaCoste&Romberg gravity meter and the analogue free mode filter are being discussed in detail. For a broadband data stream dissolving the free mode signal at 72 db, the total dynamic range requires 130 db. For a broader understanding the basics of digital data acquisition, the sampling theorem and what is

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called oversampling are discussed in more detail. Furthermore, a digital recording system with dynamic range of 130 db (at a frequency resolution of 0.02 Hz) and its performance under field conditions is described. In conclusion the configuration of the system as single channel station (gravity meter) or multichannel station (tiltmeter) is demonstrated. In addition to the already existing data channels other channels are available to include meteorological data. Finally, a few original recordings are presented to demonstrate the quality of the raw data sets.

The computation of the noise spectra shows that it was possible to achieve the necessary resolution. The noise amplitudes of the gravity meter at the range of 10^2 - 10^4 seconds were less than 2 ngal/ $\sqrt{\text{Hz}}$ and about 10 ngal/ $\sqrt{\text{Hz}}$ for the pendula.

Parts of this work were published by Asch & Jentzsch (1986) and as a comprehensive study by Asch, (1988).

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Calibration of Askania - Borehole - Tiltmeters

by

A. Weise *

1. Introduction

Three Askania - borehole - tiltmeters are installed in Scandinavia, each equipped with a computer controlled recording system developed by ASCH (1988). An aliasfilter integrates over 10 seconds. The resulting values, one per second, are reduced to a sample rate of 20 seconds by numerical lowpass filtering. The filter is a symmetrical transversal operator of 358 seconds length.

The three tiltmeters are working in two different projects with two fundamental different subjects:

1. Two tiltmeters are installed in the southwest of Norway, near the reservoir Blå Sjø. Mainly we are interested in the longperiod terms of the drift, caused by deformations because of the waterlevel changings of the reservoir.
2. The third tiltmeter, working in Metsähovi in Finland, is part of the three component station, together with the Earth Tide Gravimeter LaCoste Romberg ET 18. Here the aim is to determine the free oscillations of the earth in three components. The second aim is to compare the long term drift with the results of the long water-tubes of KÄÄRIÄINEN (1979) installed near by.

2. Calibration Methods

The calibration is a known topical problem. There are several possibilities of calibration, depending on the instrument:

1. In - situ ball - calibration: moving a ball of known mass is causing a well defined change of tilt for Askania - borehole - tiltmeters, or change of gravity for gravimeters.

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2. Observation along a calibration line, as for gravity for example the vertical calibration lines in Hannover, Darmstadt and Wuchang.
3. Parallel recording with a well calibrated instrument.
4. Calibration with the amplitude of the wave 01.

The in - situ ball calibration is the most elegant and effective calibration method. Up to now the accuracy of calibration of tiltmeters was ± 1 %, corresponding to the noise level of analogous recording. For the Askania tiltmeters the manufacturer gives a possible calibration accuracy of ± 0.1 %.

The questions to be investigated are:

- are the sensors of the tiltmeter as well as the recording system able to yield the requested accuracy of ± 0.1 %?
- will the constancy of the calibration factor also be in this order, or
- are there variations in the calibration factor?

3. Calibration Evaluation and Results

Fig. 1 gives an example of the ball calibration of the tiltmeter P1 in the station Mosvatnet in Norway. The oscillations directly before and after the calibration step are caused by filtering. Thus, there is no rectangular signal, but several values are affected by the filtered calibration, superimposed by tides, drift and meteorological effects. The ball calibration is carried out separately for both channels X and Y.

Calibrating one component, there is a distinct cross-coupling at the other component of 0.3 to 0.5 millivolts, that means 1 to 2 %. The reason for this cross - coupling cannot be explained clearly, yet. Possible is, that the ball in the ball case does not move exactly parallel to the corresponding component and is therefore generating a very small tilt in the other component. A deviation of 0.1 to 0.2 mm could already cause this effect (Fig. 2a).

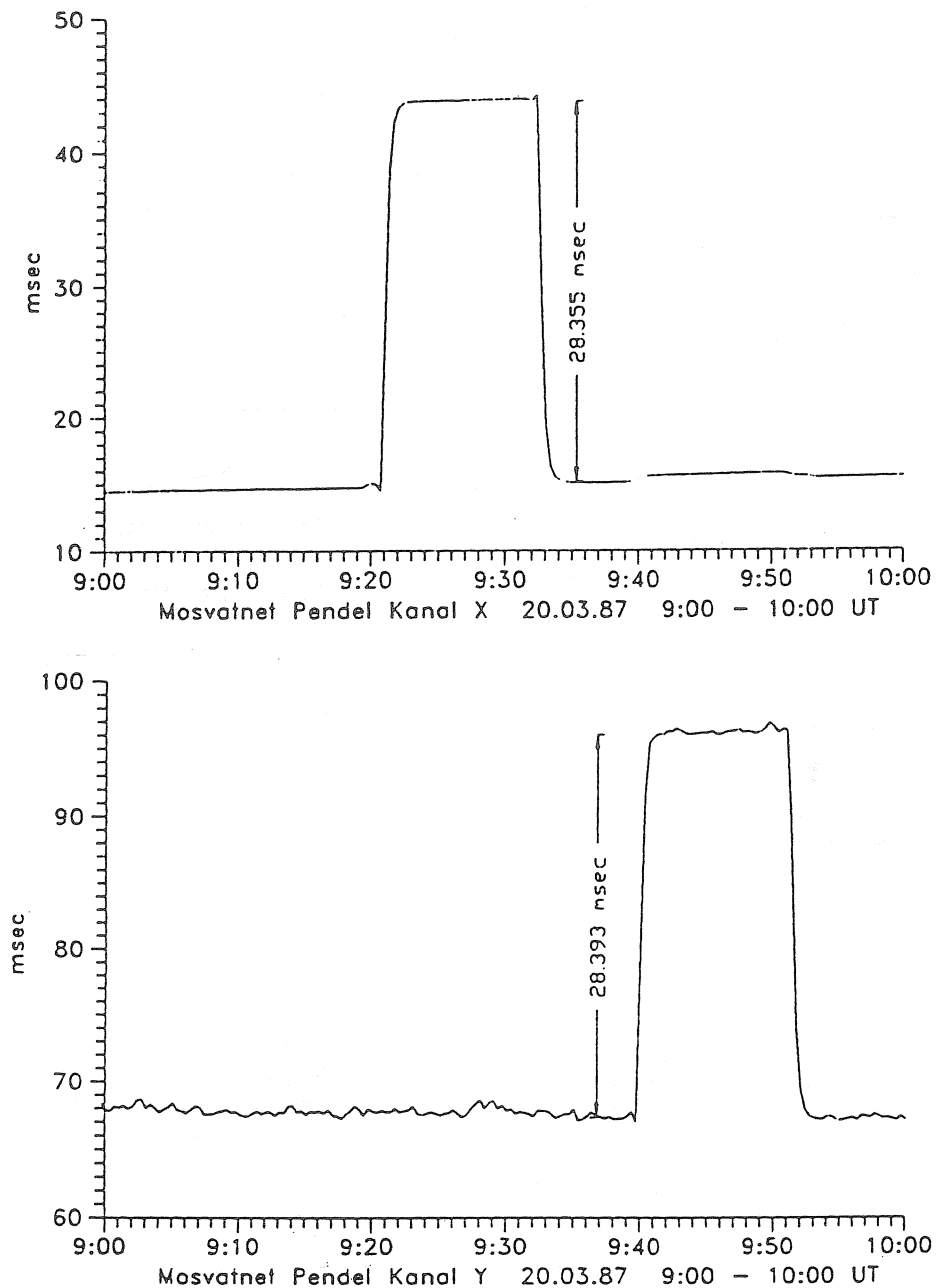


Fig. 1: Ball calibration of tiltmeter P1 in Mosvatnet in the components X and Y

The second possibility is a deviation in the orientation of the ball case and the direction of the tilt component. That would be a systematic effect (Fig. 2c).

The aim is an extensively automatic analysis of calibration pulses, free of the influence of drift and tides. Each flank of the calibration is analysed separately. Frequently there is only one flank useable.

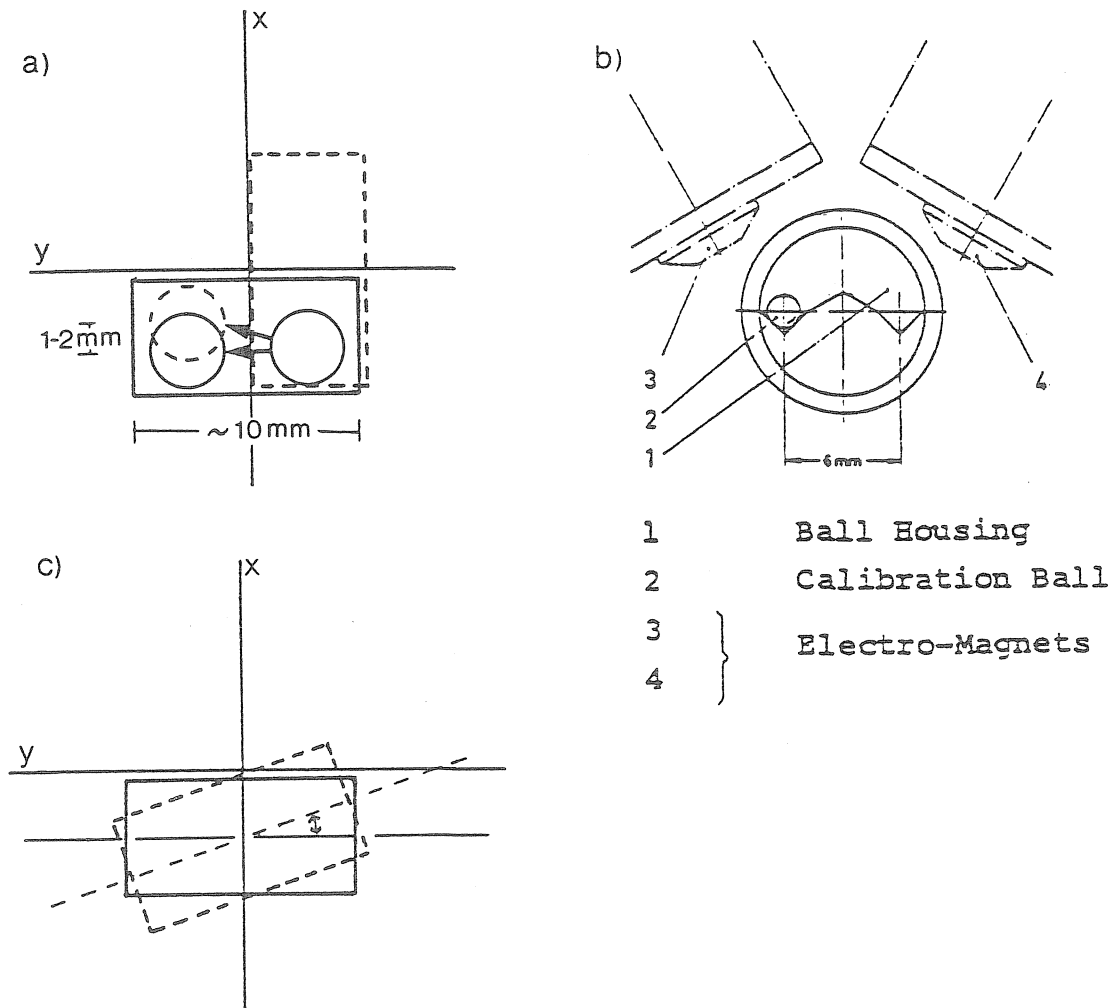


Fig. 2: a) The ball in the ball case does not move parallel to the corresponding tilt component
 b) ball case, sketch of BODENSEEWERK GEOSYSTEM GMBH
 c) The ball case in deviation from the tilt component

The classical method of analysing the calibrations is polynomial approximation of the time series just before and after the calibration step (FLACH et al., 1971, 1975, GROßE - BRAUCKMANN, 1973). The amount of the step is the difference of both polynomials in t_0 in Fig. 3. For each calibration those black crosses being recorded 20 - second-values are distributed in a different order in relation to the exact moment of the calibration t_0 . This moment t_0 is not exactly known. That means, the evaluation of the calibration factor is not possible exactly in t_0 and therefore inaccurate, but sufficient for analogous recordings with the noise level in the order of 1 %.

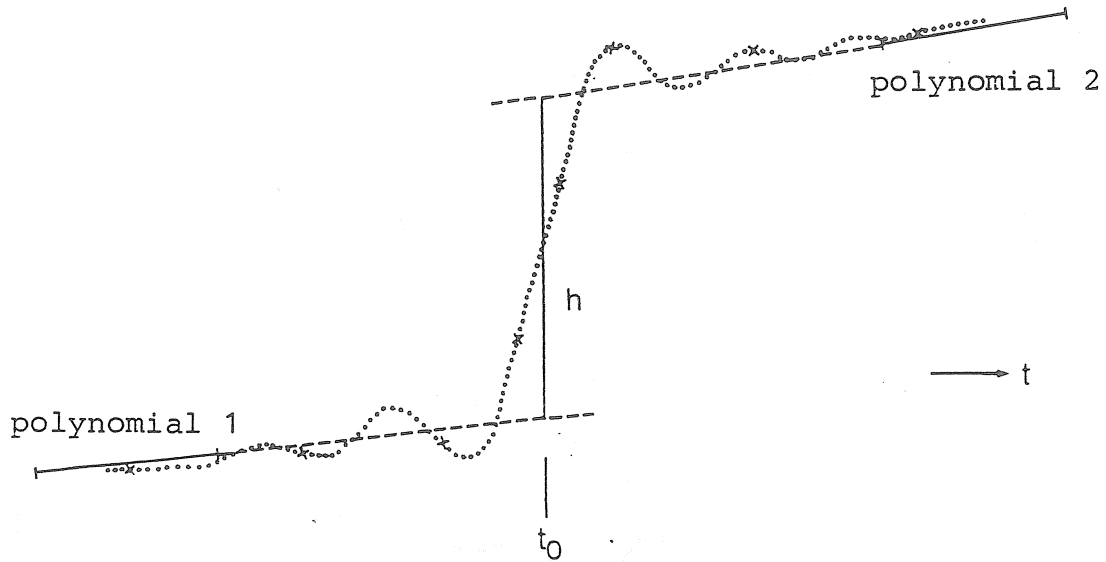


Fig. 3: Filtered calibration step

With the following modification of this method, the order of $\pm 0.1\%$ calibration accuracy could be achieved. The steps are:

1. The recorded calibration with the sample rate of 20 seconds is interpolated to a rate of 1 second with a constructed filter of the same attribute as the filter of the recording system.
2. A rectangular step is modelled with the approximate height h_0 and the sample rate of 1 second (Fig. 4a).
3. This step model is filtered with the filter the recording system is working with, keeping the sample rate of 1 second. The time t_0 is known here (Fig. 4b).
4. The modelled and the observed interpolated steps are shifted against each other in steps of one second, where the cross correlation between both is evaluated. At the point of maximum correlation the best agreement of observed and modelled step according to t_0 is obtained (Fig. 4c).
5. Finally, the amount of the calibration step h is estimated by least squares approximation of the observed step to the modelled one. Parameters to be estimated are the scale factor c ($h = c * h_0$) and the coefficients of a polynomial of second degree separating the drift.

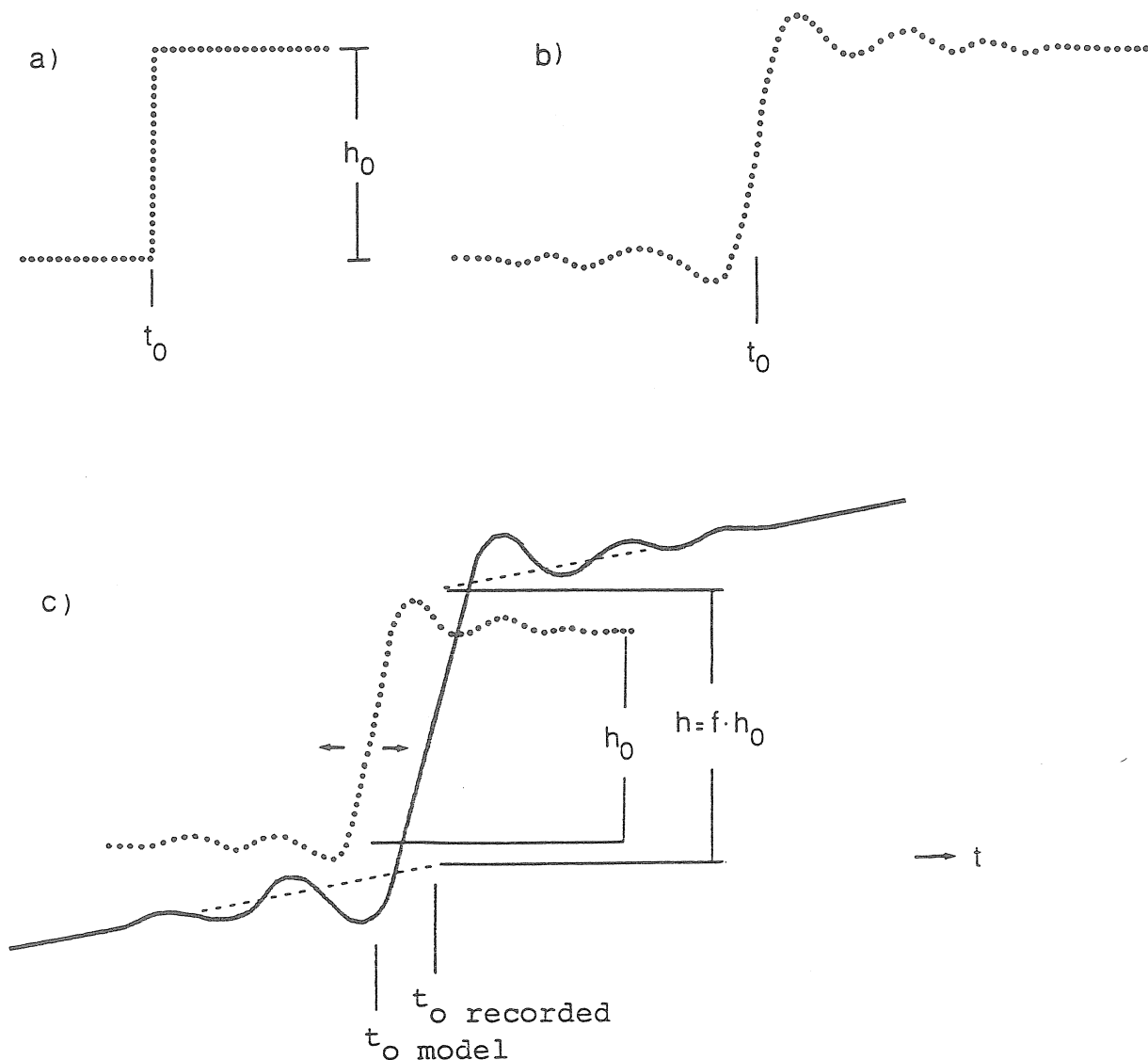


Fig. 4: Evaluation of the calibration factor
a) modelled step with approximate height h_0
b) modelled step, filtered
c) stepwise evaluation of cross correlation between modelled and interpolated calibration pulse

The disturbed values directly before and behind the step are not introduced into the adjustment. So, 5 to 7 minutes before and after a calibration data are needed for the analysis. The accuracy is of course mainly depending on the quality of the recording. The maintainers at the stations are requested to wait some 10 minutes between the calibration and changing the diskette as well as between

the calibrations of the two components. Disturbed data near the calibration are eliminated. Calibrations disturbed by earth quakes for example are not used.

4. Discussion and some Results

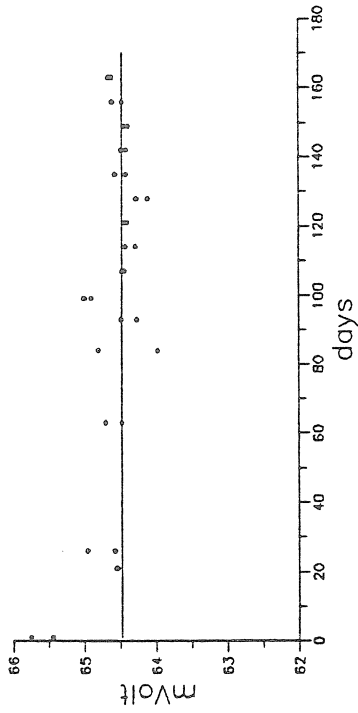
Some results of the calibrations are given in figs. 5, 6 and 7 for the stations Skipadal and Mosvatnet in Norway and Metsähovi in Finland:

In Skipadal and Mosvatnet the calibration factors for the component X are constant within the accuracy of ± 0.1 %. Because of meteorological noise the reduction of the amplification was necessary. This produced some changes of the calibration factor as well as a slight increase of the accuracy .

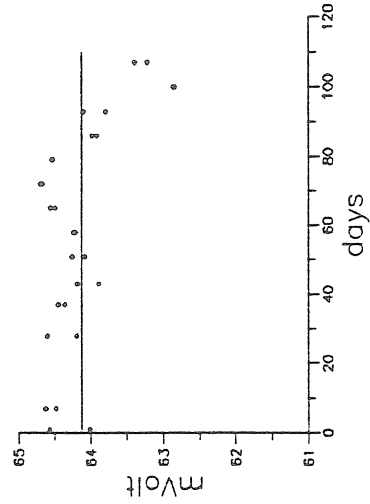
For the component Y the calibration factors show a strong running - in period in both stations referred to electronic problems which have been solved in the meantime. Generally in both stations the Y-components orientated in the direction normal to the nearby Norwegian coast are stronger effected by meteorological noise than the X-components. Since December 1987 the calibration factor is stable also for the component Y within the order of ± 0.1 % (figs.5 and 6).

For the instrument P6 in Metsähovi / Finland we have slight but significant deviations from a constant factor in X and Y. The resulting calibration functions are approximated by polynomials within $\pm 0.1 \dots 0.2$ %. The instability of about 10 % cannot be caused by the sensor. The origin of a non constant calibration factor can only be searched for in electronic problems. In the meantime the calibration factor is absolutely stable for the component X. Small variations of about 3 % during 6 months remain in the Y-component, while in both components the accuracy of ± 0.1 % is achieved (Fig. 7).

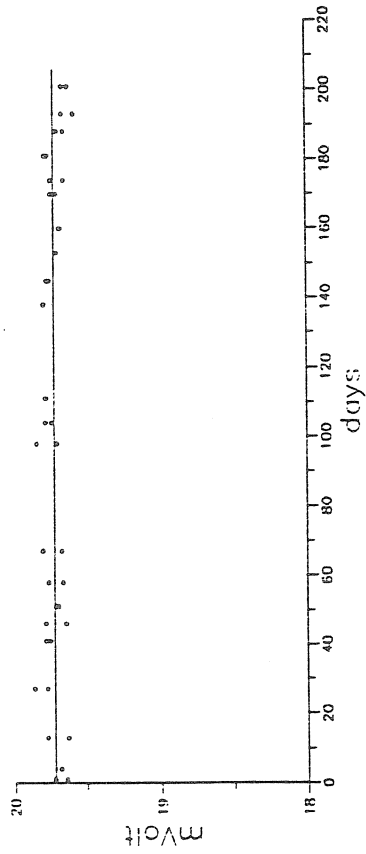
calibrations Skipadal P2 channel x 87.11.26 - 88.05.06
+/- 0.042 mV (0.06 %)



calibrations Skipadal P2 channel y 88.01.21 - 88.05.06
+/- 0.097 mV (0.15 %)



calibrations Skipadal P2 channel x 86.12.07 - 87.06.25
+/- 0.009 mV (0.05 %)



calibrations Skipadal P2 channel y 86.12.07 - 87.07.07
+/- 0.081 mV (0.45 %) and
+/- 0.054 mV (0.25 %)

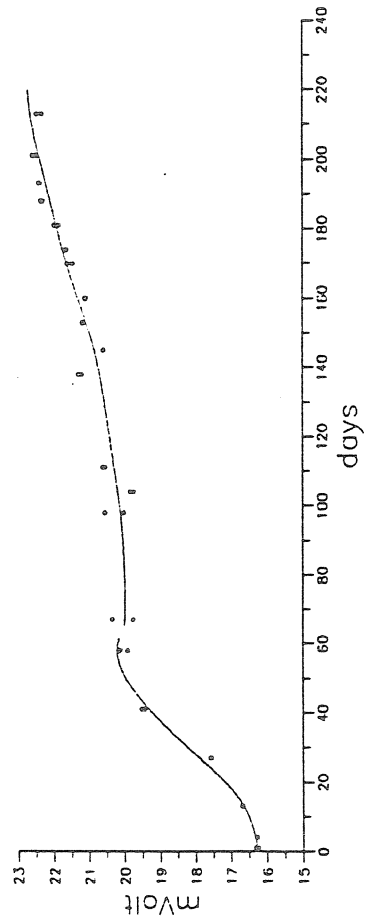


Fig. 5: Calibration factors of the tiltmeter P2 at the station Skipadal, Norway, in X- and Y-component

Summarizing, there is to state:

The sensor of the tiltmeter as well as the computer controlled recording system are able to achieve the calibration factor within $\pm 0.1 \dots 0.2$ % accuracy, as given by the manufacturer.

Concerning the sensor, the calibration factor should be stable in this order and generally is. A non constant calibration function can only be caused by electronic problems, it can even be a hint to their existence.

5. Acknowledgements

This research was carried out as part of the project of Prof. G. Jentzsch funded by the German Research Society (DFG, project no. Je 107/12-3).

The measurements were supported by our Fennoscandian partners:

The tiltmeasurements at the Blå Sjø are carried out in cooperation with Statens Kartverk and Statkraft in Norway. In Finland the project is supported by the Finnish Geodetic Institute. The "Bayrische Kommission für die Internationale Erdmessung der Bayerischen Akademie der Wissenschaften" in Munich made available the Askania - borehole - tiltmeter P6 recording in Finland, which is gratefully acknowledged.

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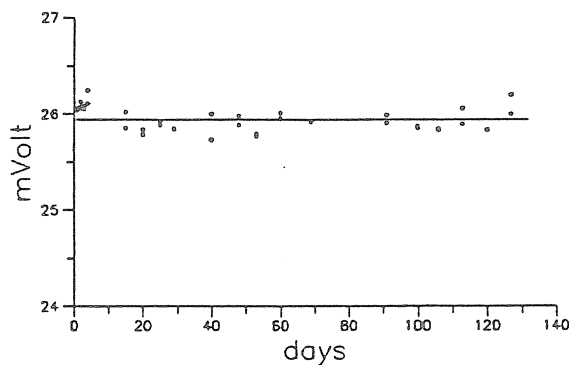
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calibrations Mosvatnet P1 Channel x 86.12.05 - 87.04.10

+/- 0.022 mV (0.09 μ)



calibrations Mosvatnet P1 channel y
86.12.05 - 87.04.10

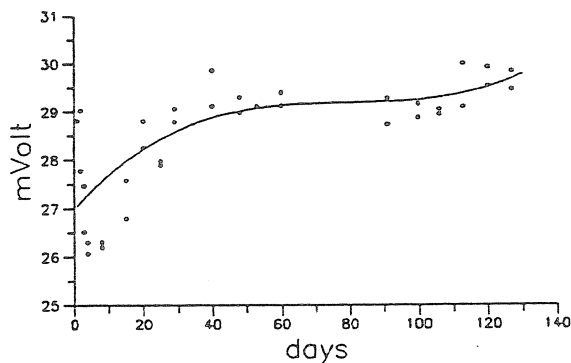
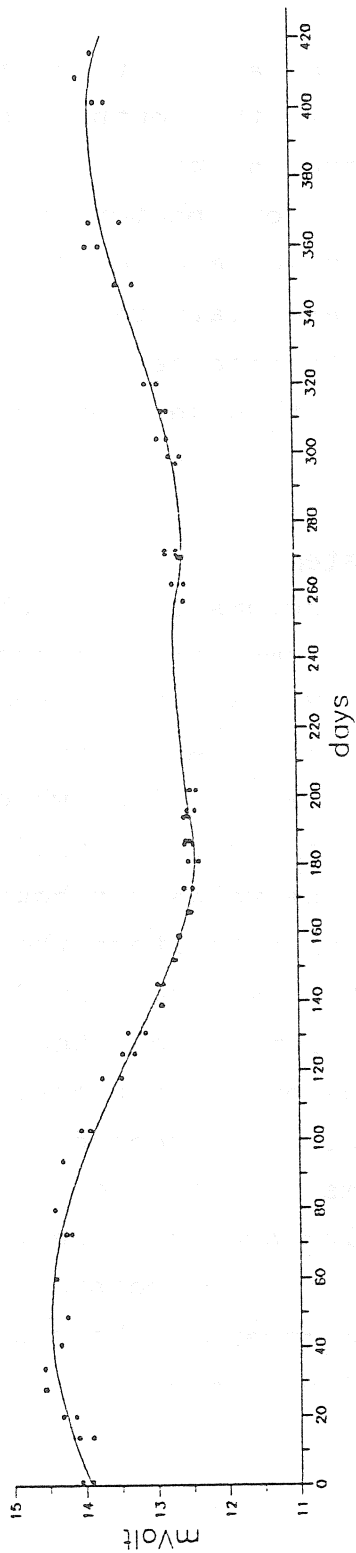


Fig. 6: Calibration factors of the tiltmeter P1 at the station Mosvatnet, Norway, in X- and Y-component

Calibrations Metsæhovi P6 channel X 86.10.04 - 87.11.23

± 0.020 mV (0.15 %) / ± 0.032 mV (0.24 %)



calibrations Metsæhovi P6 channel Y 86.10.04 - 87.11.23

± 0.015 mV (0.10 %)

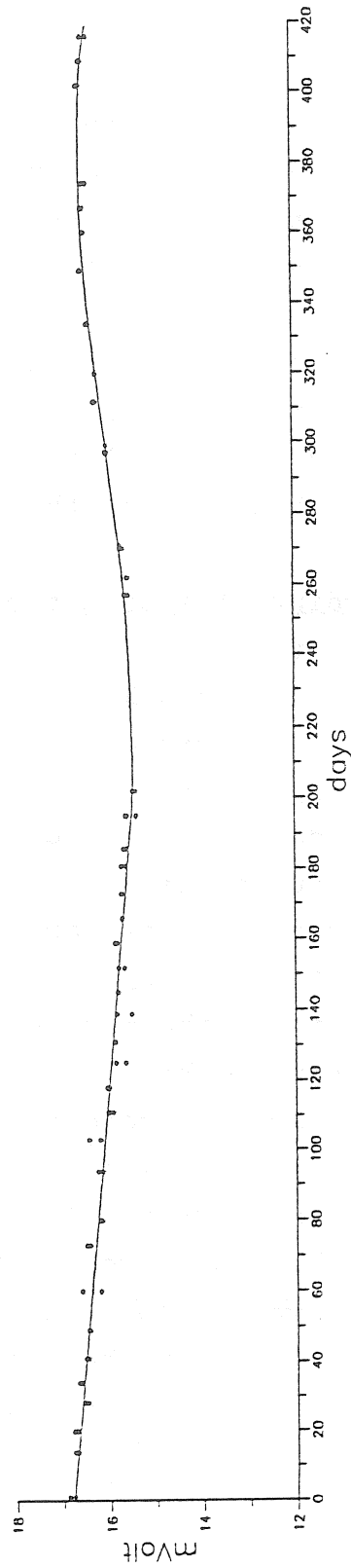


Fig. 7: Calibration factors of the tiltmeter P6 at Metsæhovi, Finland, in X- and Y-component

A complex tidal data acquisition and processing system

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Abstract

The paper deals with a complex data acquisition system for digital recording and processing of the output signals of pendulums and gravimeters. The computing part of the system accepts data from old analogous, on photopaper recording instruments as well as from newly developed and digitally recording capacitive pendulums. These latter are capable - connected to an intelligent tidal data recorder - of high-speed recording of the tidal signal after Earthquakes for detecting of the eigenvibrations of the Earth.

The recording part of the complex system

There are two pairs of horizontal pendulums in our observatory. The old Tomaschek-Ellenberger pendulums are recording on a photodrum. The capacitive pendulums [Montes, 1981] are recording electrically. These pendulums are connected to an intelligent digital tidal data recorder [Montes, 1985, 1986] which samples their output signal at a rate of 20 seconds. It prefilters the densely measured data and puts only one value per hour into the semiconductor memory. This unit is able to store the high-speed sampled data in the memory, too. This is made only after high-energy Earthquakes. The output signals of the capacitive pendulums are recorded also analogous on a strip-chart recorder for immediate control of the digitally recorded data and therefore the working of the pendulums. Of course the capacitive pendulums have a mirror on the pendulum arm for direct recording of the movement on a photodrum. This later possibility is of great importance in the phase of development. The old Tomaschek-Ellenberger pendulums are only used for control.

Pendulum development is continuous in our institute, therefore our processing system is capable of reading out analogous and photorecords, too. The digitally recorded and prefiltered data are transferred fortnightly from the semiconductor memory of the intelligent data recorder to a cassette unit and the recorded data can be transferred on a cassette to the data processing computer (Figure 1).

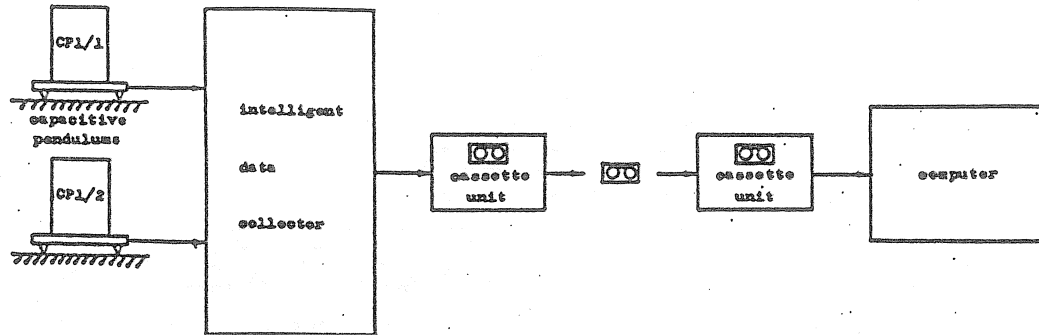


Fig. 1 Digital recording with capacitive pendulums

The data processing part of the system.

Data processing was previously based on a HP-1000 computer, which has a digitizer table among a large number of peripherals as an input peripheral for analogous and photorecords. On an IBM compatible personal computer the preparation and processing of the data is more convenient. To keep the advantages of both we have connected them together via an RS-232 C interface. To read in data from cassette into the IBM-compatible computer we developed a cassette interface.

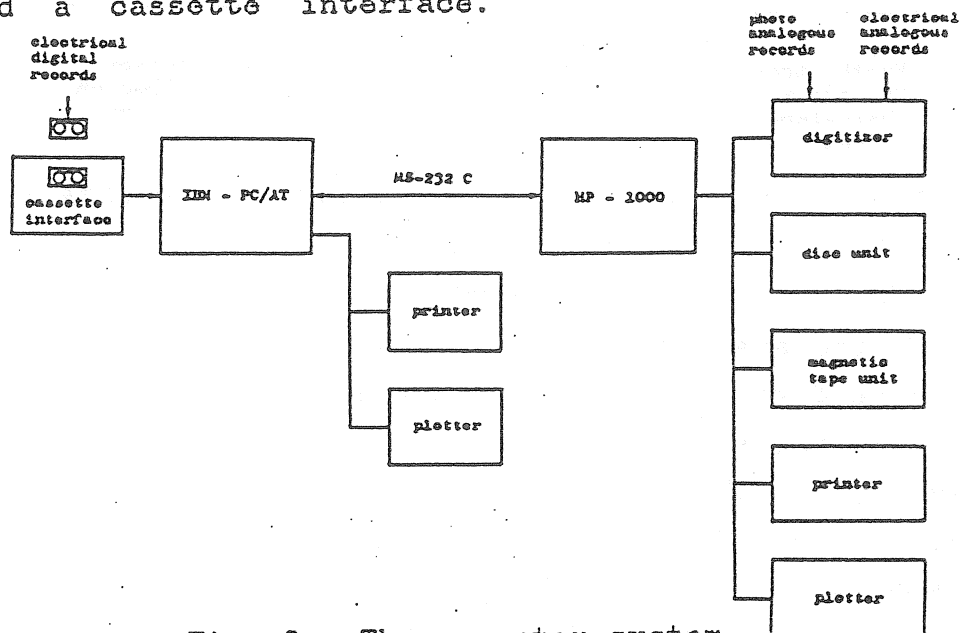


Fig. 2 The computer system

Figure 2 shows the computer system and the flow-chart on Figure 3 shows the process of the evaluation of tidal data.

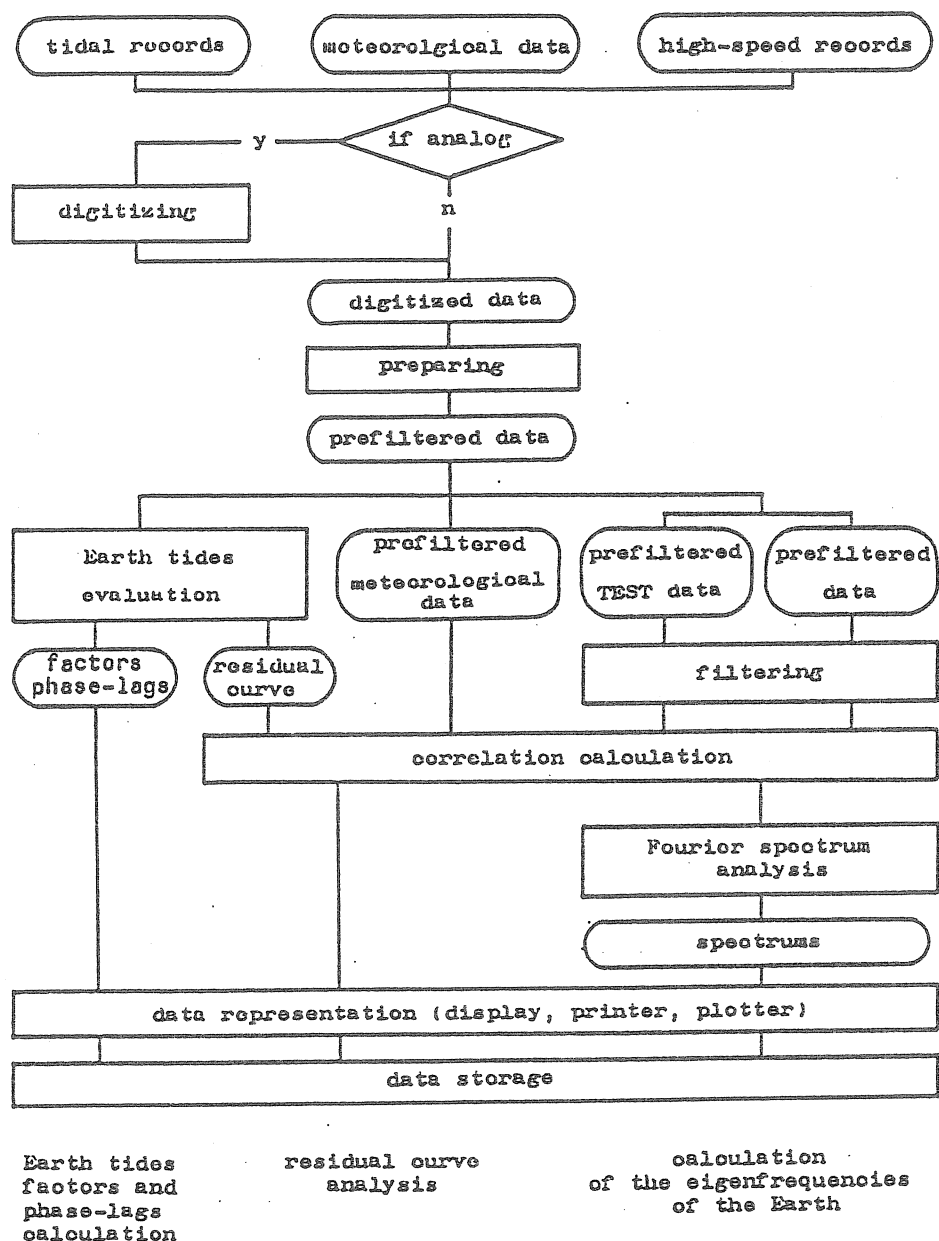


Fig. 3 The flow-chart of the data processing system

Data reach the processing system in digital or analogous form. Therefore the analogous records have to be digitized by means of a RA03-type digitizer table and a digitizer program. The digital data reach the system via the cassette interface. The receiving program writes the data in an appropriate format on a diskette. The evaluation of the data can be made both on the HP-1000 and the IBM-PC AT computer. The aim of the evaluation determines the steps of the processing. This aim can be the calculation of tidal

factors, phase-lags, the analysis of the residual curves and the detection of the eigenvibrations of the Earth.

In the first step the incoming data are represented on the display or on the plotter and by means of a preparatory program the outliers are removed.

From the methods for calculating tidal factors and phase-lags we use the Venedikov and Bartha methods. In addition the program makes possible the optional grouping of the tidal waves, the choice between Doodson's 79 and Cartwright-Tayler's 511 waves approaches. If choosing Bartha's method there is a possibility to a collective evaluation of more records. Table I shows the most characteristic waves obtained as a result of the evaluation of some records made in our observatory.

Table I
Amplitude factors and phase-lags measured in Sopron

DATE	NUMBER OF REG. DAYS	COMPO-NENTS OF E.T.	TYPE OF PEND.	AMPLITUDE FACTOR (γ)					PHASE LAG (\times)				
				O1	P1S1K1	N2	M2	S2K2	O1	P1S1K1	N2	M2	S2K2
20.07.71.-	158	E-W	VM44	0.707 ± 0.018	0.719 ± 0.012	0.747 ± 0.022	0.742 ± 0.004	0.663 ± 0.008	4.57 ± 1.44	-1.06 ± 0.97	3.90 ± 1.71	0.27 ± 0.34	1.20 ± 0.70
25.05.72.	97	N-S	VM45	1.585 ± 0.247	1.721 ± 0.173	0.619 ± 0.051	0.755 ± 0.010	0.841 ± 0.018	25.0 ± 8.97	8.96 ± 5.76	7.31 ± 4.78	-1.50 ± 0.79	5.36 ± 1.26
24.06.72.-	17	E-W	TES102	0.869 ± 0.068	0.982 ± 0.042	0.871 ± 0.082	0.750 ± 0.013	0.597 ± 0.033	-6.55 ± 3.76	-21.4 ± 2.60	6.01 ± 5.38	-4.01 ± 0.97	-1.65 ± 3.26
15.06.72.	19	E-W	VM44	0.726 ± 0.035	0.784 ± 0.022	0.787 ± 0.047	0.731 ± 0.008	0.768 ± 0.021	6.49 ± 2.70	3.60 ± 1.56	4.04 ± 3.43	3.11 ± 0.60	2.66 ± 1.57
25.08.72.-	50	E-W	TES102	0.568 ± 0.085	0.803 ± 0.066	0.983 ± 0.100	0.730 ± 0.020	0.726 ± 0.035	1.76 ± 8.62	-5.85 ± 4.72	-9.47 ± 5.78	1.83 ± 1.65	4.75 ± 2.78
13.11.72.	38	E-W	VM44	0.718 ± 0.019	0.757 ± 0.015	0.800 ± 0.024	0.738 ± 0.005	0.852 ± 0.008	1.73 ± 1.50	-2.24 ± 1.16	-8.66 ± 1.71	-6.51 ± 0.36	-8.43 ± 0.68
11.07.85.- 07.08.85.	28	E-W	CP1/1	0.707 ± 0.024	0.724 ± 0.014	0.771 ± 0.048	0.741 ± 0.006	0.535 ± 0.015	-7.74 ± 1.94	5.23 ± 1.14	-6.48 ± 3.62	-10.01 ± 0.48	-7.86 ± 1.71
07.02.87.- 07.05.87.	88	E-W	CP1/1	0.755 ± 0.211	0.779 ± 0.176	0.917 ± 0.374	0.826 ± 0.057	0.834 ± 0.089	-6.56 ± 8.05	-7.64 ± 6.92	86.00 ± 12.44	-12.53 ± 1.95	-6.38 ± 3.12

The residual curve is also obtained as a result of the tidal evaluation program. The correlation between the residual curve and the meteorological data or other quantities changing in time is obtained by means of a correlation analysis program.

The eigenvibration of the Earth can be detected by high-speed records made after Earthquakes. The obtained data are filtered first by means of a band pass filter which emphasizes the eigenfrequencies and suppresses the noise and the long period signals coming from the tide.

The above mentioned correlation program is used as a filter program. By a cross-correlation made between an inactive period and the after-quake period of the record, the periods coming from measurement noise in the band of the eigenfrequencies can be determined.

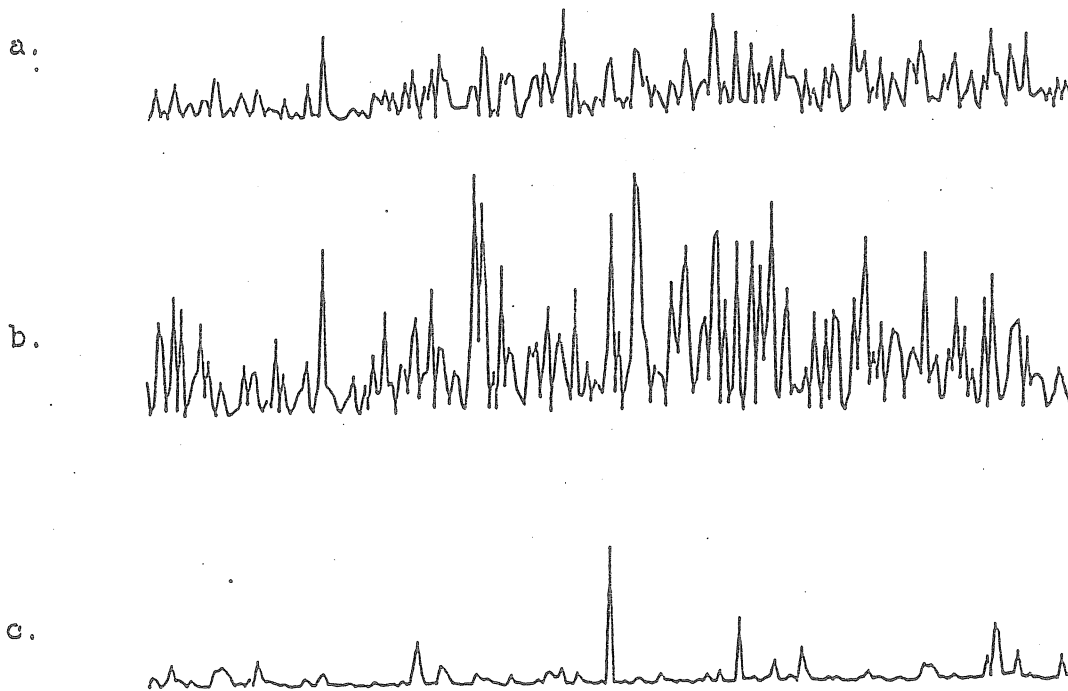


Fig. 4

- a. Power spectrum of a high-speed record (29.02.88.)
- b. Power spectrum of filtered high-speed record
- c. Power spectrum of the crosscorrelation function of the high-speed record and a TEST record

The frequencies are determined by a Fourier-spectrum analysis program. An example is given on Figure 4. Table II shows the parameters of the earthquakes after which the high-speed records were made, Table III the eigenfrequencies using the values given by Mendiguren (1973) as reference.

EARTHQUAKE	DATE	ORIGIN TIME (GMT)	EPICENTER		DEPTH (km)	MAGNITUDE
			LAT.	LONG.		
El Salvador	29.04.83	01h-02m-20s	13.43N	89.22W	92	4.8
California	02.05.83	23h-42m-38s	36.24N	120.30W	12	6.5
Japan	26.05.83	03h-00m-01s	40.37N	139.15E	33	7.6
Ryukyu Islands	04.09.84	13h-08m-16s	24.65N	124.56E	72	4.7
Taiwan	14.11.86	22h-34m-23s	24.04N	121.96E	33	6.2
Tonga Islands	06.10.87	04h-19m-06s	17.94S	172.23W	16	7.3
Alaska	30.11.87	19h-23m-20s	58.68N	142.79W	10	7.6
Komandorsky Island	29.02.88	05h-31m-41s	55.01N	167.45E	33	6.8

The last part of the processing program is printing of results and tables and plotting of curves. Previously the data were stored on magnetic tape, at present we use diskettes for this purpose.

Table III The identified periods of the spectra

[illegible]

Conclusion

The above described system is working quite well in the practice. To obtain more and better results we need more pendulums and high-precision gravimeters. At present we began to install a new quartz-tube extenzometer in our observatory. A lot of different instruments on the same place give a good possibility to diminish the measurement errors comparing the results obtained by the different instruments. Therefore we are working on new, more reliable, more accurate instruments and new processing programs.

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Determination of transfer functions
of horizontal pendulums on the basis of laboratory
measurements

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Abstract

In the practice, it is necessary to know the dynamic behaviour of the horizontal pendulums, especially when we use them for recording of the eigenvibration of the Earth. This is described by the frequency, unit step and impulse response functions. These functions are important in that case too when an electrical transducer is applied to the pendulum. In the paper the above mentioned function are given for horizontal pendulums but these functions are valid with some restrictions for vertical pendulums. The described laboratory measurements make possible to determine in a short time the different parameters of the transfer functions of the pendulums with an accuracy necessary in the practice.

Introduction

The differential equation

$$\theta \frac{d^2\phi}{dt^2} + k \frac{d\phi}{dt} + mg \sin(i) \sin(\phi) + r\phi = mgs\varphi \quad (1)$$

describing the movement of the horizontal pendulum (Mittelstrass 1966) is not useful for the practice, because the majority of the co-efficients cannot be measured directly and these co-efficients do not give any direct information about the signal transfer of the pendulum. Another disadvantage is that the equation (1) describes only the tiltmeter mode of the pendulum. Namely, there

exists also an accelerometer mode and it is very important to take it into consideration at some pendulum measurements. These two working modes cannot be separated from one another, because the pendulum is sensitive to tilts and accelerations at the same time. Fig. 1. shows the two working modes of a horizontal pendulum.

The differential equation of the pendulum in accelerometer mode is as follows

$$\theta \frac{d^2\phi}{dt^2} + k \frac{d\phi}{dt} + mgssin(i)\sin(\phi) + \tau\phi = ms \frac{d^2x}{dt^2} \quad (2)$$

This differential equation differs from that of the tiltmeter mode only in its right side.

The parameters of equations (1) and (2) are:

θ = the moment of inertia,
 k = the damping factor,
 m = the mass of the pendulum beam,
 s = the distance between the centre of gravity and the centre of rotation of the pendulum beam,
 g = the gravitational acceleration,
 τ = the restoring moment,
 ϕ = the angle displacement of the pendulum beam,
 φ = the tiltangle of the pendulum
 x = the horizontal displacement of the pendulum,
 i = the tiltangle of the rotation axis of the pendulum against the vertical.

The transfer functions of the horizontal pendulum in tiltmeter mode

Differential equation (1) can be written as follows

$$\frac{d^2\phi}{dt^2} + 2\beta \frac{d\phi}{dt} + \omega^2 \phi = \frac{g}{l_0} \varphi \quad (3)$$

where

$$2\beta = \frac{k}{\theta} ; \sin\phi \sim \phi ; \frac{\theta}{ms} = l_0 ; \omega^2 = \frac{g}{l_0} \sin i + \frac{\tau}{\theta} \quad (4)$$

From Eq. (3) Laplace-transformation yields

$$s^2 \Phi(s) + 2\beta \omega_e \Phi(s) + \omega_e^2 \Phi(s) = \frac{g}{l_0} \varphi(s), \quad (5)$$

where

s = the Laplace-operator.

From Eq. (5) the transfer function of the pendulum is obtained in the tiltmeter mode:

$$H(s) = \frac{\Phi(s)}{\varphi(s)} = \frac{K}{1 + 2\xi\tau s + \tau^2 s^2} \quad (6)$$

where

$$\tau = \frac{1}{\omega_e} \quad \text{the time constant of the pendulum,}$$

$$\xi = \frac{\beta}{\omega_e} \quad \text{the damping factor,}$$

$$K = \frac{g}{l_0 \omega_e^2} \quad \text{the mechanical gain of the pendulum at zero frequency,}$$

$$\omega_e = \frac{2\pi}{T_e} \quad \text{the angle velocity of the eigenswinging of the pendulum}$$

By formally replacing of s by $j\omega$ the frequency response function can be derived from Eq. (6):

$$H(j\omega) = \frac{K}{1 + j\omega 2\xi\tau + (j\omega)^2 \tau^2} \quad (7)$$

The unit step response of the pendulum is obtained by inverse Laplace-transformation of $H(s)$. Usually, $\xi \neq 1$ at horizontal

pendulums and in this case the unit step response is:

$$v(t) = K \{1 - e^{-\beta t} (\cos(\omega_{\beta} t) + \frac{\beta}{\omega} \sin(\omega_{\beta} t))\}, \quad (8)$$

where

t = the time

$$\omega_{\beta} = \omega_e \sqrt{1 - \xi^2}$$

$$\beta = \xi \omega_e$$

By derivation of Eq. (8) the impulse response of the pendulum follows:

$$w(t) = \frac{k}{\omega_{\beta} \tau^2} e^{-\beta t} \sin \omega_{\beta} t. \quad (9)$$

The transfer functions of the horizontal pendulums in accelerometer mode

The accelerometer transfer functions are obtained from the differential equation similarly as in the case of tiltmeter mode. The frequency transfer function is:

$$H(j\omega) = \frac{1}{l_0} \frac{(j\omega)^2 \tau^2}{1 + j\omega 2\xi\tau + (j\omega)^2 \tau^2}, \quad (10)$$

the unit step response:

$$v(t) = \frac{1}{l_0} e^{-\beta t} \cos(\omega_{\beta} t), \quad (11)$$

the impulse response:

$$w(t) = - \frac{R}{l_0} e^{-\beta t} \cos(\omega_{\beta} t) - \frac{R}{l_0} e^{-\beta t} \sin(\omega_{\beta} t) . \quad (12)$$

Measurement of the parameters of the transfer functions

The horizontal pendulum is placed directly on a concrete pillar (Fig.2.) for testing. The measurement is made as follows: At different sensitivities, set by means of the "sensitivity screw" the pendulum is tilted by means of a crapaudine placed under the drift screw. The tilt ($\varphi = 0.06746$ secs of arc) is made in some (5-10) points of the working range of the pendulum arm at the same sensitivity. The turning points of the eigenswingings of the pendulum arm are read on a scale by means of a light beam reflected of a mirror placed on the pendulum arm. In the case of ellectrically recording e.g. capacitive pendulums, the movements of the pendulum arm are recorded electrically on a strip-chart recorder at the same time, too. In both cases the damping factor ξ , the eigenperiod T_e and the angle deviation of the pendulum arm Φ_e can be determined from the measurements as it can be seen on Fig. 3. Fig. 4. shows, how to determine the mechanical deviation of the pendulum arm Φ from the scale. X_1 is the centre line of the eigenswinging of the pendulum arm in state of rest, X_2 is the one after the tilting by the crapaudine and X_3 is one after tilting back. In case of electrically recording pendulums K can be written as follows:

$$K = K_1 K_2 = \frac{\Phi_e}{\Phi} \frac{\Phi}{\varphi} \quad (13)$$

where

$$K_1 = \frac{\Phi_e}{\Phi} \quad \text{is the electrical gain of the pendulum}$$

$$K_2 = \frac{\Phi}{\varphi} \quad \text{is the mechanical gain of the pendulum}$$

From simultaneous electrical and photo measurements both gains can be determined. By means of the equations given in this paper all parameters of the pendulum can be calculated from the measuring results. Table I. shows the damping factor ξ , the instrument constant k and the reduced length of some pendulums:

Table I

Pendulum	ξ	k [1/s ²]	l [m]
CP1/1 (capacitive)	0.1890	6.215	0.0400
CP1/2 (capacitive)	0.1773	5.507	0.0451
TE 9102	0.1308	4.685	0.0550
ORB 89	0.0504	4.554	0.0530

Fig. 5-10. show the transfer functions in tiltmeter and accelerometer mode, calculated by means of the measured results.

Conclusions

Horizontal pendulums are by 2-3 orders of magnitude more sensitive instruments than other inclinometers, therefore their dynamic investigation on the usual swinging tables (Caspary and Geiger 1979, Eichholz and Schäfler 1982a, 1982b) is nearly impossible. The above described static measurement provides a sufficient accuracy (1-2 percent) for the practice. By the demonstrated measuring method we can also determine the quality of the suspension and the linearity of the transducer.

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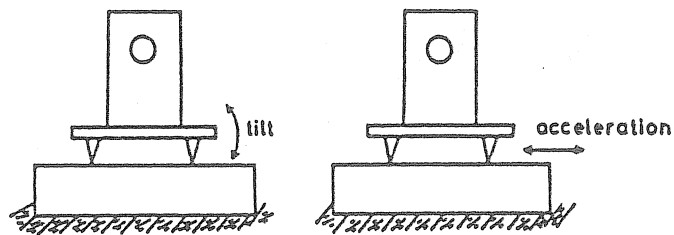


Fig.1. The tiltmeter and the accelerometer mode of the pendulum

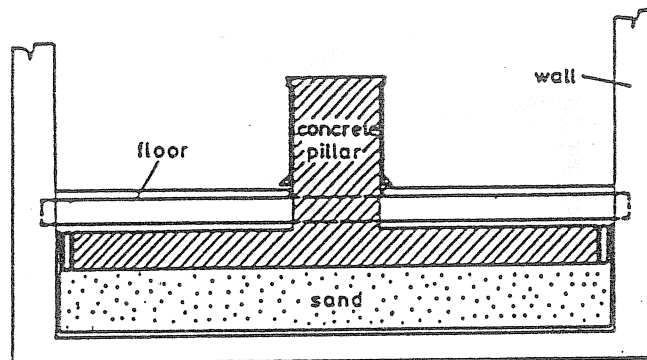


Fig. 2. The concrete pillar for testing horizontal pendulums

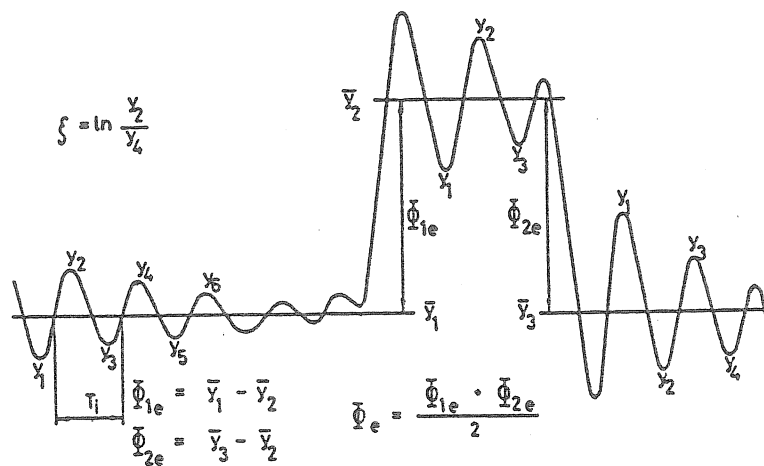


Fig. 3. Electric record of a tilting of the pendulum

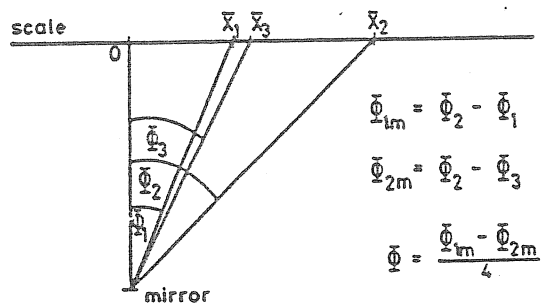


Fig.4. Determining the angle deviation of the pendulum by means of a scale

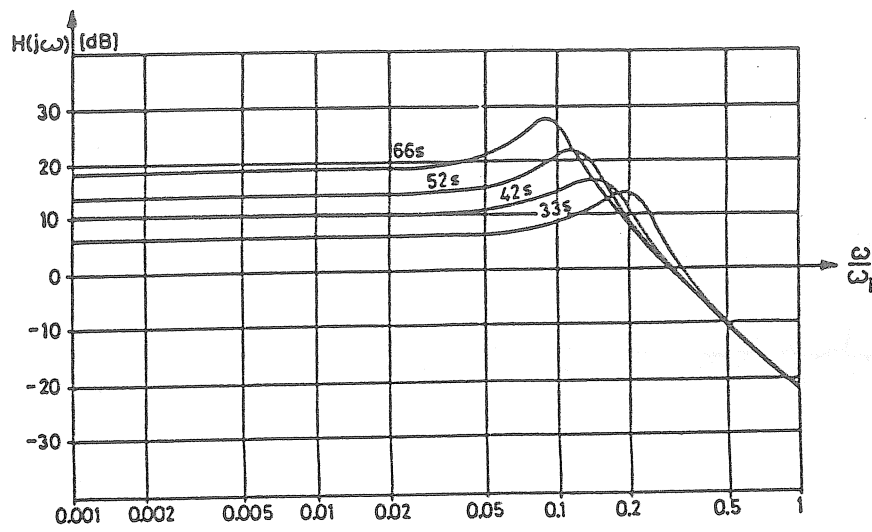


Fig. 5. The frequency response of the pendulum in tiltmeter mode

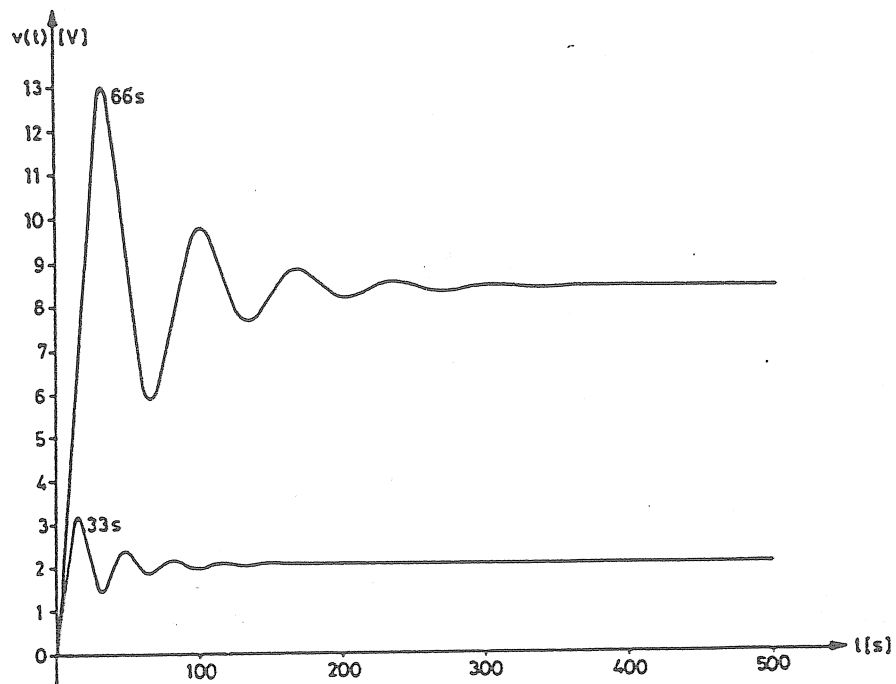


Fig. 6. The unit step response of the pendulum in tiltmeter mode

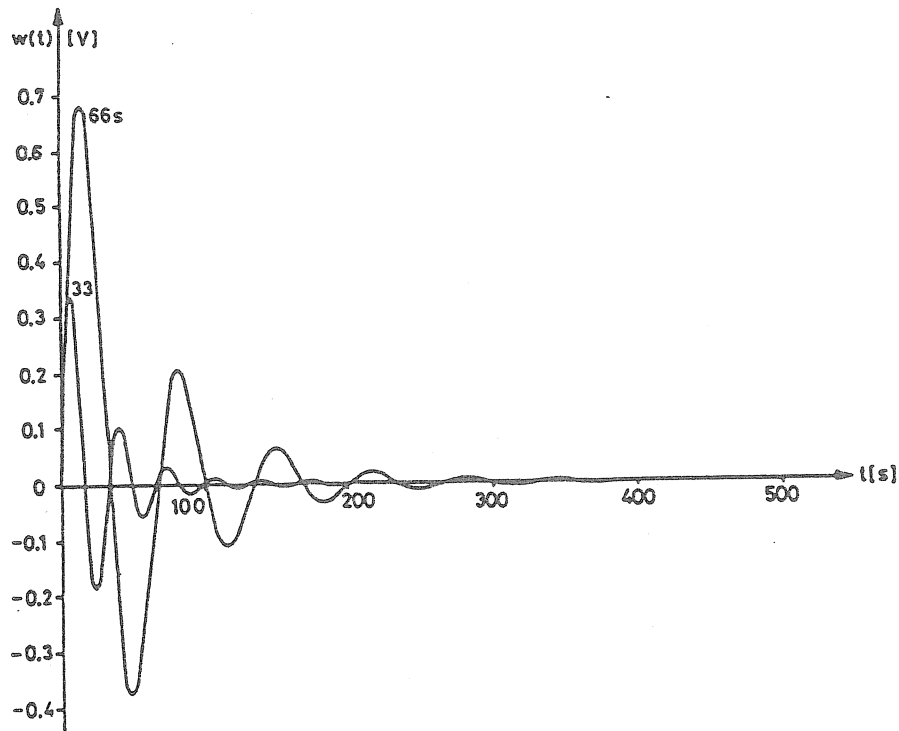


Fig. 7. The impulse step response of the pendulum in tiltmeter mode

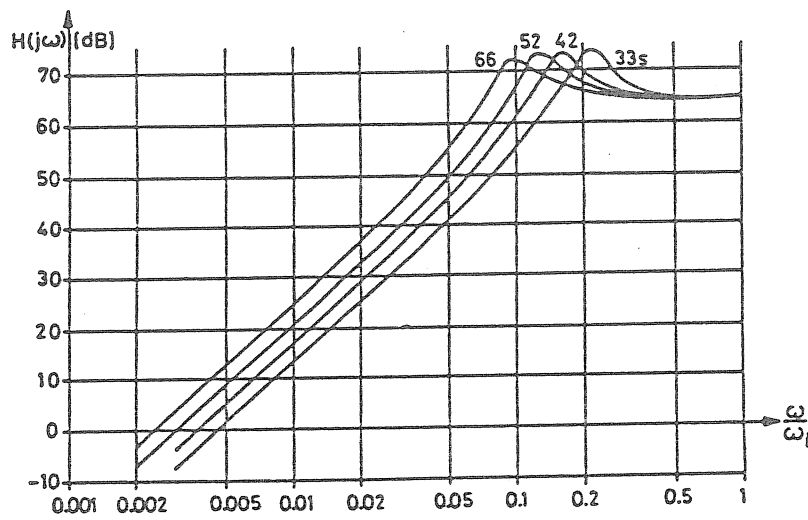


Fig. 8. The frequency response of the pendulum in accelerometer mode

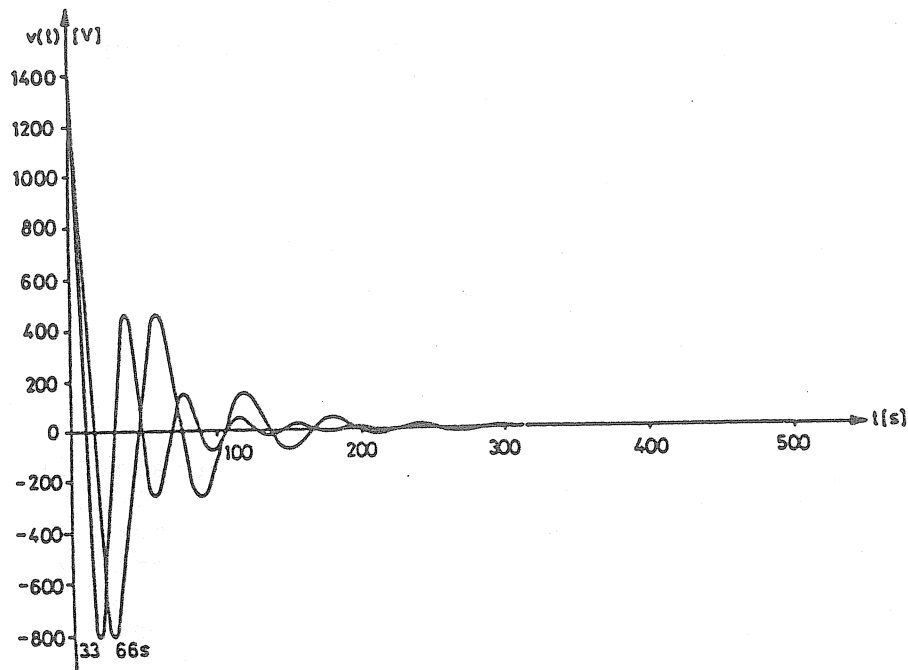


Fig. 9. The unit step response of the pendulum in accelerometer mode

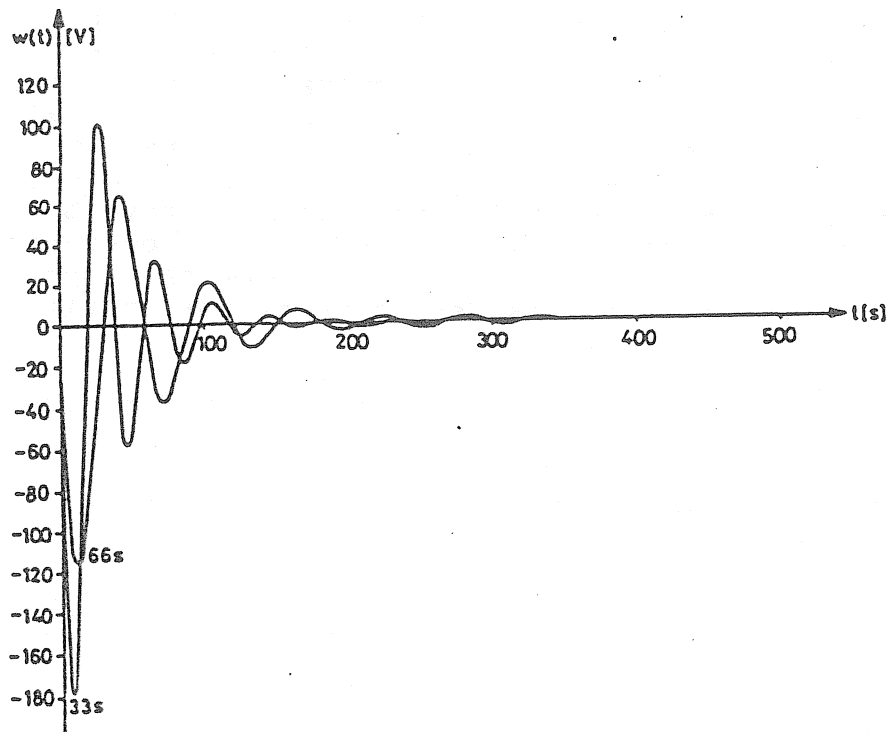


Fig. 10. The impulse step response of the pendulum in accelerometer mode

Report on the Work of the Subgroup on Tide Generating Potential Investigations at the Institute for Theoretic Geodesy Bonn

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Abstract:

During the last two years at the Institute for Theoretic Geodesy, University Bonn, a program has been developed to calculate the tide-generating potential from ephemerides. It has been used to compare the harmonic developments from Büllesfeld and Xi. The results show that in general it makes necessary great expenses to model the tide-generating forces within an error bar of about 10 [ngal]. Additionally the physical parameter system has come to a limit that doesn't allow to approximate closer than 4 [ngal].

1. Introduction

On the last meeting of the working group on high precision tidal data processing in October 1986 it was realized that new models for the tide-generating potential have been developed by Büllesfeld (Bonn), Tamura (Mizuzawa) and Xi (Beijing), because the standard model CTED is no more precise enough for the analysis of superconducting gravimeter registrations (Büllesfeld 1985). In order to compare these three models a subgroup was formed by representatives of the concerned institutes (Working Group 1987). During the last two years several investigations have been made by these institutes together with the Royal Observatory of Belgium at Brussels (Ducarme 1988). The following report will concentrate on the work that has been done at the Institute for Theoretic Geodesy at Bonn. It contains:

- estimating the nowadays available precision for the 'true values' of the tide-generating potential,
- developing a program to calculate the tide-generating potential by ephemerides on this highest accuracy,
- comparing the present tidal models with the values calculated by the developed program.

2. Estimation of the available accuracy

In the publications the authors give different precisions for their tidal models (Büllesfeld: standard deviation about 20 [ngal]; Tamura: maximum difference in the frequency-domain about 0.8 [ngal]). So our first interest was concentrated on the limit of the accuracy that can be reached by the 'true values'. This

should be the limit, up to which the approximation of the tidal force by a model can only be efficient.

Such an estimation of the available precision has just been introduced by Büllesfeld (1985). His results are shown in table I. There can be seen that the greatest errors are produced by the uncertainty of the physical parameter system itself. The other influences like the choice of the ephemeris calculation, the neglect of potentials of higher degree or of the attraction of further celestial bodies can be avoided and have partly been taken into account by the present new models.

Nevertheless an other problem has appeared and not been solved till now: the velocity of gravity is not known. If it is limited like the velocity of light, then the true places have to be substituted by the apparent places of the celestial bodies; its difference is known from optical astronomy as the aberration and will have a maximum difference in gravity of about 5 [ngal] (see figure I) (FRIEDRICH 1988c).

So we can realize that the limit for the precision of the 'true values' is mainly depending on the physical parameter system and the influence of the aberration. This leads to a standard deviation in the range of 4 [ngal].

Special error influences produced by the choice of the ephemerides calculation are discussed in the following chapter.

3. Choice of the ephemeris calculation

For several years the classical ephemeris calculations of Brown and Newcomb (Moon and Sun) have been replaced by programs that are solving the n-body problem for the sun and the planets by numerical integration.

Such a program, GLE2000, has also been developed by Schastok (1987). In addition it uses a Post-Newton-approximation according to Einstein's theory of general relativity. For our investigations this program has been extended by relativistic transformations from the barycentric system to an arbitrary local horizontal system on the Earth. This includes mainly the aberration influences of the apparent places of the celestial bodies. Then the tidal force in the direction of the three axis in this horizontal system is calculated by Newton's law of the attraction of two masses.

The used ephemeris program gives very close values in comparison to the wellknown JPL-Ephemerides DE118. The amount of the difference-vector in the Earth-Moon-system is less than 1 [cm] within a 10 years' integration time (Schastok 1987).

The precision of the ephemeris calculations (numerical integration) have been controlled by observations of the distances between the concerned celestial bodies. Their influences on the precision of the tidal forced are far below the limit of 1 [ngal] and can therefore be neglected (table I) (Friedrich 1988c).

The further investigations show that the difference between the classical and the relativistic ephemerides are below a maximum error of 5 [ngal] (see figure II.) (Friedrich 1988c). Its standard deviation of about 3 [ngal] therefore has to be taken into account, if the classical ephemeris program has been used like in Büllesfeld's and Xi's development. Tamura has taken JPL-Ephemerides as a basis.

4. Comparison of different tidal models

The extended ephemeris program has been taken to calculate 'true values' for the comparison with the harmonic tidal models. Because of the preceded investigations an interval for calculated values of about 1 [ngal] was regarded to be sufficient. Our investigations have first been concentrated on the development ETMB85 (Büllesfeld 1985). The results seemed to be the same as those being introduced in Büllesfeld's publication: maximum difference of 80 [ngal] and standard deviation of 20 [ngal] in the time-domain (see also figure V.). Thus different results have appeared in the frequency-domain which can be seen in figure III. and IV. The first picture shows the flat shape of the spectral analysis (residuels of an 18.6 years comparison of his model with the tidal force calculated from ephemerides) In contrast to Büllesfeld's work our investigations of a two month residuel data set show sharp peaks of more then 10 [ngals] for the amplitudes of the wellknown tidal force frequencies (1 cpd, 2 cpd, 3 cpd, ..). This difference is caused by the data length itself. The longer the dataset is, the more the shape of its Fourier-transform is approximating the shape of white noise, if the process of the signal is not a deterministic one. As a conclusion it can be recognized that spectral analysis of such long residuel data sets are not typical for the spectral behavior of parts of the present time interval. So in our following investigations Fourier-transforms have not been used. Further comparisons with the new Xi-3070-model (Xi 1988) have been made by Friedrich (1988c). The results of several short data sets show that even by such a long development of 3070 tides it is not possible to suppress the residuels under a limit of 20 [ngal] (see also figure VI.).

5. Conclusions

If we take the aspired precision of 10 [ngal] as a basis, it can be recognized that no present model is sufficient. Even the Xi-3070-model contains several residuels in the range of 20 [ngal]. But if we take its standard deviation of about 5 [ngal], the present Xi-model has nearly reached the limit, as is discussed in chapter 2. . .

Such a memory- and time-consuming model as the Xi-3070-model seems to be useful only for analysing super conducting gravimeter registrations. But for many users of tide-generating developments such an expense is not neccessary. So we agree with Ducarme's and Xi's suggestion to introduce a tidal model that can be extended or diminished depending on the requirement of the user.

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Table I: precision of the calculation of tidal forces
(after Büllersfeld 1985)

stand. deviation
[ngal]

1. physical parameter system:

Doodson constant	0.1 [cm ² /s ²]	~ 1.
coordinates of stations	1" , 50 [m]	~ 1.
figure of Earth	max. 2 [ngal]	~ 1.

standard deviation		~ 2.

2. ephemeris calculation:

'Brown' (Moon)		~ 2.
'Newcomb' (Sun)		~ 1.
JPL DE/LE 200	Sun: 2000 [m]	~ 0.002
	Moon: 10 [m]	~ 0.010

3. neglect of further influences:

potential degree n: Moon, n=4: max. 87 [ngal]	~ 40.
n=5: max. 2 [ngal]	~ 1.
Sun, n=3: max. 4 [ngal]	~ 2.
forces of further planets	
Venus	~ 7.
Jupiter	~ 2.

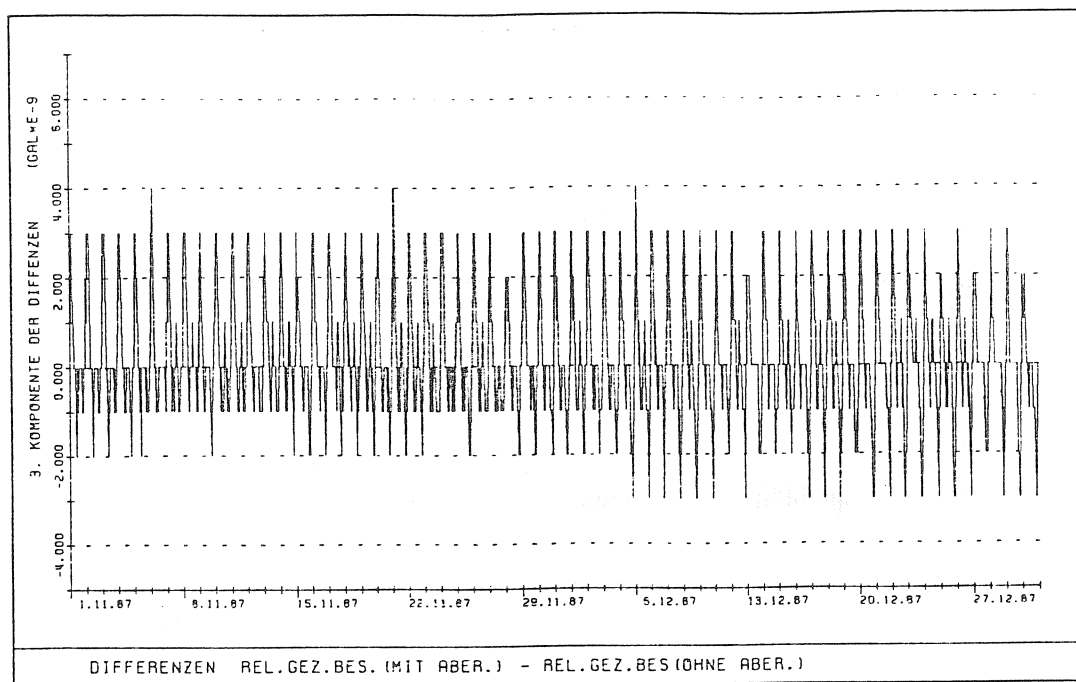


figure I.: differences of tides with and without aberation

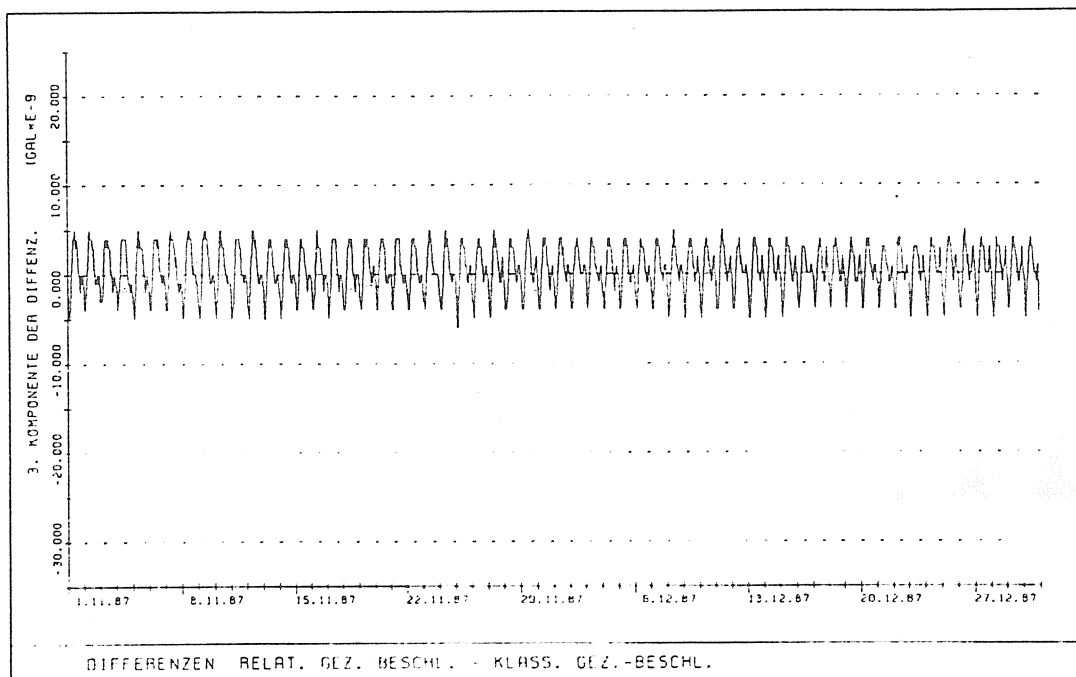


figure II.: differences of tides calculated from classsical and relativistic ephemerides

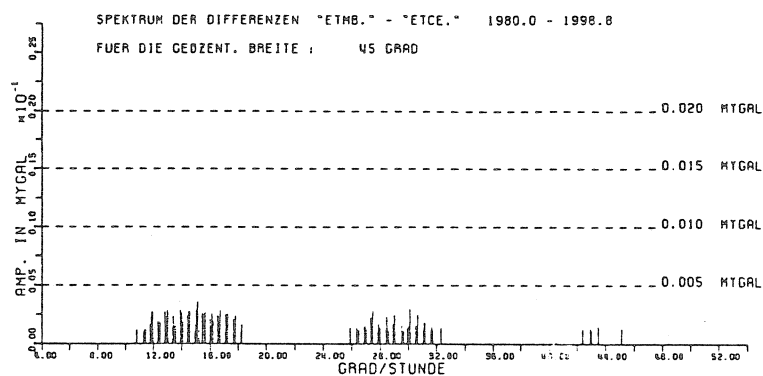


figure III.: amplitudes of the residuals of ETMB85
(from Büllersfeld 1985)

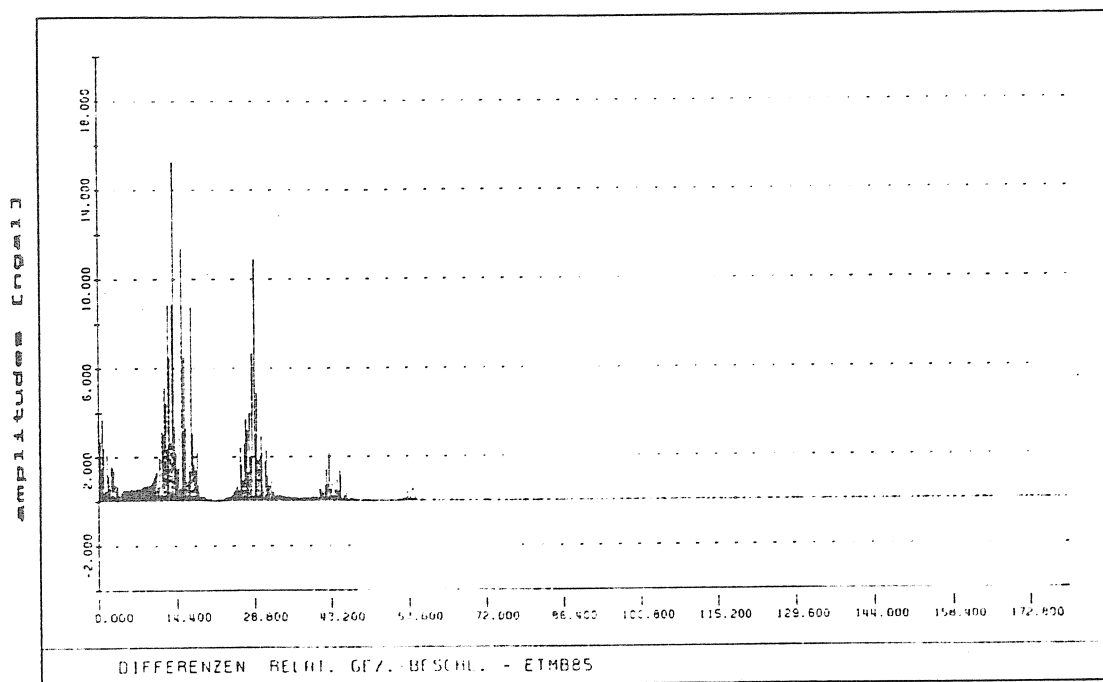


figure IV.: frequency spectrum of the residuals ETMB85

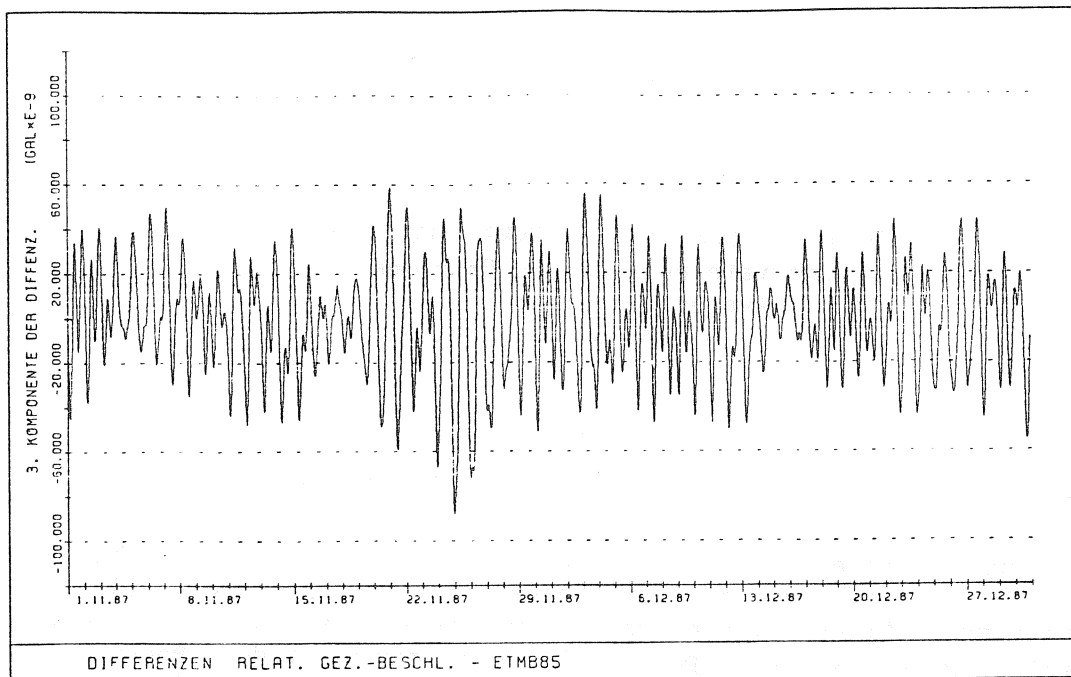


figure V.: residuals of the ETMB85-model

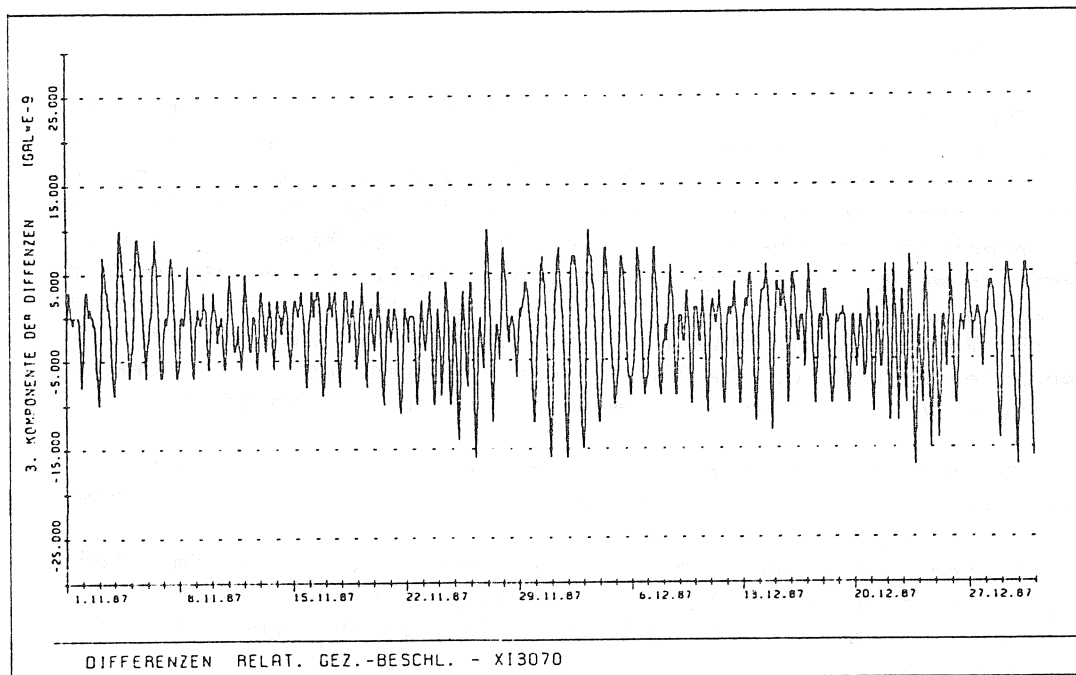


figure VI.: residuals of the Xi-3070-model

Tidal Potential Developments

for

Precise Tidal Evaluation

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SUMMARY:

The currently accepted tidal potential development (Cartwright-Tayler-Edden) insures a precision better than $1 \mu\text{gal}$ for tidal gravity predictions. The microgal level is also a limit for the modelisation of external perturbations on gravity tides. However more accurate tidal developments are necessary for dedicated applications such as tidal analysis. The development due to Bullesfeld (1985) brings already an improvement by a factor of two, but the most precise potential representations are those of Tamura (1987) and Xi Qin Wen (1988) with a maximum discrepancy of $0.025 \mu\text{gal}$ and a standard deviation of 7 nanogal.

1. INTRODUCTION

The use of a tidal potential development is not the only way to evaluate the astronomical tidal forces. It is quite easy to take the position of Moon and Sun in the ephemeris and to compute directly their influence. This global method has the same intrinsic precision as the tidal evaluation through harmonic development. However it allows only a global factor to be applied on the astronomical tides to take into account the elastic response of the Earth. It is thus not possible to take into account the resonance effect of the liquid core and even less the oceanic loading effects as these phenomena are frequency dependant.

The typical precision requirements for different applications are given in table 1 for gravity tides which are the most frequently requested.

Taking into account the fact that the peak to peak amplitude of gravity tides is of the order of $250 \mu\text{gal}$ we may consider that a $1 \mu\text{gal}$ accuracy is roughly equivalent to the one per cent precision level.

Which are the perturbations to be included in the tidal prediction?

As already pointed out there are mainly the indirect effects due to the oceanic tides which can change tidal amplitudes up to 10 per cent and tidal phases up to 5 degrees, mainly in the semi diurnal band. It corresponds to amplitude changes up to 10 μgal .

If we exclude coastal areas we can still reach the 5 μgal level of accuracy.

But we should not neglect the effects of the atmospheric pressure changes. Such effects can be corrected by considering the local pressure variations with efficacy factors experimentally determined which are comprised between $-0.3 \mu\text{gal mbar}^{-1}$ and $-0.4 \mu\text{gal mbar}^{-1}$. This simple method allows to correct ninety per cent of the effect. To reach a higher precision it should be necessary to take into account the global distribution of pressure over continents and oceans. Up to now the exact behaviour of the ocean submitted to pressure effects is not clearly understood. The inverse barometer hypothesis does not seem to match always the reality.

Table 1

Required Precision
for
Gravity Tides Evaluation

10 μgal	- Gravimetric Prospection with Worden type instruments (0.1 mgal precision)
1 μgal	- Relative gravity measurements with a precision of a few microgal (LaCoste & Romberg instruments) - Absolute gravity measurements
0.1 μgal	- microgravimetry - Earth tides analysis of spring gravimeters records
0.01 μgal	- Earth tides analysis with Superconducting gravimeters

2. PRACTICAL REQUIREMENTS

Considering again table 1 let us try to see how we can reach the different levels of precision.

At the 10 μ gal level of accuracy any method will fit while less than 100 terms are required for the harmonic development.

The 1 μ gal accuracy for the reasons explained before, can only be obtained by the harmonic development of the potential.

For what concerns the internal precision of the development it is sufficient to keep all terms with amplitude higher than 0.01 μ gal. It will evidently depend on the latitude, the largest number of terms being required at 45° degrees.

Figure 1 shows that a development reduced to 180 terms fullfills practically our requirements.

To obtain 1 μ gal accuracy we must besides modelize the tidal factors with a one per cent accuracy in amplitude and a 0°5 precision in phase. The most direct way to achieve that goal is to perform tidal gravity observations. However many authors pointed out the difficulty to calibrate gravimeters for tidal records at the one per cent level.

One can also combine a body tides model with local oceanic tidal loading computations. to obtain synthetic amplitude factors and phase lags. For inland locations, load computations using the standard Schwiderski maps are certainly meeting the one microgal level of accuracy. It should also be pointed out in that context that several body tides models are existing which may differ at the one per cent level.

From the preceeding considerations it is clear that one microgal is the present day accuracy limit, but greater internal precision can easily be achieved and is certainly useful for many applications. For example a 0.1 μ gal precision is desirable for microgravimetric measurements where only short time spans are involved.

For harmonic tidal analysis also we need the most precise representation of the tidal force to adjust the observations on it. Up to now we used the standard Cartwright-Tayler-Edden (CTE) development reaching nearly this 0.1 μ gal precision (figure 2).

To improve the internal precision below that level it is necessary to include terms derived from the fourth order potential.

Four recent developments due respectively to Bullesfeld (1985), Xi Qin Wen (1987 & 1988) and Tamura (1987) are now available. We shall compare their performances and try to determine the actual limit of precision. But it seems useful to discuss first the characteristics of the tidal potential and the most practical way to use it for tidal computations.

3. THE TIDAL POTENTIAL

Under its most general form the tidal potential of the Moon is written as

$$W = GM \sum_{n=2}^{\infty} \frac{r^n}{d^{n+1}} P_n(\cos z) \quad *$$

This expression is generally transformed into:

$$W = \underbrace{-\frac{3}{4} (GE) \mu \frac{a^2}{c^3} \frac{r}{a} (-)^2}_{(a)} \underbrace{\frac{c}{d} \frac{r^2}{d^3} \left[-P_2 + \frac{r}{d} P_3 + \dots \right]}_{(b)}$$

The first part (a) of the formula is a scale factor $D(r)$ depending only of the geocentric distance r . Through Laplace development the second part will be splitted in a latitude dependent part (geodetic coefficient $g_{nm}(\theta)$) and a time dependent part that will generate infinite series of harmonic terms.

Let us take the example of the second order potential.
We obtain by Laplace development three families of waves.

$n = 2$	$g_{nm}(\theta)$	time dependant part
$m = 0$ Long Period	$3(\sin^2\theta - 1/3)$	$(\sin^2\delta - 1/3)$
$m = 1$ Diurnal	$\sin 2\theta$	$\sin 2\delta \cos AH$
$m = 2$ Semi Diurnal	$\cos^2\theta$	$\cos^2\delta \cos 2AH$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} \cdot \frac{c}{d} (-)^3$$

We shall now examine the three constitutive elements of the Tidal Development.

4. THE SCALE FACTOR

The scale factor is directly linked to the Doodson constant. However there are presently two definitions of the Doodson constant.

The original one D_0 is due to Doodson himself and has been adopted by Melchior and Bullesfeld.

We can write

$$D(r) = D_0 \frac{r}{a} (-)^2$$

* all symbols and notations are explained in appendix 1.

with

$$D_0 = -\frac{3}{4} \mu \bar{g} \bar{a}^4 \left(\frac{\sin \pi}{a} \right)^3$$

and

$$\bar{g} = \frac{GE}{\bar{a}^2}, \quad \sin \pi = \frac{a}{c}$$

Taking into account that GE is now a fundamental constant for geophysicists and astronomers, Xi Qin Wen and Tamura adopt a more straightforward definition:

$$D(r) = D_1 \frac{r}{a}^2$$

with

$$D_1 = -\frac{3}{4} (GE) \mu \frac{1}{a} (\sin \pi)^3$$

Obviously one has the relation:

$$\frac{D_1}{D_0} = \left(\frac{a}{\bar{a}} \right)^2$$

Table 2 gives numerical values of D_0 and D_1 according to different standards.

In practice differences appear only at the 10^{-5} level i.e. a few nanogals.

For higher potential orders there is a factor $(r/d)^{n-2}$ which is again splitted

$$\frac{r}{d} = \frac{r}{a} (\sin \pi) \cdot \frac{c}{d}$$

Only r/a is included in the scale factor $D(r)$ and $(\sin \pi) c/d$ is entering the harmonic development. It follows an amplitude decrease of the harmonic terms by a factor $(\sin \pi)^{n-2}$.

Table 2

Computation of Doodson Constants

	IUGG67	IAU1976	MERIT
a	6378160m	6378140m	6378137m
\bar{a}	6371023.60m	6371003.78m	6371000.79m
GE(10 ⁻¹⁴)	3.9860297m ³ s ⁻² *	3.986005m ³ s ⁻²	3.98600448m ³ s ⁻²
μ	0.0123000123	0.01230002	0.012300034
sin π	3422"451	3422"451	3422"448
D ₁	2.633613m ² s ⁻² **	2.6335838m ² s ⁻²	2.6335811m ² s ⁻²
(a/ \bar{a}) ²	1.002241523	1.002241473	1.002241471
D ₀	2.627723m ² s ⁻²	2.6276939m ² s ⁻² ***	2.6276912m ² s ⁻² ***
	Melchior Bullesfeld	Xi Qin Wen	Tamura

* from GE = $\bar{g} \bar{a}^2$ with $\bar{g} = 9.82024 \text{ m s}^{-2}$

** obtained by the relation $D_1 = D_0 (a/\bar{a})^2$

*** obtained by the relation $D_0 = D_1 (\bar{a}/a)^2$

5. THE GEODETIC COEFFICIENTS

The geodetic coefficients are not only latitude dependent but also unnormalized (*). When wishing to develop a given potential order at a given threshold level the different families will be multiplied by different factors. But it is at least possible to obtain that their maximum amplitudes are directly comparable.

It is obtained by multiplying the amplitudes of the terms coming from a given family by

$$\Gamma_{nm} = |g_{nm}(\theta)|_{\max}$$

Obviously the corresponding geodetic coefficients will have to be divided by these Γ_{nm} .

For example at order 2

$$\Gamma_{20} = 2, \Gamma_{21} = 1, \Gamma_{22} = 2$$

and the geodetic coefficients become $g_{20} = 3/2 (\sin^2 \theta - 1/3)$, $g_{21} = \sin 2\theta$, $g_{22} = \cos 2\theta$.

For higher order the same normalisation system is still valid.

Of course, when dealing no more with the potential itself but with its derivatives, direct comparison between terms coming from different potential orders is no more possible.

It is a reason why Bullesfeld proposed an additional normalisation by a factor $2/n (a/a)^{n-2}$ which has the advantage to normalize correctly the vertical component of the tidal force.

On the other hand the problem is getting worse as Doodson originally put by mistake

$$\Gamma_{30} = \frac{\sqrt{5}}{2}$$

Up to now only Bullesfeld wanted to correct it and we should not recommend to do it.

For our tests we recomputed the amplitude of Bullesfeld's development in the standard normalisation system.

(*) Cartwright Tayler and Edden are using a completely different system of normalisation.

6. THE HARMONIC DEVELOPMENT

For each celestial body we have three quantities which depends on time i.e.

AH : the hour angle

δ : the declination

c/d : the ratio between the mean and instantaneous distance to the Earth.

There are obviously two methods to produce an harmonic development.

We can develop the functions of these arguments with respect to variables linearly depending from time. It is the analytical method.

We can also start from the luni-solar ephemeris, compute hourly numerical values of the tidal potential which is expressed as a function of r , δ and z , and, by harmonic analysis, identify all terms taking into account that all possible frequencies are known beforehand.

The analytical method was used by Doodson and by Xi Qin Wen. The spectral analysis method was used by CTE, Bullesfeld and Tamura.

It is well known that, following the classical definition of Doodson, the argument of any tidal wave can be expressed as a linear combination of Moon and Sun arguments i.e.

$$t = n_1 \tau + n_2 s + n_3 h + n_4 p + n_5 N' + n_6 p_s$$

with $\tau = 15^\circ \cdot t + h - s +$: lunar time
$t = UT$: universal time
s	: mean longitude of the Moon
h	: mean longitude of the Sun
p	: mean longitude of the perigeum of the Moon
$N' = -N$: mean longitude of the lunar node
p_s	: mean longitude of the perihelium

The lunar and solar arguments can be evaluated according to Brown's and Newcomb's formulas expressed in fractions of julian century.

This original definition has to be improved.

It is necessary to replace UT in the Brown's and Newcomb's formulas the dynamical time TD which is replacing the ephemeris time ET since 1984. The difference depends on the epoch considered e.g. in 1987 one has

$$TD = UT + 57s$$

It produces a slight offset of the time scale. This improvement is already proposed by Bullesfeld (1985).

Tamura proposes other improvements for the computation of the arguments.

He is introducing periodic terms s and h in the computation of Moon and Sun longitude and proposes a revised definition of the lunar time

$$\tau = 15^\circ \cdot t + \alpha_m - s$$

Where α_m differs from h by:

- the subtraction of the aberration ($20''5$) from the apparent position of the Sun in order to introduce true position.
- the use of t in the computation of the fraction of Julian century.

These corrections modify the computed value of the wave arguments. The difference reaches up to ± 0.1 for terms with large $|n_2|$ and $|n_3|$.

As demonstrated by our tests these new definitions are slightly improving the tidal predictions whatever be the developments used (table 3, fig. 3).

Moreover, Tamura introduced two supplementary arguments to take into account the perturbations of the Earth's orbit by Venus and Jupiter. This produces 8 additional terms by modulating the main solar terms.

7. PROCEDURE OF TEST

We can use the different tidal developments to generate tidal predictions and compare them to a standard data set independently computed. This data set has obviously to be generated by direct computations from the Moon and Sun ephemeris but not the same one as those used by Cartwright, Bullesfeld or Tamura.

According to these criteria the only data set available to us was one year of gravity tides computed by Wenzel from Moon and Sun ephemeris for the year 1971 at the station Hannover. However the values are given in $0.01 \mu\text{gal}$ units and the precision of the best tidal developments would require values expressed in nanogal.

The final comparison is made by subtracting the hourly values of the tidal predictions from the hourly values of the standard data set and by performing a Fast Fourier Transform on the residuals to obtain amplitude and power spectra. However the mere inspection of a direct representation of the residues allows already to compare the precision of the different tidal developments (fig. 1 to 8). The different results are summarized in table 3.

In figure 1 the reduced development intends to keep all waves with amplitude greater than $0.01 \mu\text{gal}$ at 45° of latitude. One sees that it will ensure a precision of $\pm 1 \mu\text{gal}$.

The complete development increases the precision to ± 0.25 μgal . Bullesfeld was the first to introduce terms derived from the 4th order potential (fig.2). This is sufficient to reduce the largest residues by a factor of two (± 0.125 μgal).

Similar conclusions are drawn from the comparison of the power spectra of the residuals. In figure 4 one can see that the introduction of the fourth order potential is reducing drastically not only the fourth diurnal spectrum but also the ter-diurnal one. The large peaks in long period, diurnal and semi-diurnal bands drop one order of magnitude.

Up to now we are still using the standard formula for the computation of the wave arguments.

The corrections introduced by Tamura are producing a ten per cent decrease of the largest residues (fig. 3) and is also reducing the standard deviation (table 3).

The Xi Qin Wen development (1178 waves) computed for the year 2000 is constructed by the analytical method.

It is also including the fourth order potential but not the small fourth order terms which coincide with some of the largest constituents in LP, D and SD bands.

By discarding all terms with coefficient of amplitude equal to 1.10^{-5} we obtain a potential reduced to 839 terms.

Although there are already more terms than in Bullesfeld's development the residuals are slightly larger (table 3). Even the complete development hardly reduces the residues. It is due to the fact that the supplementary terms with amplitude coefficients equal to 1.10^{-5} are largely undetermined. As their real amplitude is in fact ranging from $0.5 \cdot 10^{-5}$ to $1.4 \cdot 10^{-5}$ the noise level is nearly equal to the signal. It is clear that such a development should require one more significant digit.

This step was made by Tamura for his development. He is giving amplitudes to the sixth decimal digit. Only the constituents with amplitudes larger than 10^{-5} (second order), $7 \cdot 10^{-6}$ (third order) and $5 \cdot 10^{-6}$ (fourth order) are taken into account. These amplitudes thresholds are corresponding to 0.8 nanogal for each order of the potential.

A reduced development has been obtained following a rule similar to the criterion used for Xi Qin Wen. If we eliminate the terms with amplitude coefficient lower than $1.5 \cdot 10^{-5}$, this Tamura reduced potential TAM 920 does not show up residues larger than 0.06 μgal which represents a reduction of fifty percent with respect to Bullesfeld (figure 5).

However the neglected terms do significantly improve the precision as only a few residues exceed 0.03 μgal when using the complete Tamura potential TAM 1200 (figure 5).

In figure 6 we can see that Tamura potential is reducing all peaks by one order of magnitude.

In 1988 Xi Qin Wen computed a new tidal development giving the amplitudes to the sixth decimal place. He refined also the computation of the lunar and solar arguments introducing additional perturbations to s and principally h .

For comparison purposes we created reduced developments by eliminating all terms with amplitude lower than $1.5 \cdot 10^{-5}$ and $5 \cdot 10^{-6}$ respectively. We obtained potential developments with 910 terms and 1398 terms directly comparable with TAM920 and TAM1200. The level of the residuals is quite similar as seen from table 3 and figure 7.

The very slight improvement obtained by Xi Qin Wen (1988) is mainly due to the reduction of the power in ter and quaterdiurnal bands (figure 8).

It should be noted that the best precision is only obtained by using the new set of formulas for the computation of the lunisolar arguments. A test made with Tamura's formula showed a ten percent increase of the standard deviation for XI 910. In practice however the introduction of short period corrections is restricting the use of recurrence formula in the computation. At the nanogal level the recurrence should be reinitialized after a few days. With Tamura on the contrary no difference does appear after one year of use of the recurrence with respect to the direct computation.

As explained before the smallest terms with amplitude $1 \cdot 10^{-6}$ are largely undetermined and produce only a kind of noise. It is a reason why we also computed residuals for a reduced potential XI 2026 in which all terms with amplitude coefficients $1 \cdot 10^{-6}$ were suppressed. As seen from table 3 there is no difference with respect to the complete development XI 2920.

Taking into account the rounding error of the Wenzel data set the standard deviation of Tamura and Xi (1988) developments reaches thus 10 nanogal.

8. DIRECT COMPARISON

As the accuracy of the test series of Wenzel is not sufficient for our purpose, the direct comparison of the two developments with similar precision will give the best estimation of the accuracy really achieved.

From table 4 it is clear that the peak to peak amplitude of the hourly differences can reach 50 nanogal i.e. $2 \cdot 10^{-4}$ of the tidal range. The standard deviation is close to 7 nanogal.

We also performed a comparison between XI2920 and XI2026. The additional waves with amplitude $1 \cdot 10^{-6}$ can produce differences up to ± 5 nanogal with a standard deviation of 1.6 nanogal.

More striking is the comparison of tidal predictions obtained by computing the lunisolar arguments either with the Tamura's formula either with the Xi Qin Wen ones. Here the bigger differences are reaching ± 10 nanogal with a standard deviation of 3 nanogal.

Table 3

Comparison of Different Tidal Developments
With the
Wenzel Standard Data Set at Hannover

Repartition of the Residuals

	n	Rmin	Rmax	Range	σ
a) CTE	118	-1.36	1.21	2.57	0.3382
	505	-0.19	0.25	0.44	0.0416
Bul	656	-0.10	0.14	0.24	0.0262
<hr/>					
b) Bul	656	-0.09	0.13	0.22	0.0252
Xi(87)	839	-0.12	0.14	0.26	0.0305
Tam	920	-0.06	0.05	0.11	0.0152
	1200	-0.04	0.03	0.07	0.0109
Xi(88)	910	-0.09	0.05	0.14	0.0169
<hr/>					
c) Xi(88)	910	-0.09	0.05	0.14	0.0156
	1398	-0.06	0.03	0.09	0.0113
	2026	-0.06	0.02	0.08	0.0107
	2920	-0.06	0.02	0.08	0.0108

n : number of terms

Rmin, Rmax : extreme values

Range : peak to peak amplitude

σ : standard deviation

The mean value of the residual R = - 0.015

All data are expressed in microgal

a) arguments computed according to Melchior 1973

b) arguments computed according to Tamura 1987

c) arguments computed according to Xi Qin Wen 1988

Table 4

Gravity Tides Computations
for
Station Hannover

Comparison of Different Models

	Rmin (μ gal)	Rmax (μ gal)	Range (μ gal)	R (μ gal)	σ (μ gal)
Xi 1398 - Tam 1200	-0.019	0.028	0.047	0.002	0.0065
Xi 2920 - Xi 2026	-0.005	0.006	0.011	0.000	0.0016
Xi 2920 - Xi 2920 (arg TAM) (arg Xi)	-0.009	0.009	0.018	0.000	0.0029

Rmin, Rmax : extreme values of differences

Range : Rmax - Rmin

R : mean value of the differences

σ : standard deviation of the differences

9. CONCLUSIONS

The resolution of the standard set of Wenzel is not sufficient to establish the real precision of Tamura's and Xi Qin Wen developments. We can only give an upper limit for the standard deviation reaching 0.01 μ gal.

The real accuracy of the two best models is quite better with peak to peak differences reaching only 50 nanogal and a standard deviation of 7 nanogal.

Xi Qin Wen (1987) development should be obviously replaced by the more recent one (1988).

Bullesfeld's development (656 terms) is quite economical in computer time and memory and is already exhibiting the most important features of more complete developments. It insures an improvement by a factor of two with respect to Cartwright, Tayler & Edden. It could be recommended for most of the applications except for tidal analysis.

Both Tamura and Xi Qin Wen which are much more time consuming (especially the second one) and should be reserved to dedicated applications, such as tidal analysis.

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APPENDIX

a : equatorial radius of the Earth

\bar{a} : radius of the equivolume sphere

c : mean distance to the Moon

d : instantaneous distance to the Moon

\bar{g} : mean gravity

$g_{nm}(\varnothing)$: geodetic coefficient associated to P_n^m

h : mean longitude of the Sun

p : mean longitude of the perigium of the Moon

p_s : mean longitude of the perihelium

r : radial distance to the center of the Earth

t : universal time

z : zenithal distance

E : mass of the Earth

G : constant of gravitation

M : mass of the Moon

N : mean longitude of the lunar node

P_n : Legendre polynomial of order n

P_n^m , $m \leq n$: Associated Legendre polynomial

W : tide generating potential

δ : declination

\varnothing : geocentric latitude

: longitude

$\mu = \frac{M}{E}$: reduced mass of the Moon

τ : lunar time

Γ_{nm} : normalisation factors for the amplitudes of the tidal waves

AH : hour angle

$\sin \pi$: sine parallax of the Moon

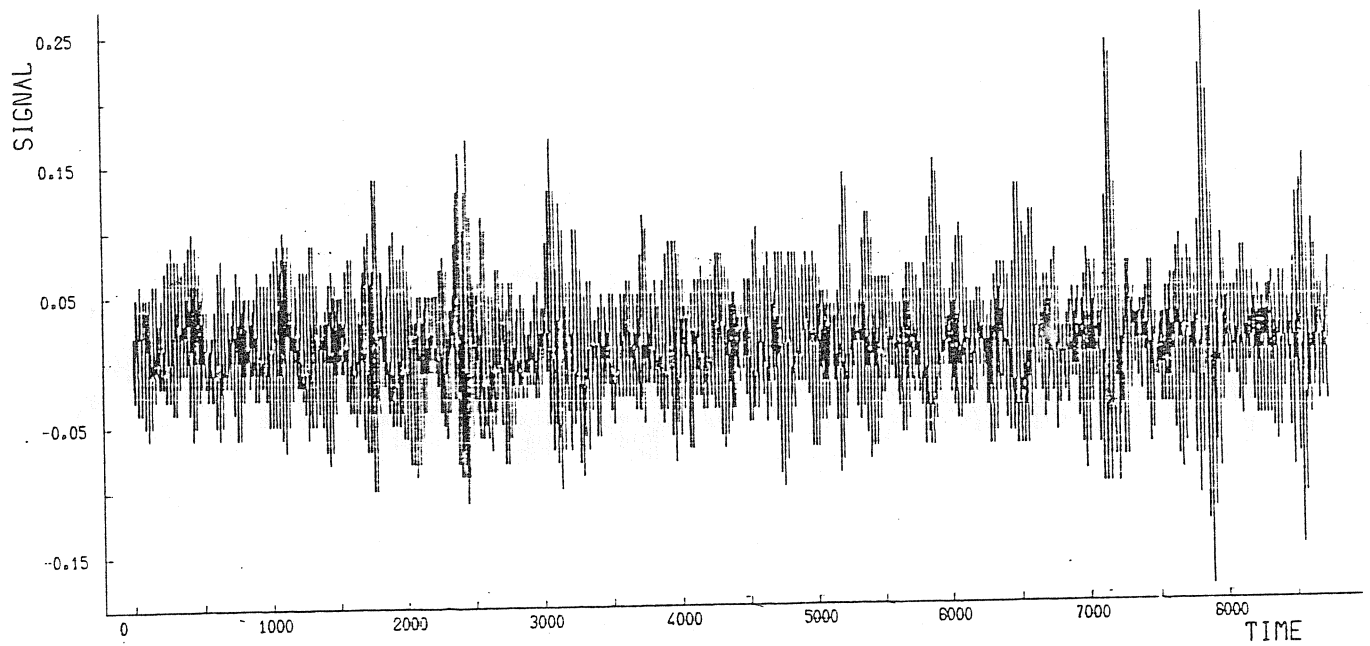
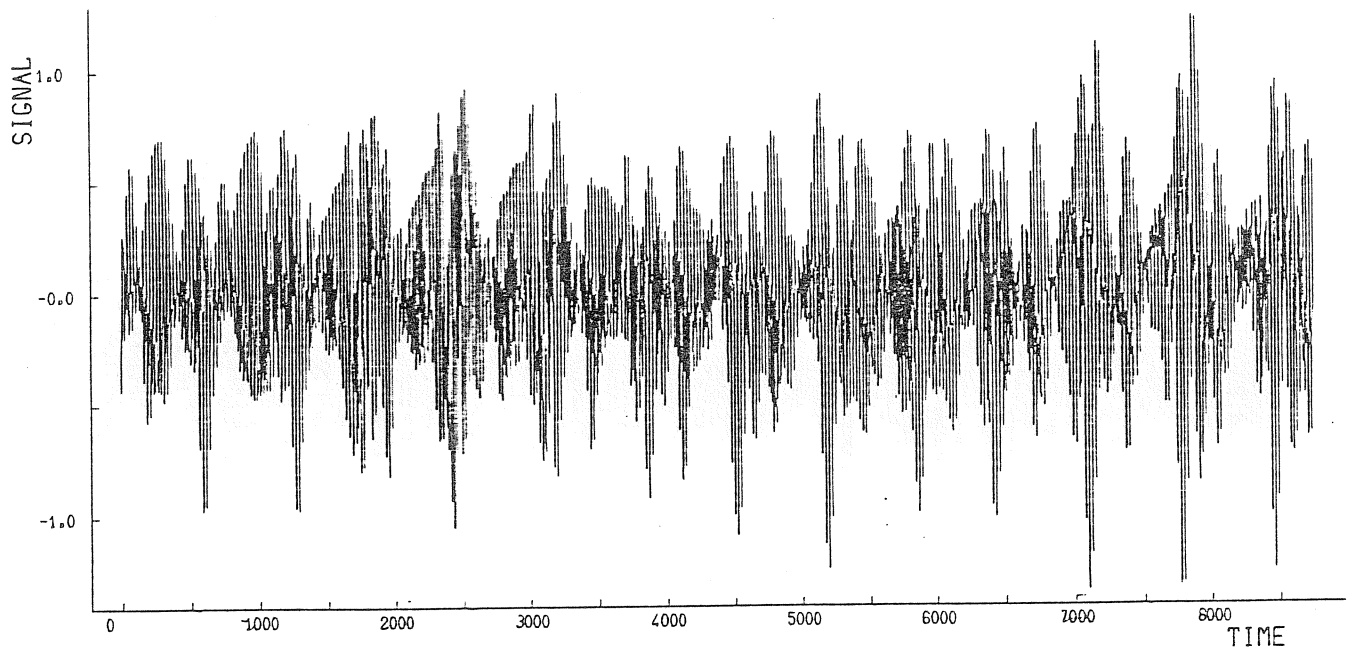


Figure 1: Hourly residues. (μgal)
UPPER reduced CTE development (118 waves)
LOWER complete CTE development (505 waves)

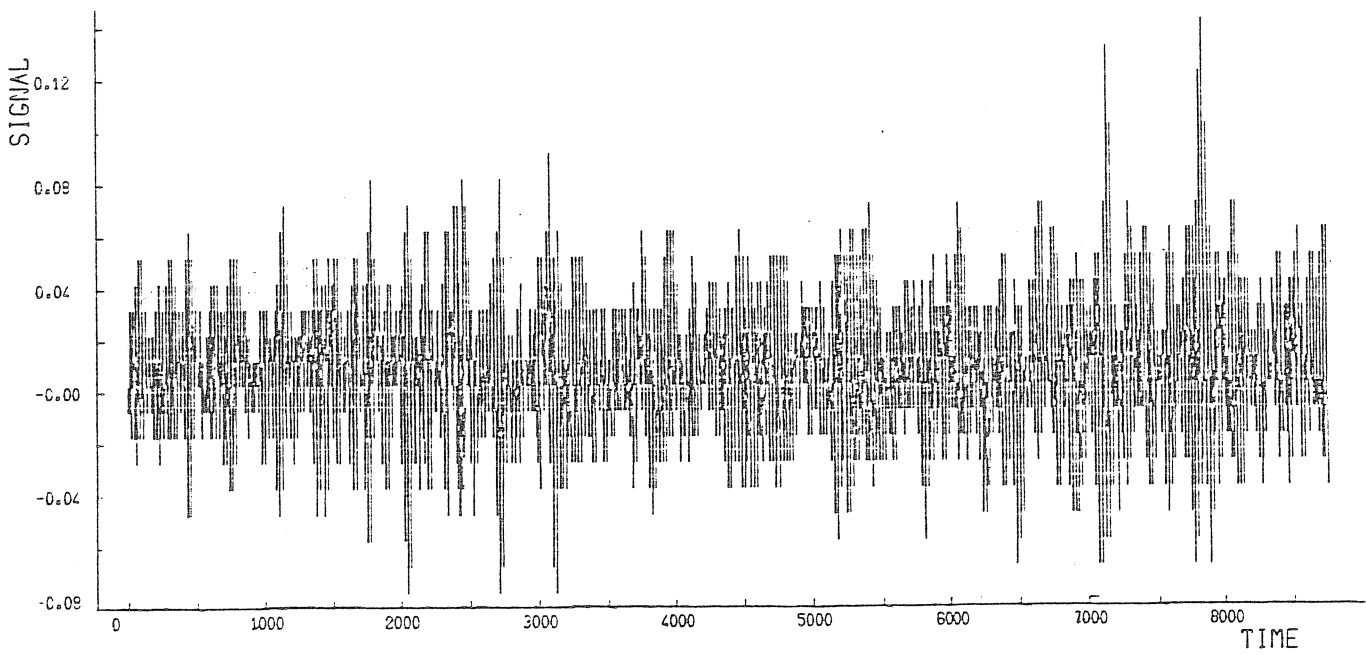
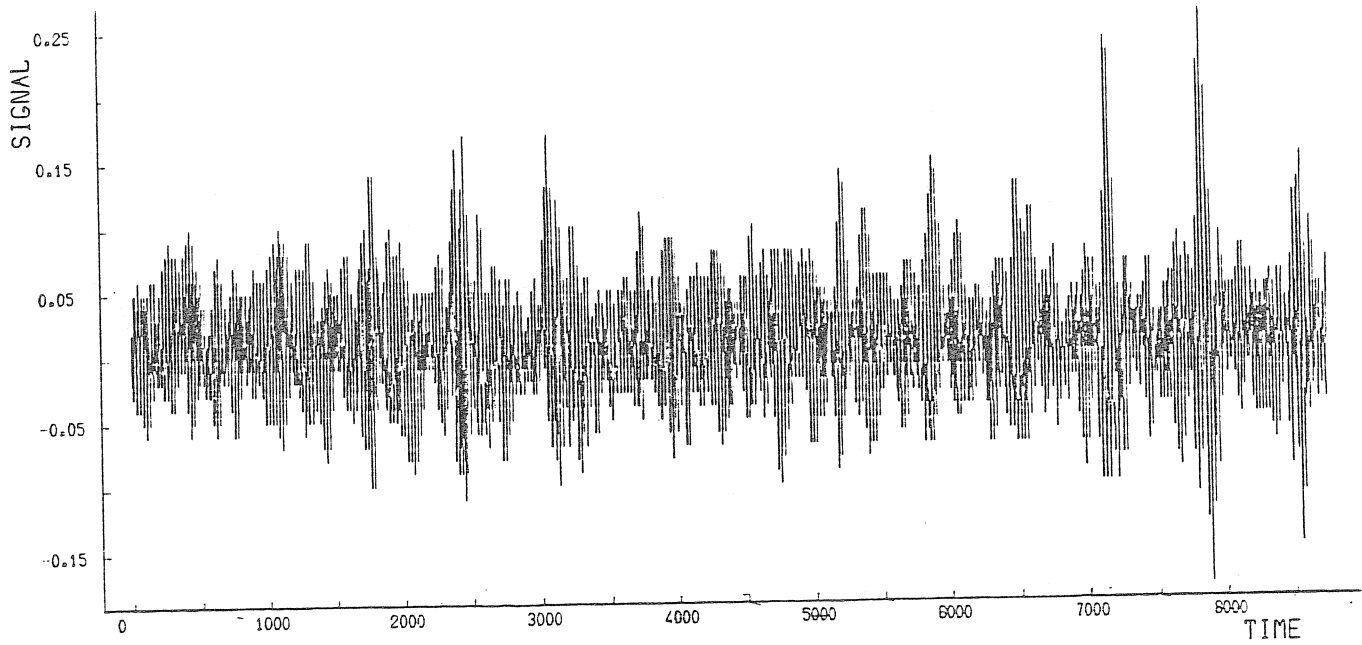


Figure 2: Hourly residues. (μgal)
UPPER standard CTE development (505 waves)
LOWER Bullesfeld's development (656 waves)

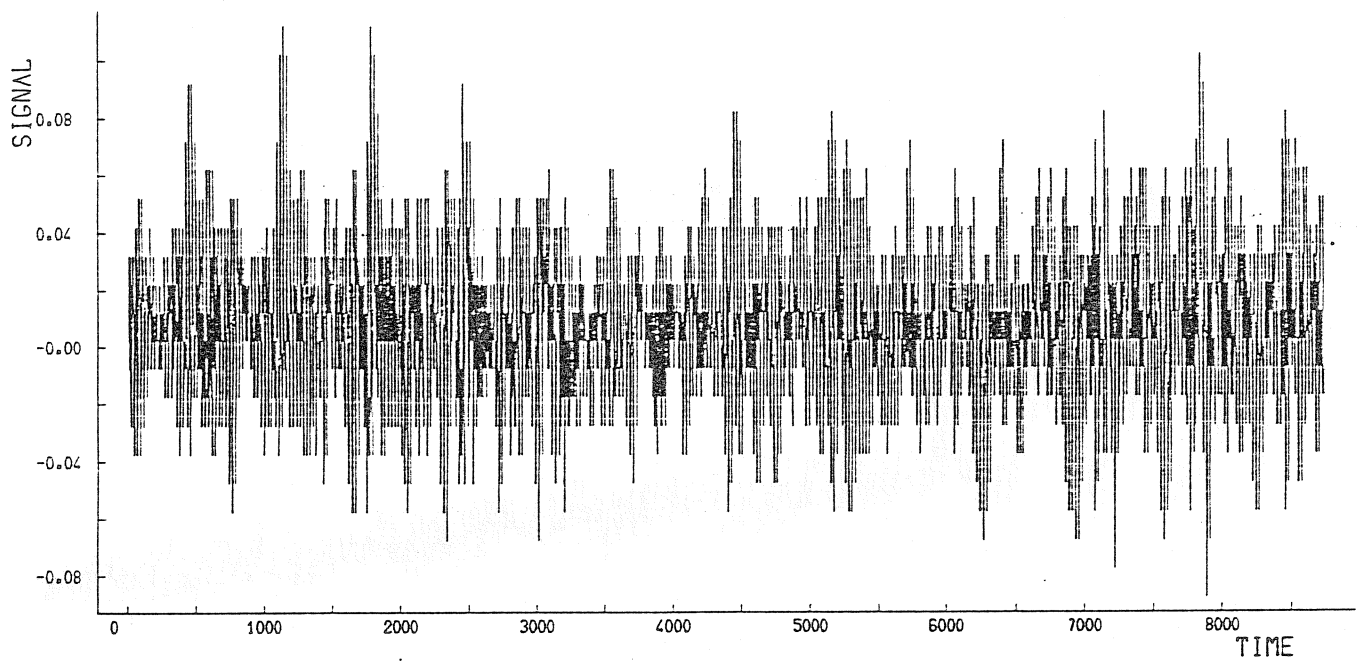
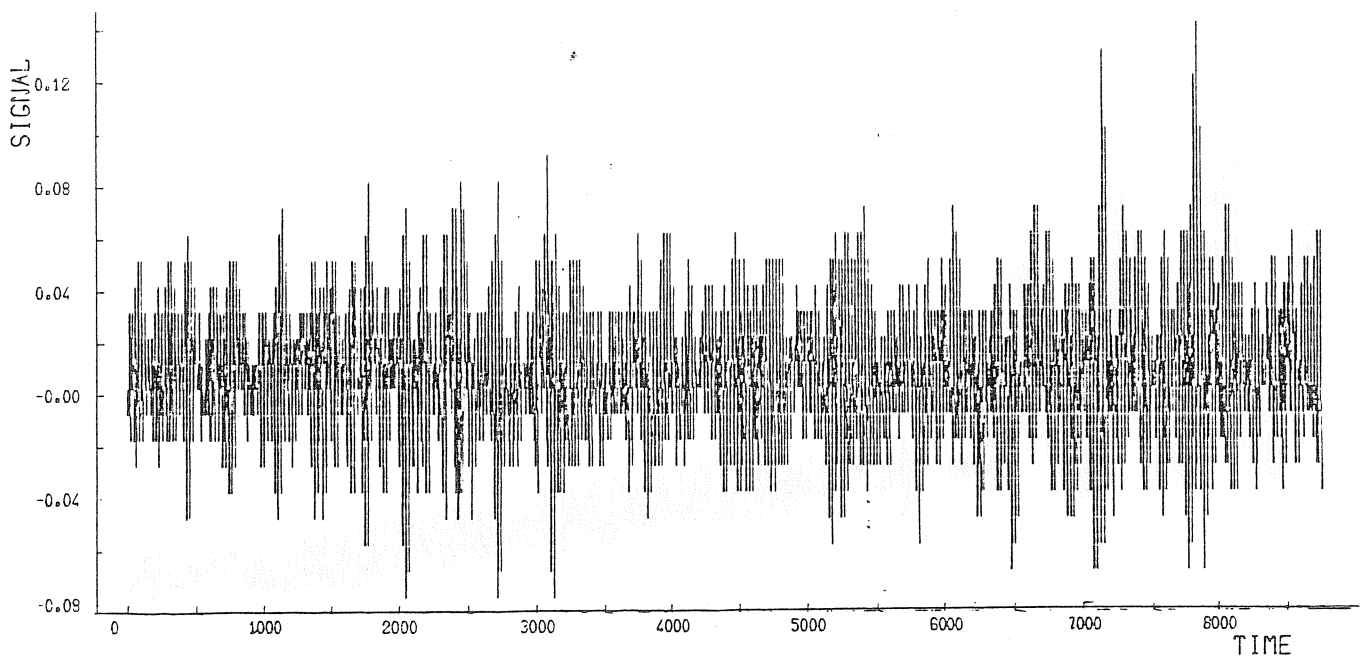


Figure 3: Hourly residues, different evaluations of the wave arguments. (pgal)
UPPER Bullesfeld with standard formula.
LOWER Bullesfeld with improved formula (Tamura).

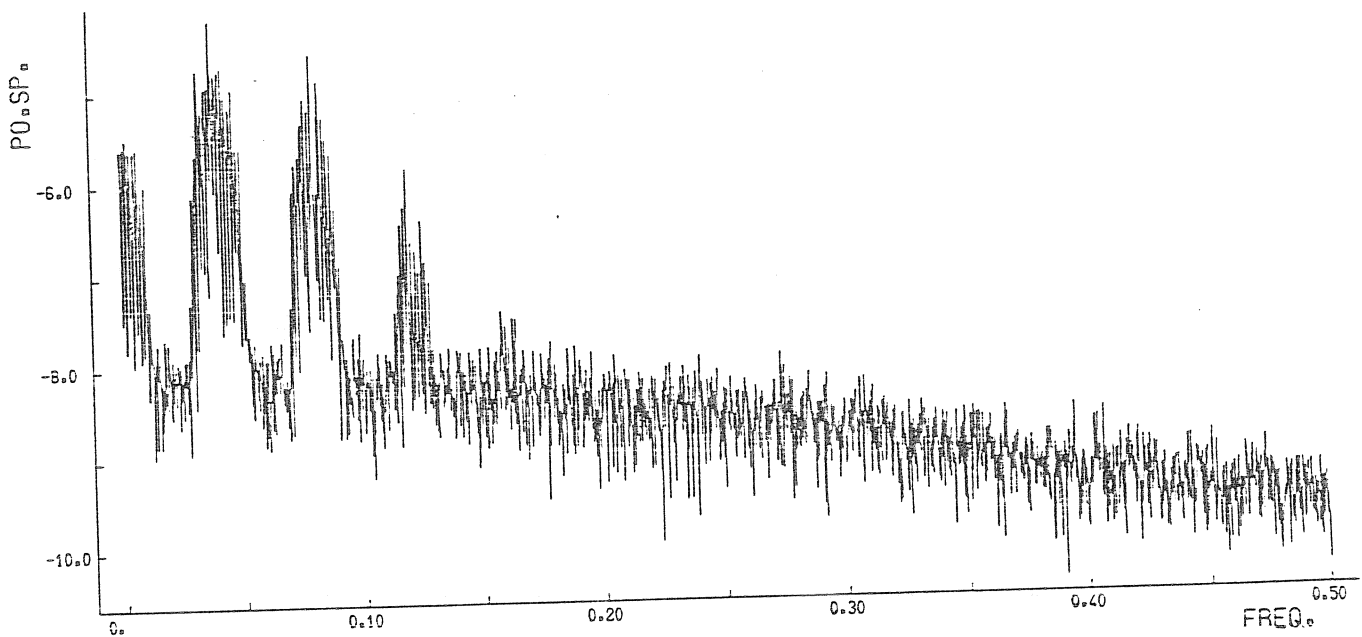
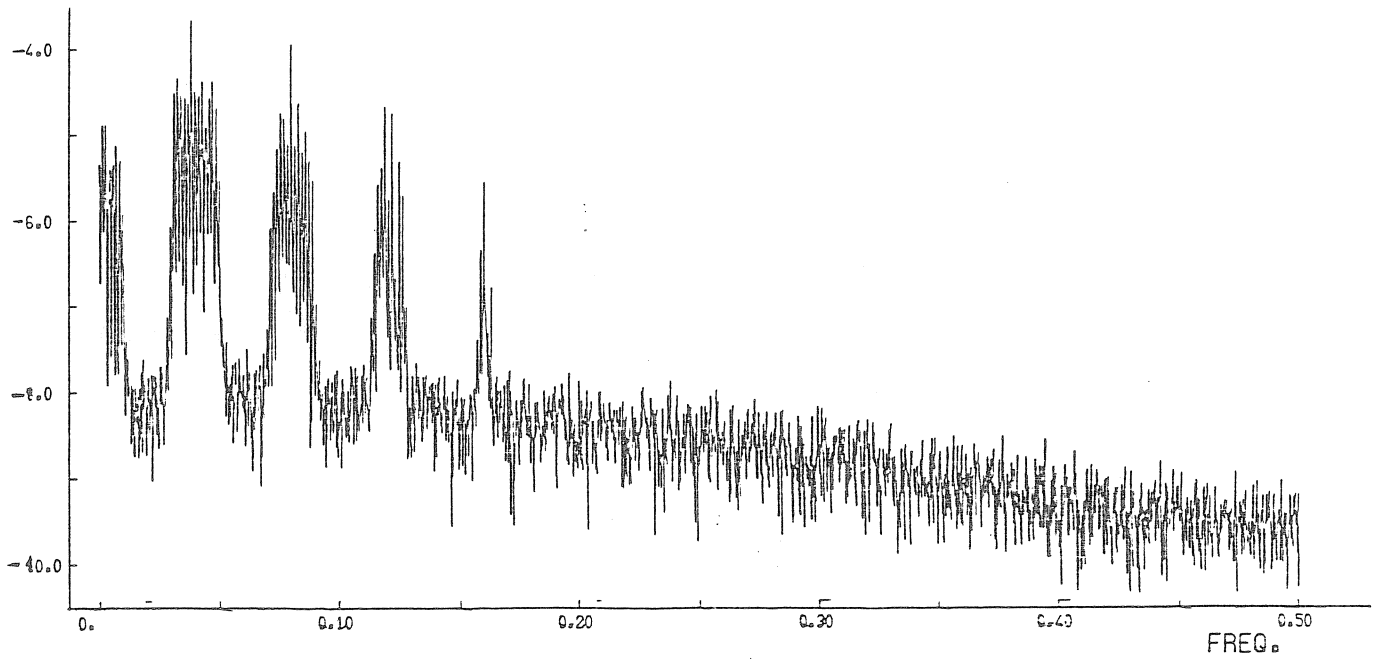


Figure 4: Power spectrum in logarithm of microgal squared as a function of frequency in cycle per hour.
UPPER Cartwright-Tayler-Edden
LOWER Bullesfeld

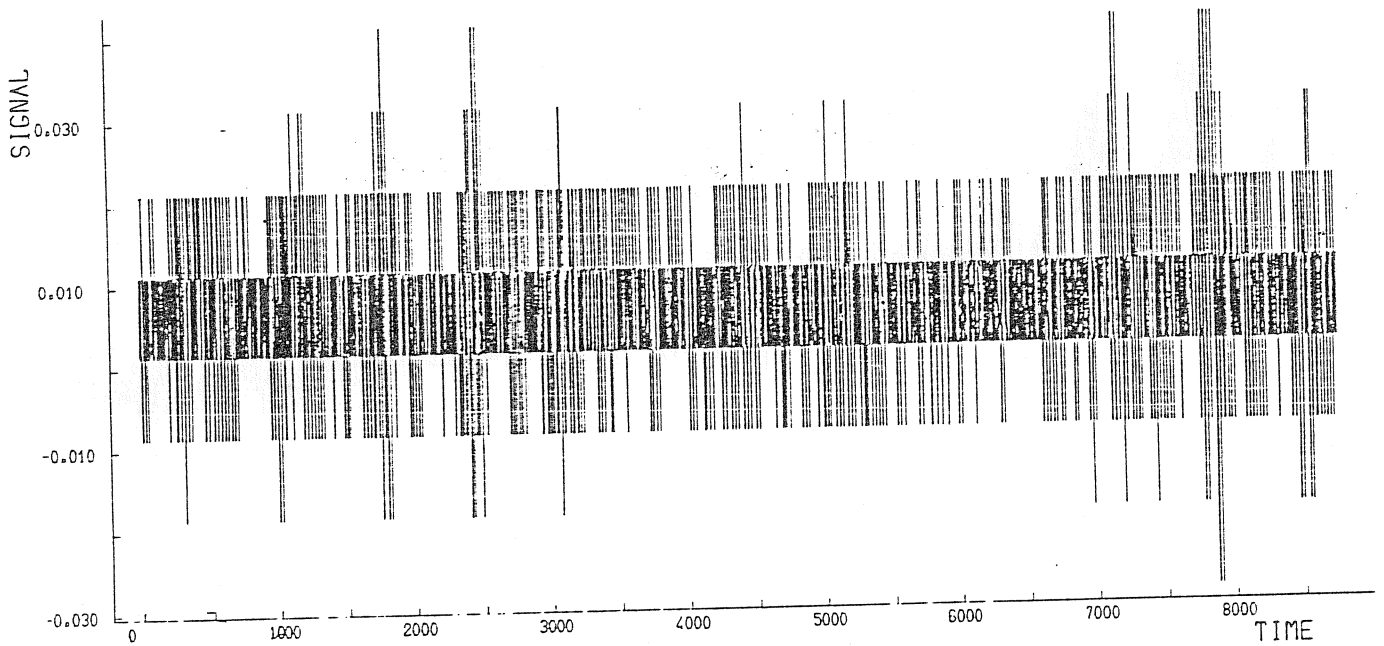
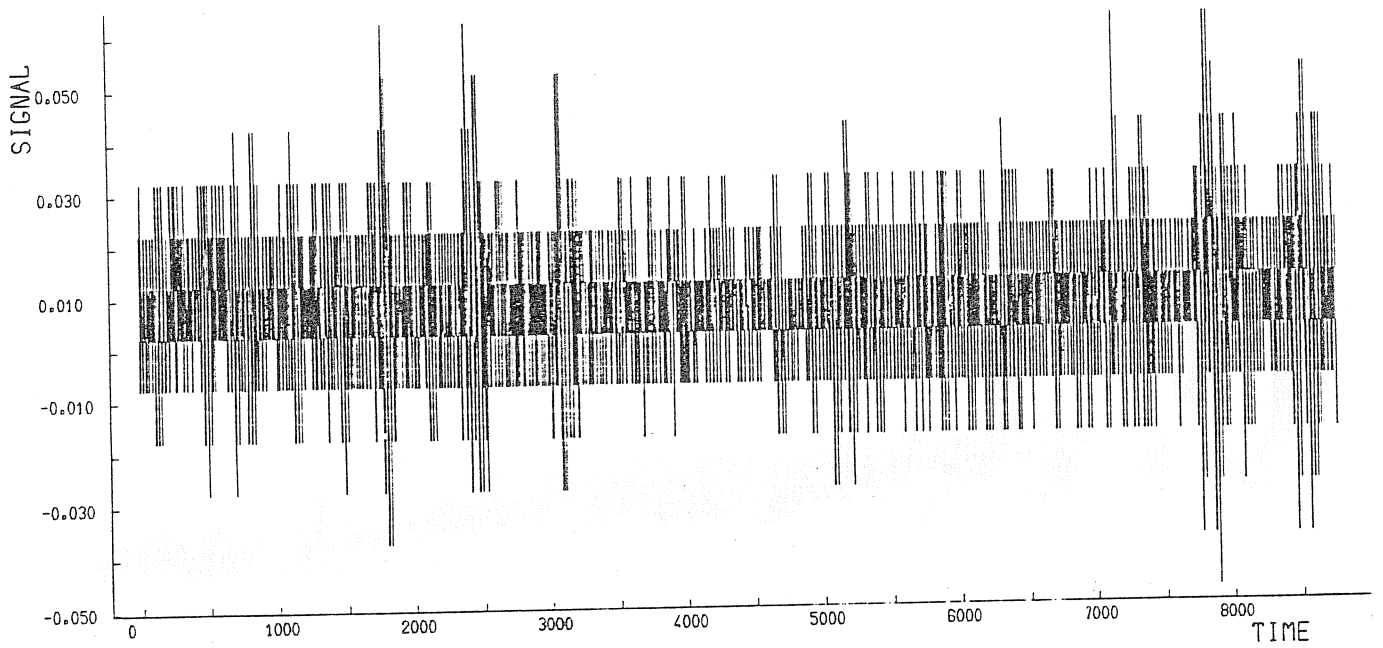


Figure 5: Hourly residues. (μgal)
UPPER Tamura reduced to 921 terms
LOWER Tamura complete (1200 terms)

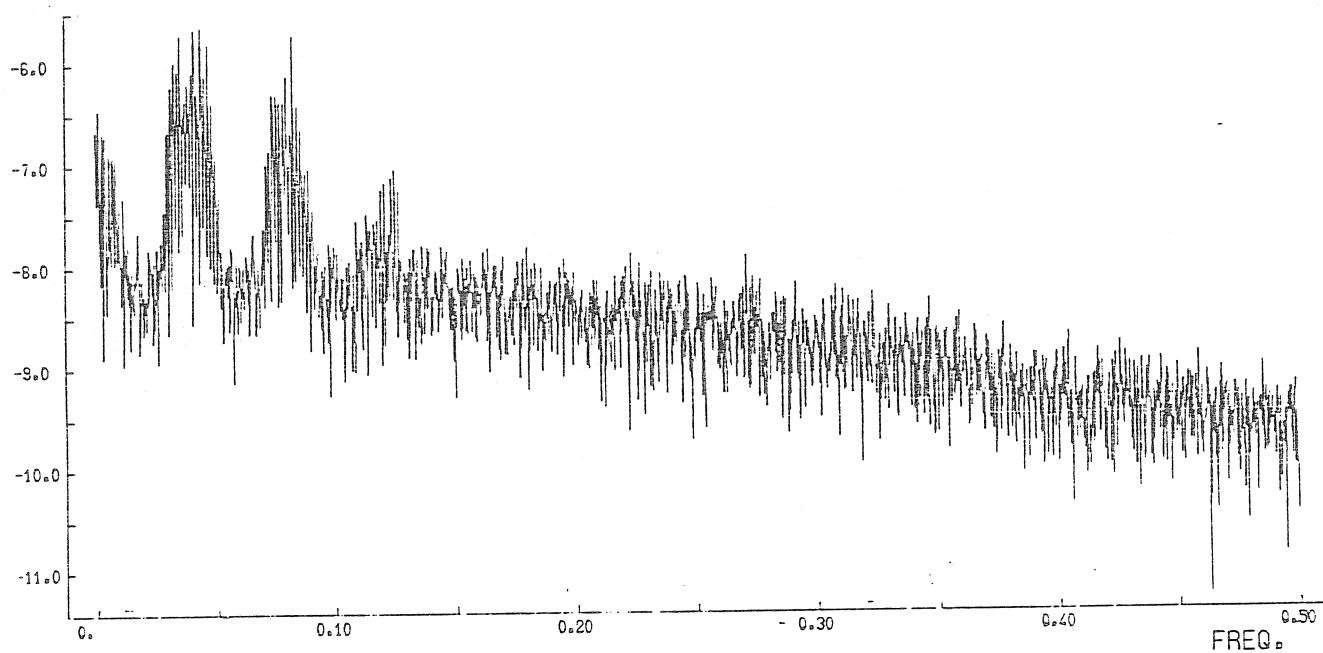
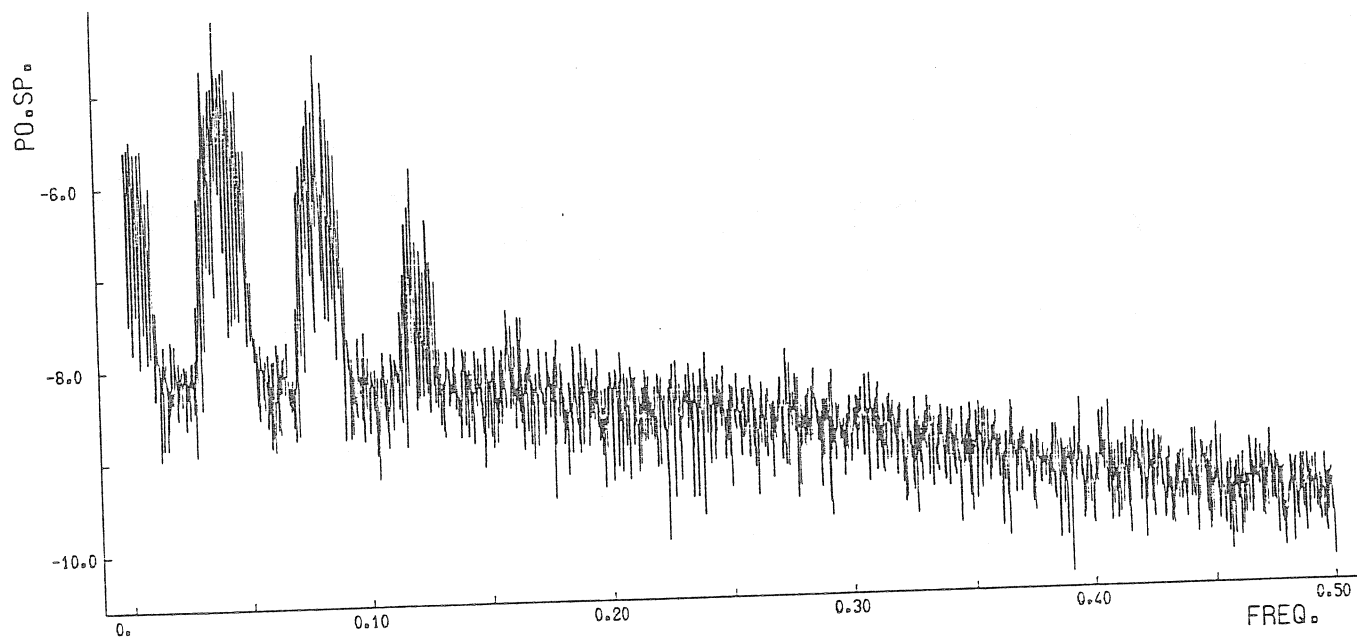


Figure 6: Power spectrum in logarithm of microgal squared as a function
of frequency in cycle per hour.
UPPER Bullesfeld
LOWER Tamura

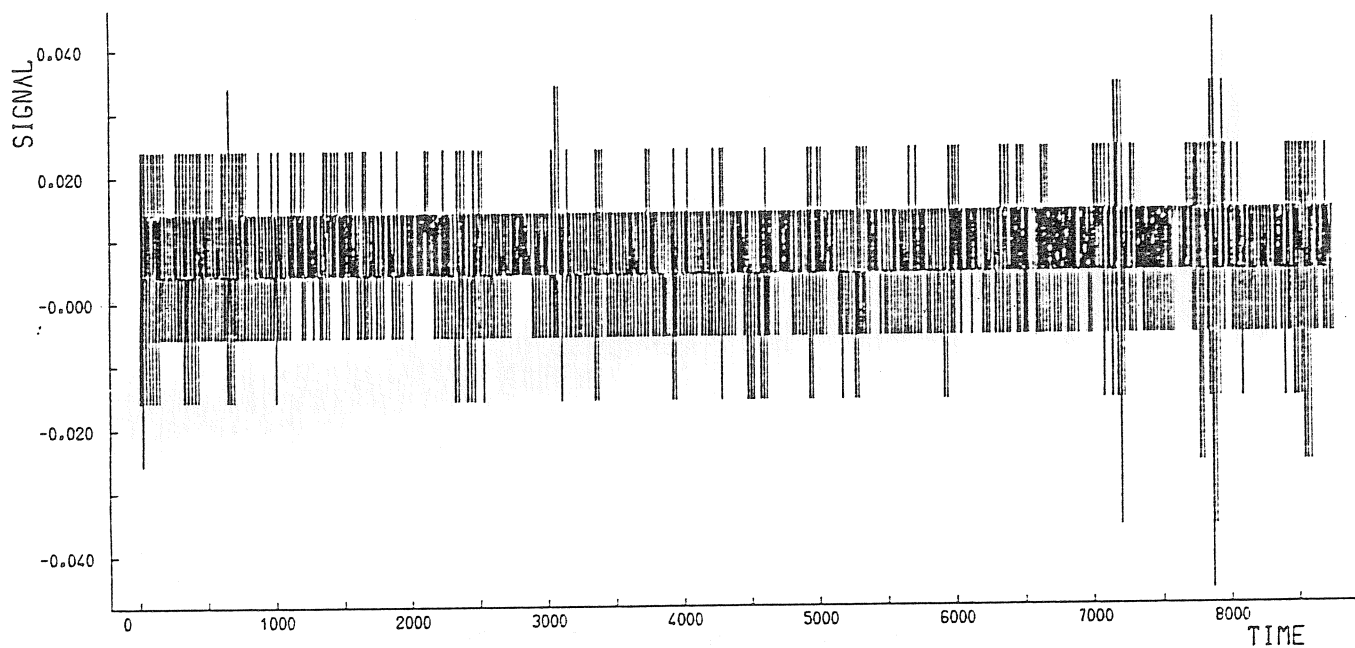
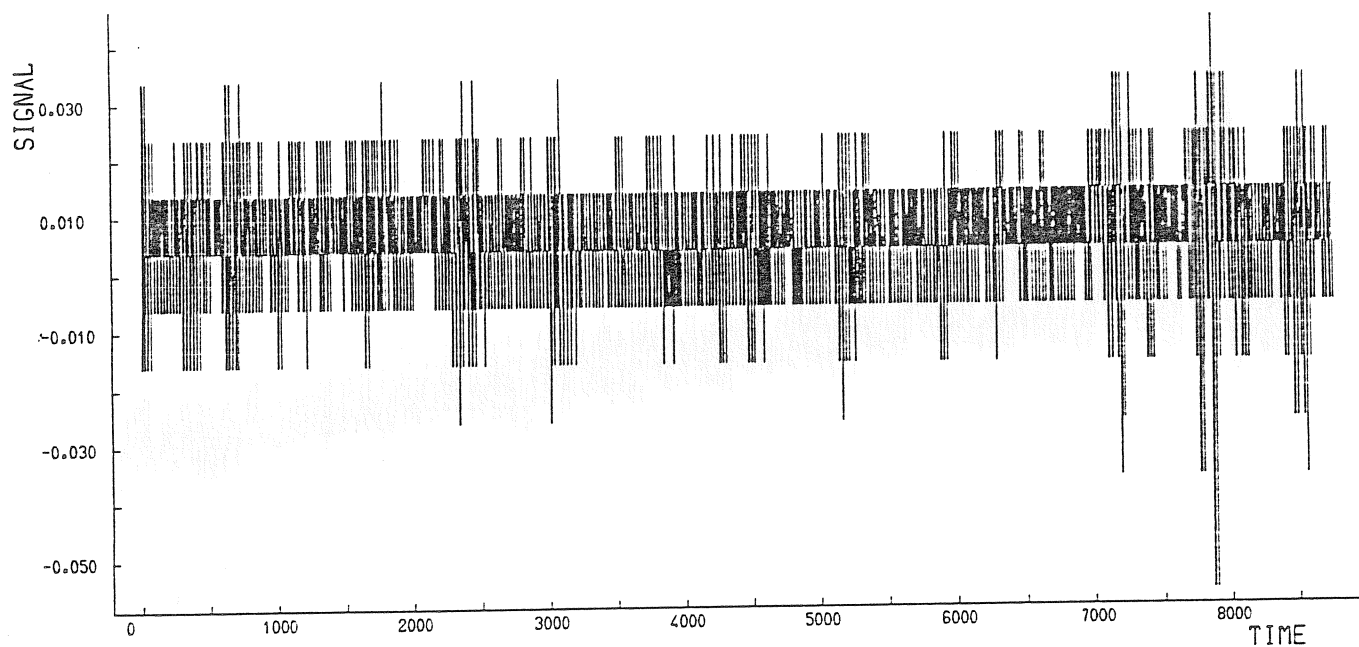


Figure 7: Hourly residues. (μgal)
UPPER Xi 1988 reduced to 1398 terms
LOWER Xi 1988 complete (2920 terms)

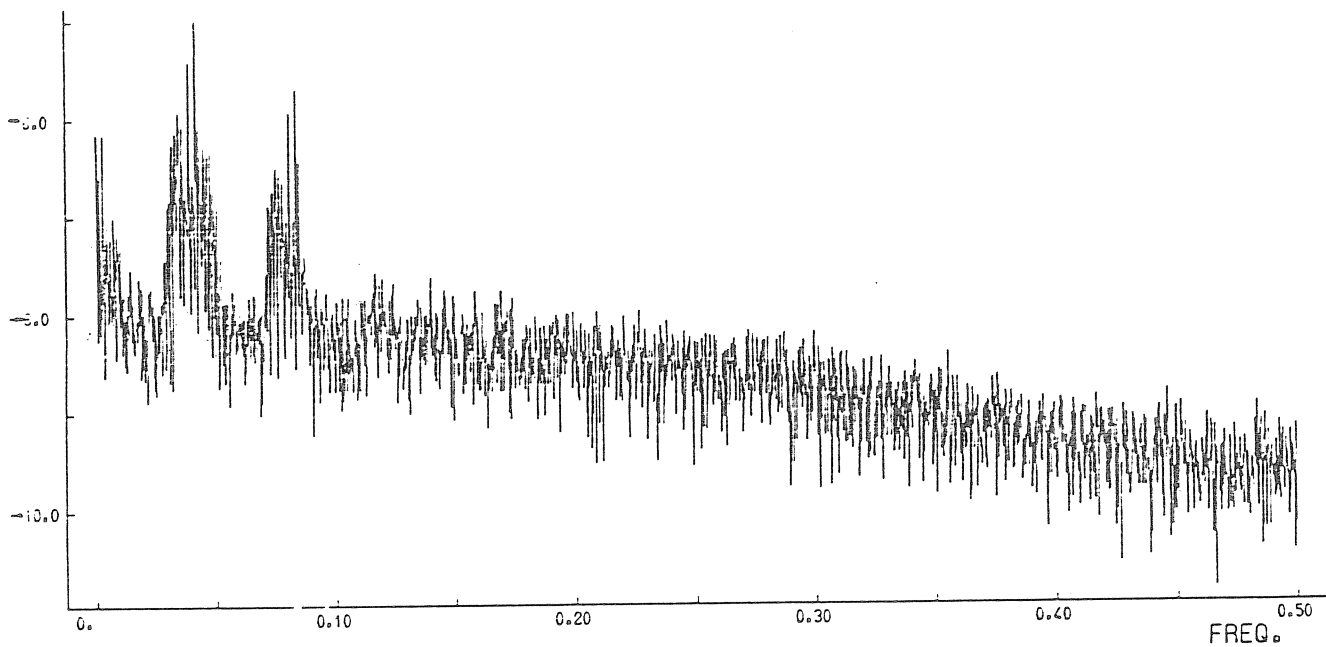
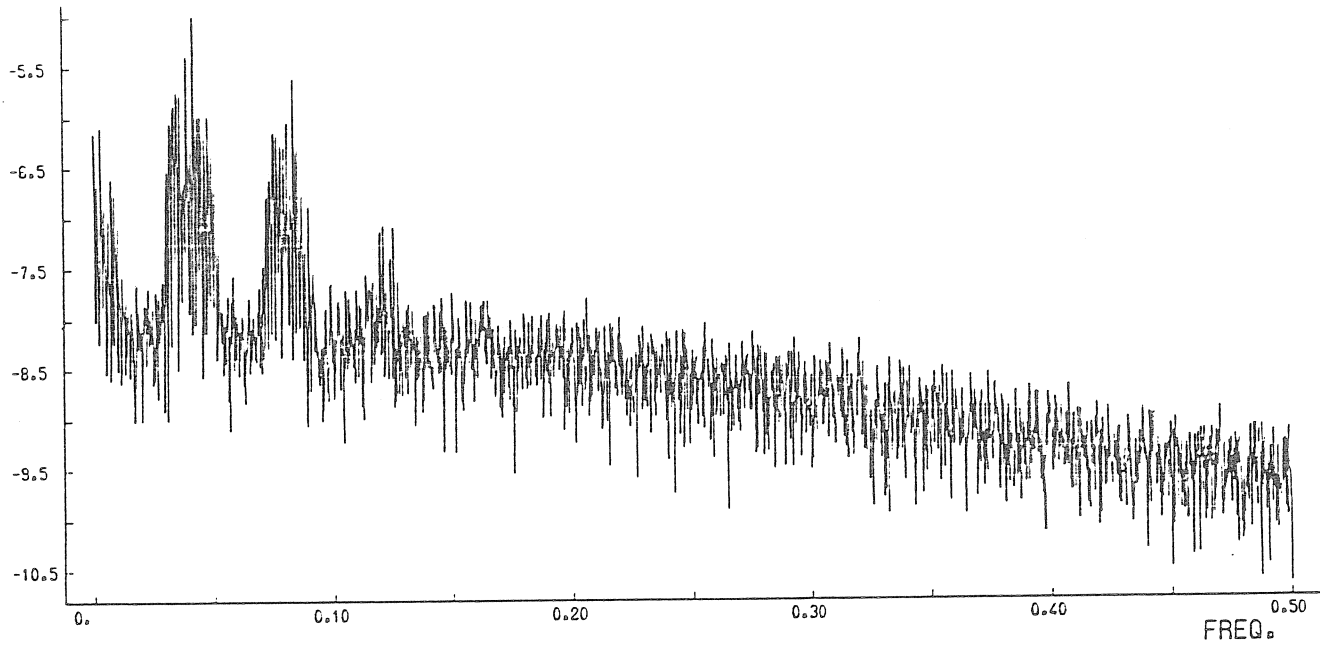


Figure 8: Power spectrum in logarithm of microgal squared as a function of frequency in cycle per hour.
UPPER Xi 1988 reduced to 1398 terms
LOWER Xi 1988 complete (2920 terms)

On the Tidal Parameters at
the Brussels Fundamental Station

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As the superconducting gravimeter installed at the Royal Observatory of Belgium has no built in calibration system we must rely on other observations to scale its records.

However the internal precision of the instrument is so high (table 1) that the amplitude ratios of the tidal parameters put very strong constraints on the modelisation of the Earth's response to the tidal forces.

In table 2 we show that the results of the superconducting gravimeter scaled on the accepted $\delta(O_1)$ value at Brussels are fitting within a constant factor $\delta_o/\delta_m = 1.01$ the modelisation obtained using the Wahr-Dehant model (Dehant, 1987, Ducarme & Dehant, 1987) for the elastic response of the Earth and the tidal loading computations based on Schwiderski's cotidal maps.

The large dispersion of the ratios δ_o/δ_m for the semidiurnal waves is due to the imperfections of the Schwiderski cotidal maps which do not have a sufficient resolution for North sea and Irish sea. It is encouraging to note that for M_2 we obtain a ratio δ_o/δ_m identical to the diurnal one by using a detailed map for these seas.

The diurnal waves are much more coherent as the indirect oceanic effects are ten times smaller in the diurnal band than in the semidiurnal one.

On the other hand a preliminary analysis of a two months record of the gravimeter LCZ3 calibrated by inertial acceleration (Van Ruymbeke, 1989) confirms that the accepted tidal gravimetric factors should be reduced. Table 3 summarizes the results. For O_1 the best adjustment corresponds to a reduction of 0.8 % for the amplitude factor. For M_2 the reduction reaches 1.1 %.

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T A B L E 1

SUPERCONDUCTING GRAVIMETER STABILITY OF TIDAL FACTORS IN SUCCESSIVE YEARLY ANALYSIS

Year	$\delta(O_1)$	$\delta(K_1)$	$\delta(M_2)$	$\delta(M_3)$
82-83	1.16256	1.14926	1.19354	1.0634
83-84	1.16300	1.14948	1.19353	1.0862
84-85	1.16292	1.14971	1.19363	1.0647
85-86	1.16254	1.14926	1.19271	1.0904
86-87	1.16243	1.14927	1.19344	1.0768
Mean of yearly analysis	1.16249 ± 0.00017	1.14940 ± 0.00009	1.19337 ± 0.00019	1.0763 ± 0.0055
Standard deviation	0.00038	0.00019	0.00042	0.0122
Global analysis	1.16269 ± 0.00013	1.14942 ± 0.00010	1.19336 ± 0.00010	1.0760 ± 0.0053

T A B L E 2

MODELISATION OF GRAVITY TIDES
AT UCCLE - BRUXELLES

	δ_o	α_o	δ_m	α_m	δ_o/δ_m	$\alpha_o - \alpha_m$
Q ₁	1.1614 ±0.0007	-0.427 ±0.035	1.1507	-0.23	1.0093	-0.20
O ₁	1.1627* ±0.0001	-0.045 0.007	1.1499	0.07	1.0111	-0.12
P ₁	1.1628 ±0.0003	0.076 ±0.016	1.1498	0.24	1.0113	-0.16
K ₁	1.1494 ±0.0001	0.143 ±0.004	1.1356	0.24	1.0121	-0.10
N ₂	1.1749 ±0.0005	3.039 ±0.026	1.1766	3.09	0.9985	-0.05
M ₂	1.1934 ±0.0001	2.512 ±0.005	1.1882 1.1816**	2.69 2.98**	1.0044 1.0100**	-0.18 -0.47**
S ₂	1.2072 ±0.0002	0.909 ±0.010	1.1955	1.22	1.0097	-0.31
K ₂	1.2097 ±0.0007	1.118 ±0.031	1.1917	1.12	1.0151	0.00

δ_o, α_o observed amplitude factors and phase differences from a global analysis of five years of the Superconducting gravimeter (without inertial correction)

δ_m, α_m modeled amplitude factors and phase differences (Wahr-Dehant model + indirect effects according to Schwiderski)

* the accepted value including the inertial correction is
 $\delta(O_1) = 1.161$

** Flather's map is used for North sea and Irish sea.

T A B L E 3

TIDAL GRAVITY RECORDING
AT UCCLE - BRUXELLES

GRAVIMETER LACOSTE - ROMBERG
LCZ 3

n	amplitude factors		efficiency	mean square errors in microgal	
	$\delta(O_1)$	$\delta(M_2)$		diurnal	semidiurnal
1584	1.1469 ±0.0070	1.1782 ±0.0036	92 %	6.14	2.58
1392	1.1561 ±0.0053	1.1809 ±0.0026	81 %	4.28	1.64
1104	1.1535 ±0.0032	1.1807 ±0.0023	66 %	2.26	1.22

n : number of hourly readings

efficiency : ratio between the number of hourly readings and the total number of observations

SOME REMARKS ON THE EARTH TIDE MODELS

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Introduction

The models of the Earth's body and ocean tides are of a great importance for the determination of corrections not only for gravimetric, but also for other geodetic measurements (VLBI, SLR, GPS, satellite altimetry).

At present, checking these models can be done only by comparison with the results of gravimetric tidal stations (Melchior, De Becker 1983 and Dehant, Ducarme 1987). Because the "Standard Earth Tide Model" recommended at the XVIII IUGG General Assembly in Hamburg, 1983 (Rapp 1983) is not quite satisfactory, the problem of a suitable Earth tide model is still relevant.

Experimental Check

The majority of the experimental data used in (Melchior, De Becker 1983 and Dehant, Ducarme 1987) for comparison with the models depend on the Fundamental World Station Bruxelles (Ducarme 1975). Therefore, the results of independent stations are of importance, as that of KAPG (Committee of the Academies of Sciences of the Socialist Countries for Planetary Geophysical Research) stations.

The main criterion for the quality of gravimetric tidal station results is the accuracy of the record calibration. At

the station Pecný (Czechoslovakia) a great attention was paid to this matter. The Askania gravimeters are being used for tidal measurements here and the principal calibration method is the record calibration by the measuring screw. The measuring scale of the instruments was calibrated on the Czechoslovak gravimetric base line with a relative accuracy better than 0.04 % (Šimon et al. 1977). With respect to the small range of the recording device, the linear subdivision of the calibrated scale is the most important problem. Therefore, the function of the optical micrometers of all the used gravimeters has been carefully investigated. It was found that their non-linearity may reach up to several percent and evidently, this is the main cause of systematic errors of the δ factors in the results of Askania gravimeters.

The methods for elimination of this effect from the record calibration were elaborated (Brož et al., in print). The efficiency of this technology may be shown on the results of long-term measurements with three different Askania gravimeters, see Table 1. All the measurements were analysed in the Computing Laboratory in Sofia using the Venedikov's method M 74. The agreement of the δ values for three main waves (O_1 , K_1 , M_2) is better than 1×10^{-3} . Although this agreement may be random to a certain extent, we may conclude that Askania gravimeters are able, when correctly operated, to yield for the main waves an accuracy of the δ factors $1 - 2 \times 10^{-3}$ from long-term measurements. Comparative measurements at fundamental station are not necessary. Naturally, the reliability of the results implies the question, if there are no systematic errors associated with all the instruments of this type. But in every case, the Askania gravimeters have not been obsolete up to now. Only the accuracy of the measured phase lags α must be improved as it does not reach the accuracy corresponding to that of the δ factors.

We consider the representative values of the tidal parameters for the station Pecný to be the results of the Gs 15 No. 228 gravimeter. Compared with the Molodensky's model I (Molodensky 1961) the δ values are by 0.0026 less on an average, compared with the Wahr's model (Wahr 1981) they are by 0.0106 greater (Holub et al., in print). Now, we compare in Table 2 these results with the Earth tide model Wahr-Dehant (Wahr's model improved by Dehant in Dehant, Ducarme 1987). All the nine waves are being used for which the loading and attraction vectors \vec{L} were evaluated in (Venedikov et al. 1986) with the use of the Schwiderski's ocean tide model (Schwiderski 1980). In the table δ_m and α_m are the measured values of tidal parameters corrected for the instrumental effects only, A_m and A_t are the measured and theoretical amplitudes of tidal wave, δ_c and α_c are the parameters corrected for the ocean tide effect, δ_{w-d} is the model value of the gravimetric factor. The weighted mean (weights inverse proportional to the squares of MSE of the δ values from the analysis) of the differences $\delta_c - \delta_{w-d}$ from all the diurnal and semidiurnal waves is only 0.0005 ± 0.0005 . Thus, the Earth tide model Wahr-Dehant combined with the Schwiderski's ocean tide model and the results of the Pecný station are in full agreement. It seems this model may be satisfactory (for the Middle- and East-Europe at least), because also other results of the KAPG stations differ from it only very little:

Potsdam (Gs 15 No. 222)	$\delta_c - \delta_{w-d} = 0.0028 \pm 0.0007,$
Budapest (Gs 15 No. 228)	$\delta_c - \delta_{w-d} = 0.0018 \pm 0.0007,$
Sofia (Gs 15 Nos 220, 221, 228)	$\delta_c - \delta_{w-d} = -0.0017 \pm 0.0007.$

In Table 3 the end residual vectors \vec{X} (amplitude X , phase χ) are evaluated for the station Pecný using the Wahr-Dehant model. The measured vectors are denoted \vec{A} (amplitude $A \equiv A_m$, phase $\alpha \equiv \alpha_m$) and the model vectors \vec{R}

(amplitude $R \equiv R_{w-D}$, phase $\varphi = 0$). The residual vectors \vec{B} (amplitude B , phase β) were evaluated as the difference $\vec{A} - \vec{B}$, the end residual vectors as the difference $\vec{B} - \vec{L}$. They are altogether very small. The arithmetical mean of the amplitudes X from all 8 diurnal and semidiurnal waves is only $0.054 \mu\text{gal}$.

In Figure 1 the relation between the sine and/or cosine components of the vectors \vec{L} and \vec{B} for these 8 waves at Pecny is demonstrated. In the graph of the cosine components the differences between the measured and model amplitudes should be manifested by a shift of the points from the quadrant axis. On the contrary, in the graph of the sine components the differences in phases could be seen. This is the case of the Pecny station - the majority of the points lies under this axis. The reason is that the measured vectors of the tidal waves have a phase lag with respect to the theoretical tide (see Table 1) and the model vectors are without any lag. The lags of the measured waves correspond approximately to a time lag -0.049 h of the tidal bulge on the Earth body. Perhaps, the measured X values have a systematical error connected with the use of an analogous record. But also the elastic Earth model without any phase lag may not be quite realistic. After elimination of this systematical difference (by correcting the measurements or the model values) the sine components of the vectors \vec{X} diminish (particularly with the M_2 wave) and the mean value of the amplitudes X is only $0.038 \mu\text{gal}$. We can conclude that the vectors \vec{X} are not suitable for the check of elastic tide models and for the study of the lateral heterogeneities. It is better to use the differences between the measured and model δ values, which do not depend either on the measured phase lag or on the amplitude of the wave.

Constant Part of Tides

The theory of the Earth figure needs the perturbing potential be a harmonic function in the outer space i.e. the gravity anomalies must not contain the influence of the masses outside the determined surface (geoid, quasigeoid). Therefore, the "Standard Earth Tide Model" (Rapp 1983) also includes that part of the constant tide which is directly caused by the masses of the Moon and the Sun (theoretical tide). In this way, the effect of these masses will be eliminated from the geodetic quantities when applying the tidal corrections. However, the effect of the constant deformation of the Earth body will be kept in them; we are not able to determine it exactly. But this solution is not quite logical because it saves the consequence the cause of which has been removed. So the characteristics of the gravity field and all geodetic quantities would not be in agreement with the physical reality. The actual gravity acceleration would be changed by $30 \mu\text{gal}$ at the equator and by $-61 \mu\text{gal}$ at the poles and the actual equipotential surfaces would be displaced by -99 mm and by 198 mm respectively. Particularly, the unavoidable alteration of the heights would cause great difficulties in the geodetical praxis, because in levelling the tidal corrections are not being used or only periodical components of tides are being introduced in the corrections.

In (Yurkina et al. 1986) there is shown that all tasks of the Earth figure theory may be solved when saving the actual geodetic quantities. It is sufficient to correct the gravity anomalies used by the correction $4P/R$ (P is the constant part of the theoretical tidal potential, R the mean Earth radius). This correction reaches the value $61 \mu\text{gal}$ at the equator and $-122 \mu\text{gal}$ at the poles. It consists of two equal parts. The first part corrects the measured gravity

values, the second one the normal gravity values - as a consequence of the height changes. The resulting geoidal heights (height anomalies) must then be corrected by the quantity P/γ (γ is the normal gravity). This correction restores the effect of the outer masses removed in advance.

New Definition of the Earth's Normal Gravity Field

Other alternative solution of the problem of eliminating the effect of external masses, generating the constant part of tidal field, from the perturbing potential, is following. The solution is founded on a new definition of the normal gravity field which contains this part of the tidal field. It is proved that two material circles in the plane of the Earth's equator, whose radii R_M and R_S are approximately the mean distances of the Moon and the Sun from the Earth, can be considered as the source of this field. These circles contain the uniformly distributed masses of the Moon and the Sun.

The gravitational potential of the mass element dm of the circle, lying in the plane of the Earth's equator, at a point on the Earth's surface is

$$(1) \quad dV = G dm/r = (G M/2\pi r) d\alpha,$$

where G is the Newton's gravitational constant, M is the mass of the Moon or Sun, $\alpha \in \langle 0, 2\pi \rangle$ and r is the distance from the point on the Earth's surface to the mass element dm . Expression $1/r$ can be developed up to 2nd degree by means of zonal spherical harmonics and the potential of the mass circle at a point on the Earth's surface can then be determined by integration from 0 to 2π . Omitting the constant term, whose force effect is zero at every point of the space, we arrive at

$$(2) \quad dV = -(GM\rho^2/2R^3)P_2^{(0)}(\sin\varphi),$$

where ρ is the geocentric radiusvector of the point on the Earth's surface with geocentric latitude φ .

To eliminate the effect of the constant part of tidal potential from the perturbing potential T , which is defined as the difference between the potential of the gravity field W of the Earth in an external point, rotating along with the Earth, and the gravity potential U of the normal Earth at the same point, we shall proceed in the same way as in the case of the centrifugal force, i.e. we shall include it in the normal gravity field of the Earth.

Therefore, if we proceed in the way outlined above, we shall be able to express, in accordance with (Heiskanen, Moritz 1967), the new normal gravity potential of the elliptical Earth on the surface of the ellipsoid

$$(3) \quad U_0^* = \sum_{n=0}^{\infty} A_n P_n^{(0)}(\sin\beta) + \frac{1}{3} \omega^2 (b^2 + E^2) - \frac{1}{3} \omega^2 (b^2 + E^2) P_2^{(0)}(\sin\beta) - \\ - \frac{1}{2} G (b^2 + E^2 \cos^2\beta) [(M_e/R_e^3) + (M_o/R_o^3)] P_2^{(0)}(\sin\beta),$$

where $E = (a^2 - b^2)^{1/2}$; a and b are the semimajor and the semiminor axes of the biaxial ellipsoid, β is the reduced latitude. We must first modify the 4th term on the r.h.s. of Eq. (3) but after providing it we are able to restrict ourselves, preserving sufficient accuracy, to expression

$$(4) \quad -\frac{1}{2} G a^2 [(M_e/R_e^3) + (M_o/R_o^3)] P_2^{(0)}(\sin\beta).$$

On the surface of the level ellipsoid the normal gravity potential must be constant for all points and then

$$(5) \quad (A_0 + \frac{1}{3} \omega^2 a^2 - U_0^*) P_2^{(0)}(\sin \beta) + A_4 P_4^{(0)}(\sin \beta) + \left\{ A_2 - \frac{1}{3} \omega^2 a^2 - \right. \\ \left. - \frac{1}{2} G a^2 [(M_d/R_d^3) + (M_\odot/R_\odot^3)] \right\} P_2^{(0)}(\sin \beta) + \sum_{n=3}^{\infty} A_n P_n^{(0)}(\sin \beta) = 0.$$

Equation (5) implies that, considering simplification (4), the constant normal gravity potential U_0 on the surface of the level ellipsoid is not changed by the new definition of the normal gravity field and, consequently, in accordance with (Heiskanen, Moritz 1967), it holds

$$(6) \quad U_0^* = U_0 = (GM_\oplus/E) \tan^{-1}(E/b) + \frac{1}{3} \omega^2 a^2.$$

Equation (5) can then be used to determine the coefficients A_n . The normal gravitational potential of the Earth is obtained by substituting for constants A_n in the first term of Eq. (3) and the normal gravity potential by adding the potential of the centrifugal force and the constant tidal potential

$$(7) \quad U_{(u,\beta)}^* = (GM_\oplus/E) \tan^{-1}(E/u) + \left\{ \frac{1}{3} \omega^2 a^2 + \frac{1}{2} G a^2 [(M_d/R_d^3) + (M_\odot/R_\odot^3)] \right\} \times \\ \times (q/q_0) P_2^{(0)}(\sin \beta) + \frac{1}{2} \omega^2 (u^2 + E^2) \cos^2 \beta - \frac{1}{2} G (u^2 + E^2 \cos^2 \beta) \times \\ \times [(M_d/R_d^3) + (M_\odot/R_\odot^3)] P_2^{(0)}(\sin \beta),$$

where $q = \frac{1}{2} \{ [1 + 3(u/E)^2] \tan^{-1}(E/u) - 3(u/E) \}$ and $q_0 = \frac{1}{2} \{ [1 + 3(b/E)^2] \tan^{-1}(E/b) - 3(b/E) \}$, see (Heiskanen, Moritz 1967). On the surface of the ellipsoid, the elliptical coordinate $u = b$.

If we continue to follow the procedure set out in (Heiskanen, Moritz 1967), and again adopt the simplification given by (4), we arrive at the following expression for the normal

acceleration of gravity on the level ellipsoid:

$$(8) \quad \gamma_0^* = (GM_\oplus/a)(a^2 \sin^2 \beta + b^2 \cos^2 \beta)^{-1/2} \left\{ \left[1 + \left\{ \frac{1}{3} \omega^2 a^2 + \frac{1}{2} G a^2 [(M_d/R_d^3) + (M_o/R_o^3)] \right\} (E/GM_\oplus)(q'_0/q_0) P_2^{(0)}(\sin \beta) - (\omega^2 a^2 b/GM_\oplus) \cos^2 \beta + (a^2 b/M_\oplus)[(M_d/R_d^3) + (M_o/R_o^3)] P_2^{(0)}(\sin \beta) \right] \right\},$$

where $q'_0 = 3[1 + (b/E)^2][1 - (b/E)\tan^{-1}(E/b)] - 1$. If we now introduce the notations

$$(9) \quad m = \omega^2 a^2 b/GM_\oplus, \quad m' = (a^2 b/2M_\oplus)[(M_d/R_d^3) + (M_o/R_o^3)],$$

express the second excentricity as $e' = E/b$, and modify Eq. (8) so that all terms are multiples of $\sin^2 \beta$ and $\cos^2 \beta$, we obtain the equation

$$(10) \quad \gamma_0^* = (GM_\oplus/a)(a^2 \sin^2 \beta + b^2 \cos^2 \beta)^{-1/2} \left\{ \left[1 + 2m' + \left(\frac{1}{3}m + m'\right)(e'q'_0/q_0) \right] \sin^2 \beta + \left[1 - m - m' - \frac{1}{2} \left(\frac{1}{3}m + m'\right)(e'q'_0/q_0) \right] \cos^2 \beta \right\}.$$

The normal acceleration of gravity on the equator ($\beta = 0^\circ$) then comes out as

$$(11) \quad \gamma_e^* = (GM_\oplus/ab) \left[1 - m - m' - \frac{1}{2} \left(\frac{1}{3}m + m'\right)(e'q'_0/q_0) \right],$$

and at the poles ($\beta = 90^\circ$) as

$$(12) \quad \gamma_p^* = (GM_\oplus/a^2) \left[1 + 2m' + \left(\frac{1}{3}m + m'\right)(e'q'_0/q_0) \right].$$

The difference as compared to the normal acceleration of gravity on the equator and at the poles, as defined by the Geodetic Reference system (Moritz 1980), is given by the terms with m' in Eqs. (11) and (12).

It follows that the solution based on the new definition of the normal gravity field enables all the quantities measured in the real gravity field to be preserved and the condition that the perturbing potential should be a harmonic function to be satisfied. The changes in the normal heights are also insignificant (Zeman 1987) and, as proved in (Yurkina et al. 1986), the elevations of the quasigeoid do not change either. Only the field of normal gravity and the external gravity potential are affected, in very the same way as if the field of the centrifugal force of the Earth's rotation was included in the normal gravity field.

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Table 1. Station Pecný. Results of three gravimeters

Instrument	Gravimetric factor δ				Phase lag α			
	O_1	K_1	M_2	S_2	O_1	K_1	M_2	S_2
Period								
Gs 11 No. 131 70 09 02 - 80 02 04	1.1490 + - 12	1.1362 + - 7	1.1847 + - 6	1.1806 + - 13	0.14 ⁰ +0.06 -	0.42 ⁰ +0.04 -	1.28 ⁰ +0.03 -	0.88 ⁰ +0.07 -
Gs 15 No. 228 76 04 22 - 84 09 25	1.1486 + - 4	1.1362 + - 3	1.1848 + - 2	1.1861 + - 4	-0.03 +0.02 -	0.10 +0.01 -	1.03 +0.01 -	0.30 +0.02 -
BN-26 83 04 17 - 86 06 28	1.1484 + - 16	1.1362 + - 11	1.1852 + - 7	1.1787 + - 13	-0.06 +0.08 -	0.25 +0.05 -	1.03 +0.03 -	-0.07 +0.07 -
Weighted mean	1.1486 + - 1	1.1362 + - 0	1.1848 + - 1	1.1851 + - 17	-0.02 +0.04 -	0.12 +0.06 -	1.05 +0.05 -	0.31 +0.13 -
Mean	1.1487 + - 2	1.1362 + - 0	1.1849 + - 2	1.1818 + - 22	0.02 +0.06 -	0.26 +0.09 -	1.11 +0.08 -	0.37 +0.28 -

Table 3. Station Pecný. Gs 15 No. 228 (76 04 22 - 84 09 25)

Wave	$A \equiv A_m$ μgal	$\alpha \equiv \alpha_m$	L μgal	λ	R_{w-d} μgal	B μgal	β	X μgal	χ
M _f	5.61 +0.20 -	4.774 ⁰ +1.951 -	0.09	179.3 ⁰	5.64	0.47	96.0 ⁰	0.47	85.0 ⁰
Q ₁	6.76 +0.01 -	-0.283 +0.110 -	0.04	212.8	6.77	0.03	-106.8	0.03	-15.8
O ₁	35.18 +0.01 -	-0.027 +0.020 -	0.13	157.7	35.35	0.17	-174.4	0.08	-126.6
P ₁	16.41 +0.01 -	0.096 +0.039 -	0.04	73.2	16.38	0.04	42.5	0.02	-32.2
K ₁	48.94 +0.01 -	0.097 +0.014 -	0.11	58.8	48.82	0.15	34.6	0.07	-7.6
N ₂	7.03 +0.01 -	1.430 +0.049 -	0.23	62.7	6.92	0.21	58.4	0.03	-79.9
M ₂	36.91 +0.01 -	1.029 +0.009 -	1.14	45.6	36.10	1.04	39.5	0.15	-88.2
S ₂	17.19 +0.01 -	0.295 +0.020 -	0.37	17.3	16.80	0.40	12.8	0.04	-30.2
K ₂	4.66 +0.01 -	0.420 +0.081 -	0.10	13.7	4.57	0.10	20.8	0.01	107.2

Mean 0.054

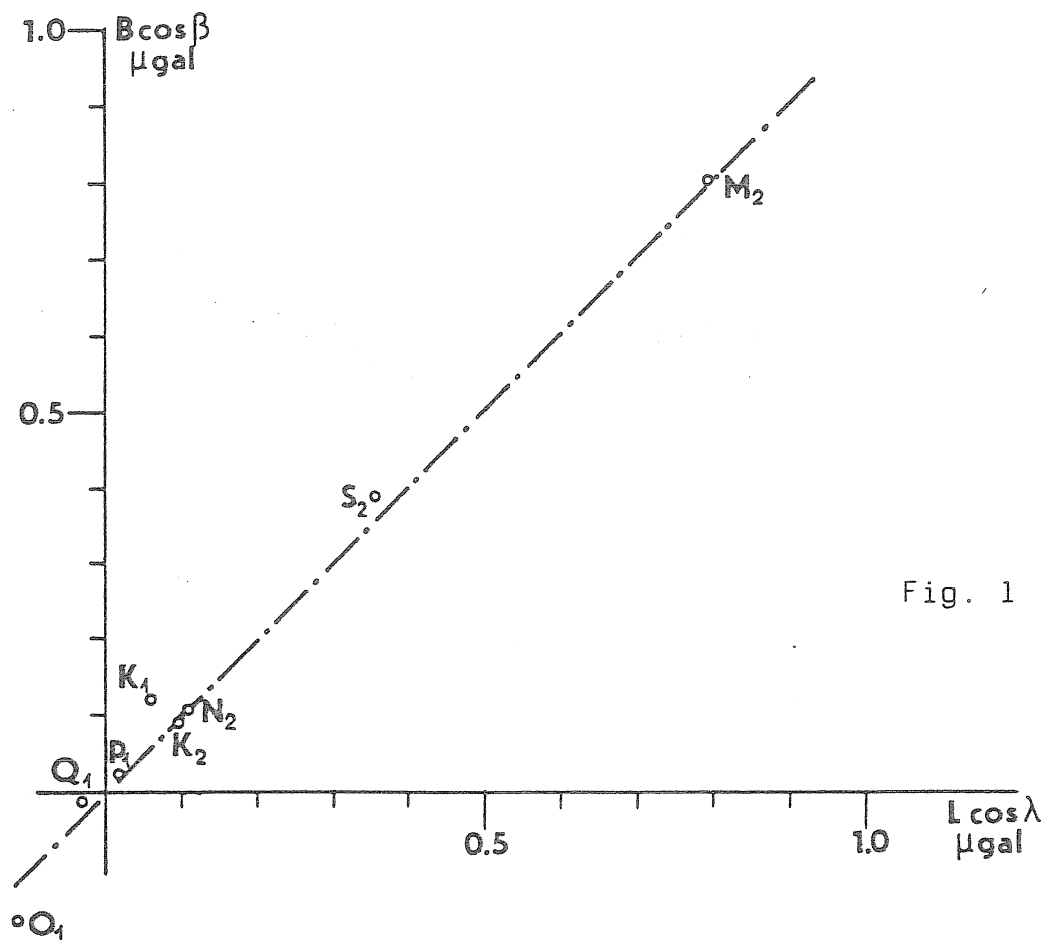
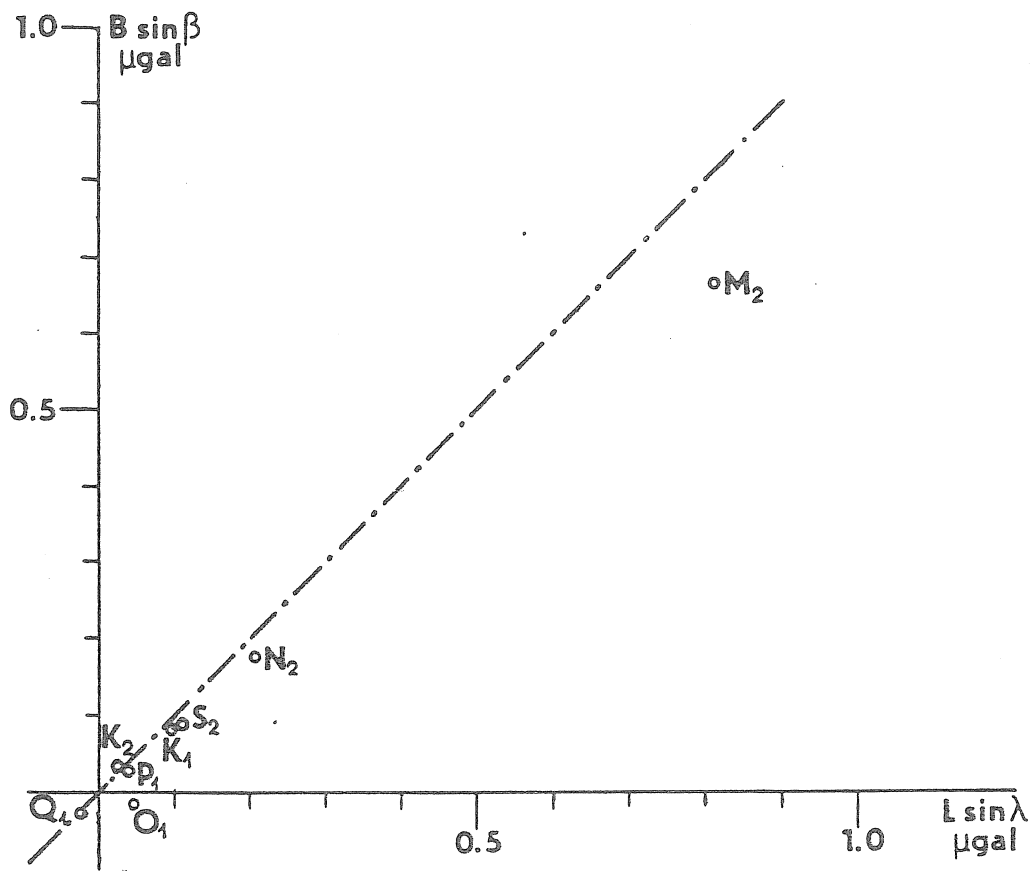


Fig. 1

COMPARISON OF GRAVITY TIDE OBSERVATIONS BY ET16 AND ET21
AT WUCHANG STATION OF CHINA

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0. INTRODUCTION

In order to check and complete the available theoretical earth and ocean tide models, improve the tidal correction accuracy of astronomical and geodetic observation data, and push the other geoscience work forward, it will be of great significance to establish a high accuracy global standard earth tide observation system.

The main problem is the accurate determination and unification of the scale of present and future observation results throughout the world. For the mainland of China Wuchang is intended to serve as a reference station. This is accomplished by carrying out long term observations with permanently installed instruments and by parallel registration with several other gravimeters in order to verify the scale. In addition a vertical base line was installed in order to check the calibration of the meters used.

In 1985 the Institute of Geodesy and Geophysics, Chinese Academy of Sciences, and the Institute of Physical Geodesy, Technical University Darmstadt, started a joint research project for investigating the topics mentioned above. It was decided to install the ET21 and ET16 LaCoste earth-tide meters together in the Wuchang station to carry out a side by side comparison. In addition a one year series with ET15 from the Institute of Oceanographic Sciences, Bidston, U.K. was recorded and since 1986 the GWR TT70 superconducting gravity meter is operating in Wuchang. Here we present the preliminary analysis of the results by ET16 & ET21 for Wuchang. Starting 1989 it is then planned to move to other selected sites in China for the validation of different earth models and for studies of ocean tidal loading computations.

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1. WUCHANG TIDAL STATION AND INSTRUMENTAL SET-UP

The station in Wuchang consists of two places. One is located in a basement 22 m deep in the ground, about 300 m from the entrance of the tunnel, see Fig. 1.

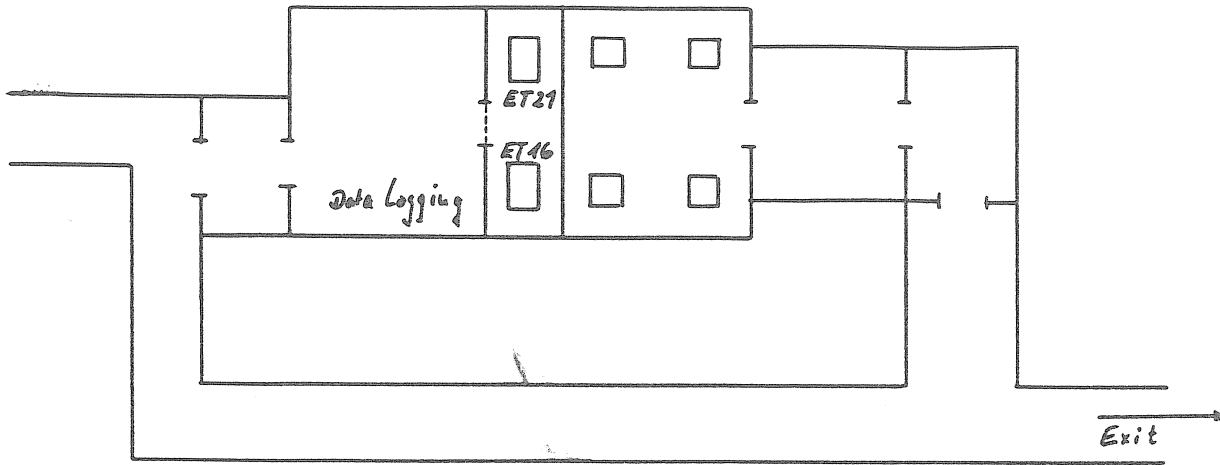


Fig. 1 Wuchang underground laboratory with 6 pillars for tidal observation

Pillars are fixed to the rock and perennial temperature and humidity variations are rather small. There are almost no environmental perturbations due to vibrations or other man made disturbances. The ET-meters are installed in that location. The other site is located in the ground floor of the Institute of Geodesy and Geophysics and is used for the superconducting gravimeter GWR TT70. It is set up on a pillar founded about 15 m deep. However, due to traffic interference of a highway nearby and due to some temperature variation the observation conditions are not as good as in the basement.

ET21 meter was imported in Aug. 1982, and equipped with a mechanical feedback zero reading system, the gear clearance of mechano-transmission is very small. The gravity tidal record is made by using the chart recorder with a record sensitivity of $1.56 \mu\text{gal}/\text{mm}$. This meter has worked for several years, and had no serious breakdown.

Prior to the installation in China the ET16 was converted from mechanical feedback to electrostatic feedback. This is due to the fact that the mechanical system suffered from a severe backlash-problem of about $3 \mu\text{gal}$ leading to phase-lags of 1.4° and 0.7° in semidiurnal and diurnal waves respectively, see (Gerstenecker and Schüller, 1983). Therefore the Harrison-Sato feedback-electronics were installed in ET16. Linearization of the system was made by unbalancing the fixed plate gains rather than by moving the outer condensor plates. The range of the feedback is about 1.2 mgal . A microcomputer controlled data logging system records and stores 7 channels including feedback, beam position, temperature and pressure. The resolution for the feedback voltage is better than $0.1 \mu\text{gal}$., see Fig. 2 for the complete diagram of the modified ET16 and the data logging.

A sample averaged from measurements in 5 sec. intervals over two minutes is stored every 10 minutes on a tape drive. Controls are put out on a chart recorder and a printer. Due to problems with the power supply there were interruptions in 1986, and in beginning of 1987 the tape drive broke down so that an interruption

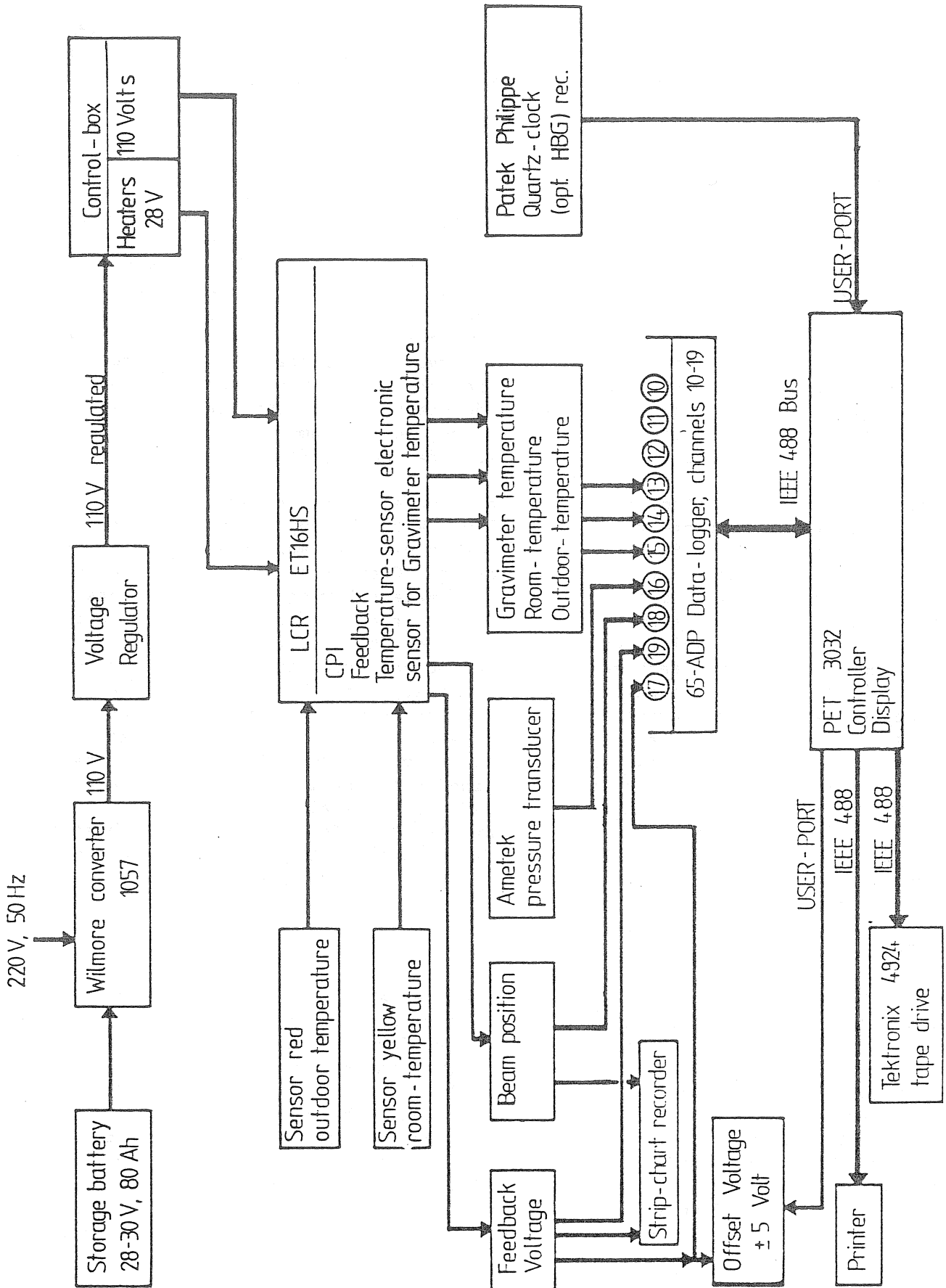


Fig. 2 Block-diagram of modified ET16

occured. Since Sept. 1987 a floppy drive is used for storage and the recordings are running well.

The superconducting gravity meter GWR TT70 was set up in 1986. Due to a failure in the digital data aquisition system the first year of registration was made by analogue records for the GWR TT70. So in spite of the good overall features like constant sensitivity, almost zero drift and small temperature influence the data do not yet have the resolution suitable for a superconducting instrument.

2. CALIBRATION AND WUCHANG BASELINE

In all types of LaCoste gravimeters, i.e. model G, D and ET, the manufacturer's scale factor has to be improved. The calibration in general can be described by a function containing linear, nonlinear and periodic terms. For the earth-tide meters, however, the periodic terms are below the detectable level in general and with electrostatic feedback they are only affecting the calibration and can be eliminated by a suitable step size. Therefore only the nonlinearity of the screw and lever system remains. As found in various calibrations of ET-meters, see e.g. (Edge et al., 1985) the LaCoste factors are only accurate to about 1% and a carefull calibration is needed for determining the complete transfer-function that is the calibration coefficients and the instrumental phase lag. The accuracy required should be about 0.2% for tidal amplitudes and 0.1° in phase.

At present the only suitable way to achieve these values is the observation at a high precision short range vertical baseline. With the support and help of Chinese colleagues in 1986, a vertical gravity baseline was established in the office building of the Institute of Geodesy and Geophysics. Gravity difference is about 6 mgal, 17 gravimeters G and 2 gravimeters D were employed for baseline observations, results are shown in Table 1. All gravimeters G have been calibrated on the national absolute gravity baseline of China in order to evaluate a linear scale factor for each meter. The periodical error influence of G meter can be considered as an occasional error so that it may be removed in taking the average of several instruments. The observation results of two Model D gravimeters are close to the average observation value of the 17 Model G gravimeters. The weighted average is taken as the final gravity value of the vertical baseline. The mean square error of this baseline is $\pm 2 \mu\text{gal}$, i.e. a relative accuracy 0.03%, which meets the accuracy demand for the scale calibration of ET meter. The Institute of Geodesy and Geophysics has made an absolute calibration for ET21 using this baseline. The scale calibration of ET21 meter is made in two steps.

2.1 Record scale determination.

By use of the reading variation of meter counter and the corresponding record displacement we can determine the record scale based on the manufacturer counter scale. During the observation of ET21 meter we found out that from 1985 to Aug. 1986, the longitudinal level was set up in the middle position, but from Oct. 1986 up to now, we changed it to minimum tilt sensitivity position, so the record scales of these two different places have to be measured respectively, they are indicated in Table 2. It is discovered that the calibration record scale is larger than the manufacturer's one. For these two longitude level positions (bulb displaced 1.5 division) the record scale difference is 0.2%, the relation between record sensitivity and longitude level position is the same as for gravimeters G and D. When dealing with the 2 years data together we choose the mean value of both positions.

observed free- calibrated				observed free- calibrated			
Inst.	Δg (μgal)	dom	factor	Inst.	Δg (μgal)	dom	factor
G589	5745	6	1.000505	G570	5741	5	1.000470
G795	5762	8	1.000538	G776	5746	2	1.00064
G796	5757	8	1.000648	G794	5754	2	1.00066
G797	5748	8	1.000576	G842	5754	2	1.00054
G799	5757	8	1.000612	G837	5746	2	1.00066
G800	5748	8	1.000341	G836	5749	2	1.00027
G808	5765	3	1.000556	G828	5747	2	1.00059
G818	5753	3	1.000472	D96	5753	7	
G820	5744	7	1.000527	D122	5759	6	
G147	5743	5	1.000366	Mean 5751.1 \pm 1.6 μgal			1.000528
absolute $\Delta g_M = 5754.1$							

Table 1 GRAVITY VALUES OBSERVED ON THE VERTICAL CALIBRATION LINE IN WUCHANG

2.2 Counter scale determination

At first, we measured the gravity difference of Wuchang vertical gravity baseline with usual field gravity method and according to manufacturer's counter scale, the measured baseline value can be calculated and compared with the real value (Table 2). We obtain the correction of manufacturer's counter scale. To check the linearity of different counter positions (namely different counter reading segments), two measuring range positions are taken for calibration, center readings are 42200 and 62600 respectively, the difference between both positions is 20400 counter units. Four times measurements for each measuring range is made on the baseline, the calibration result (in Table 2) finds that the discrepancy between both positions is only 0.12%. This indicates that the linearity of ET21 is very good. But the manufacturer's scale is systematically large. The correction factor is 0.9895, i.e. about 1.1%. This is a considerable reducing factor and will play an important role in gravity tidal observation.

ET21 meter uses the close circuit mechanical feedback system. To reduce the lag the meter is equipped with an additional integral servo-loop. According to the output response of meter on step input, we can derive the impulse response function of the whole meter system, through Fourier-transformation the phase lag for different tidal frequencies can be derived (Wenzel, 1976). We tested three times, the results obtained are the same. Fig. 3 shows the output response curve after drift correction. It can be seen from the figure that the meter phase lag is very small, within one minute it is in steady condition, the calculated phase lags for waves O_1 and M_2 are $0^\circ 08$ and $0^\circ 17$, respectively.

Therefore we determine ET21 gravity tide system parameter as follows:

tidal factor parameter $f = 0.99064$
 phase lag parameters $\Delta\varphi_{o_1} = 0^{\circ}08, \Delta\varphi_{M_2} = 0^{\circ}17$

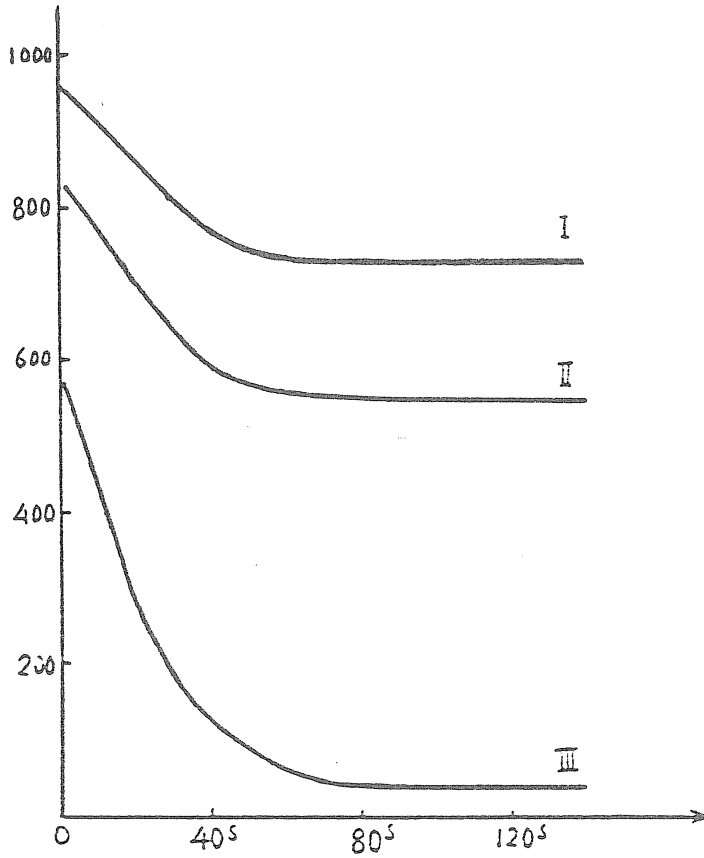


Fig. 3 Step-response of ET21

calibration of recording scale		calibration of factor for counter	
manufacturer's scale factor	1.5572 $\mu\text{gal/mm}$	gravity line ΔE_M	5754.1 ± 1.6
calibrated scale factor	1.5590 $\mu\text{gal/mm}$ 0.0001	reset position I 20400 - 64000	5818.53 ± 0.95
scale correction factor	1.00116	reset position II 40700 - 84500	5811.58 ± 0.47
		I + II/2	5815.06 ± 0.53
		counter correction factor	0.9895 ± 0.0003
final calibration factor: 0.99064			

Table 2 RESULTS FROM CALIBRATING SCALE FACTOR OF ET21

3. OBSERVATION RESULTS AND COMPARISON

ET21 observation data have been calculated by the Institute of Geodesy and Geophysics, Chinese Academy of Sciences with the Cartwright-Edden tidal development and Venedikov's harmonic analysis. Considering that after Venedikov's filter the residual drift still causes a leakage influence to other spectral bands the following modification has been made for this method. After finishing the preliminary analysis, we set up a recovering tide curve by means of the preliminary tidal factors, afterwards, the recovering tidal curve value is subtracted from original observation so that drift and noise curves are obtained. Because this curve has basically excluded the tidal variation, drift and residual (Fig. 4 and 5) are easily separated with a Pertsev filter, the spectrum analysis is made for the residual (Fig. 6). In terms of the noise peak level in various spectral bands, the accuracy of various tidal factors can be estimated precisely. After removing the drift of the observations, through a second Venedikow harmonic analysis the final result shown in Table 3 is obtained. These results have to be corrected for calibration and phase lag, the final results after correction are listed in Table 5, they don't include the inertial correction.

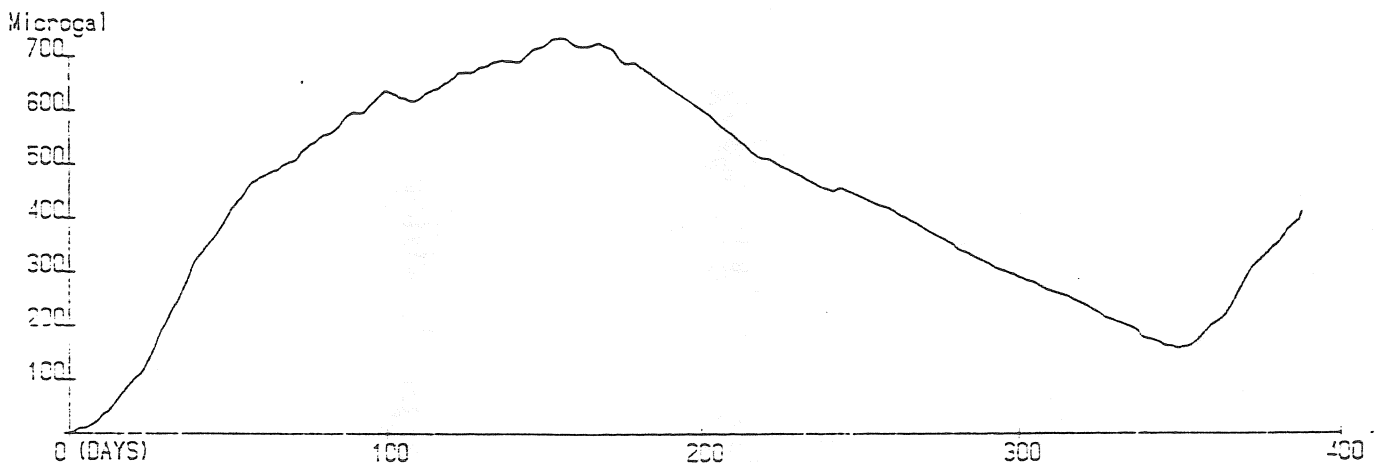


Fig. 4 The drift of ET21 from 1986/11/03 to 1987/11/25

The ET16 observations are processed at the Institute of Physical Geodesy. The 10 minute samples are used for a spline interpolation of the full hour values. The analysis is made by the Hycon program. Up to now for ET16 there is no absolute calibration available and the results given in Table 4 and 5 are referred to the manufactures scale and no instrumental phase lag is corrected for. Table 4 shows the results of the preliminary analysis for all available data since 1986. There is a quite reasonable inner accuracy of ET16 with an overall standard error of 0.38 μ gal for the residuals. The record of ET16 for 1987 and 1988 is shown in Fig. 7.

In Table 5 the results of several gravimeters are compared including the preliminary analysis of the superconducting gravimeter. Data for GWR TT70 were analyzed in Wuchang with the method described for ET21. In the meantime we obtained the results from ET15 too (Baker, T.F. and Hsu, H.T. et al., private communication) It is obvious that the scale factor for ET16 is wrong by about 1%, so for the time being we have to scale the results to ET15 using the mean difference of O1 and M2 wave. Discrepancies are now in the order of 0.1% and also the phases agree well. It is expected that after the correction for instrumental phase lags there will be a further improvement.

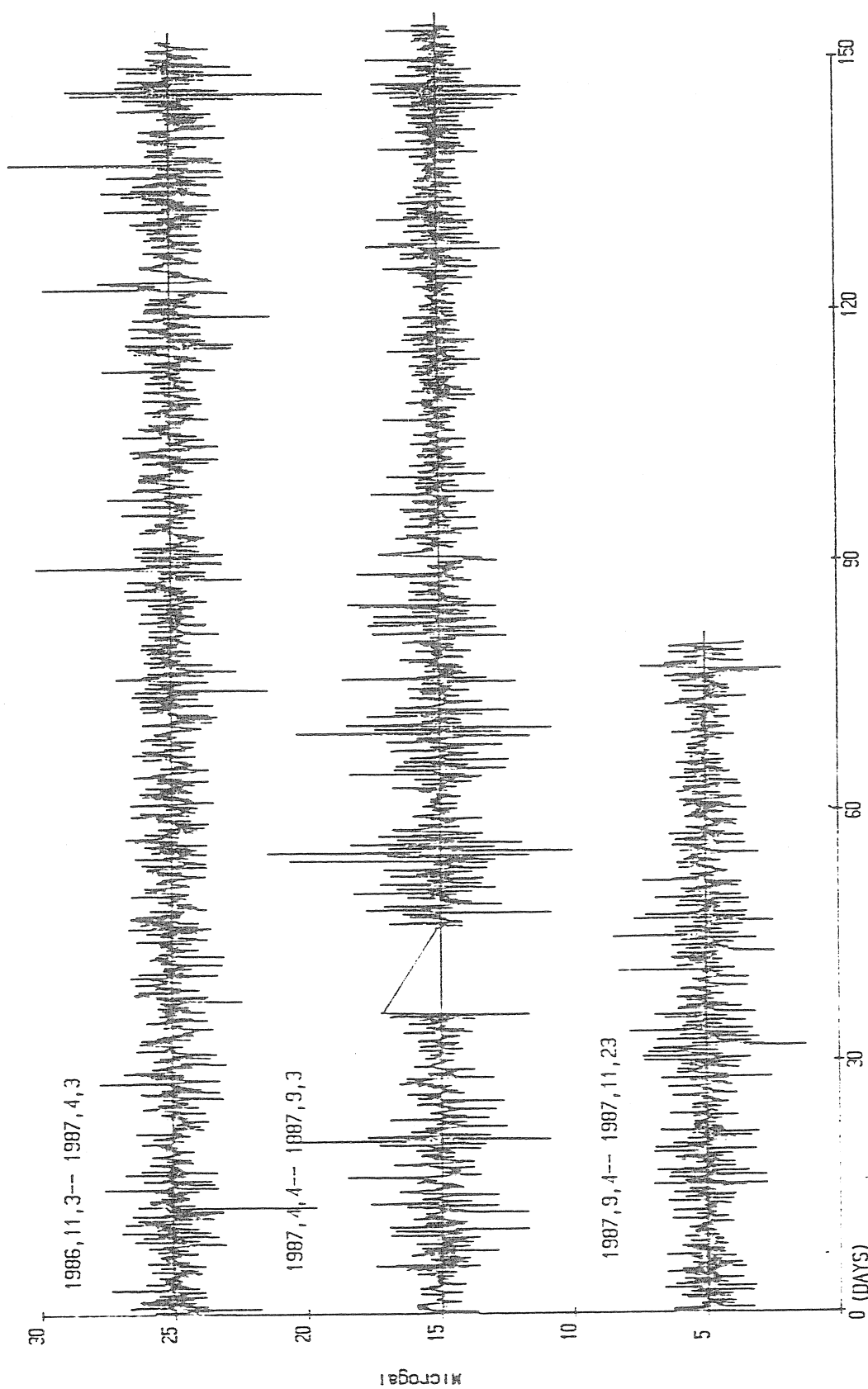


Fig 5 THE RESIDUAL OF ET-21 WITHOUT DRIFT (1986, 11, 3 -- 1987, 11, 23) AT WUCHAN STATION.

WUCHANG STATION			L = 114.339 B = 30.524		
ET21	1985, 5,30	- 1987, 10.30	23 BLOCK	1 mm = 1.5572 microgal	
	Tidal fact	error	obs. ampl.	phase lag	error
Q1	1.1959	0.0045	7.3492	- 0.4706	0.2164
O1	1.1898	0.0008	37.8820	- 0.4782	0.0417
M1	1.1776	0.0118	2.6442	0.1585	0.5759
PI1	1.2327	0.0373	0.9035	1.3765	1.7357
P1	1.1791	0.0022	14.7887	- 0.5118	0.1068
S1	1.1421	0.1360	0.2318	-18.1307	6.3223
K1	1.1674	0.0006	49.2297	- 0.6454	0.0321
PSI1	1.3391	0.0892	0.4142	- 4.0510	3.8186
PHI1	1.2800	0.0553	0.6347	0.2188	2.4760
J1	1.1921	0.0111	2.9109	- 0.7248	0.5337
001	1.1985	0.0140	2.3358	0.0248	0.6714
MSR = 1.71					
2N2	1.2019	0.0189	2.1980	1.3194	0.9005
N2	1.1966	0.0038	11.6399	- 0.6972	0.1797
M2	1.1867	0.0007	63.8332	- 0.6610	0.0322
L2	1.1599	0.0138	2.9357	- 0.2801	0.6810
S2	1.1874	0.0014	30.6538	- 0.3703	0.0657
K2	1.1847	0.0039	10.7534	- 0.8717	0.1878
MSR = 2.23					
M3	1.1066	0.0166	1.0343	- 1.0627	0.8600
MSR = 0.93					

TABLE 3 RESULTS OF VENEDIKOV ANALYSIS FOR ET21

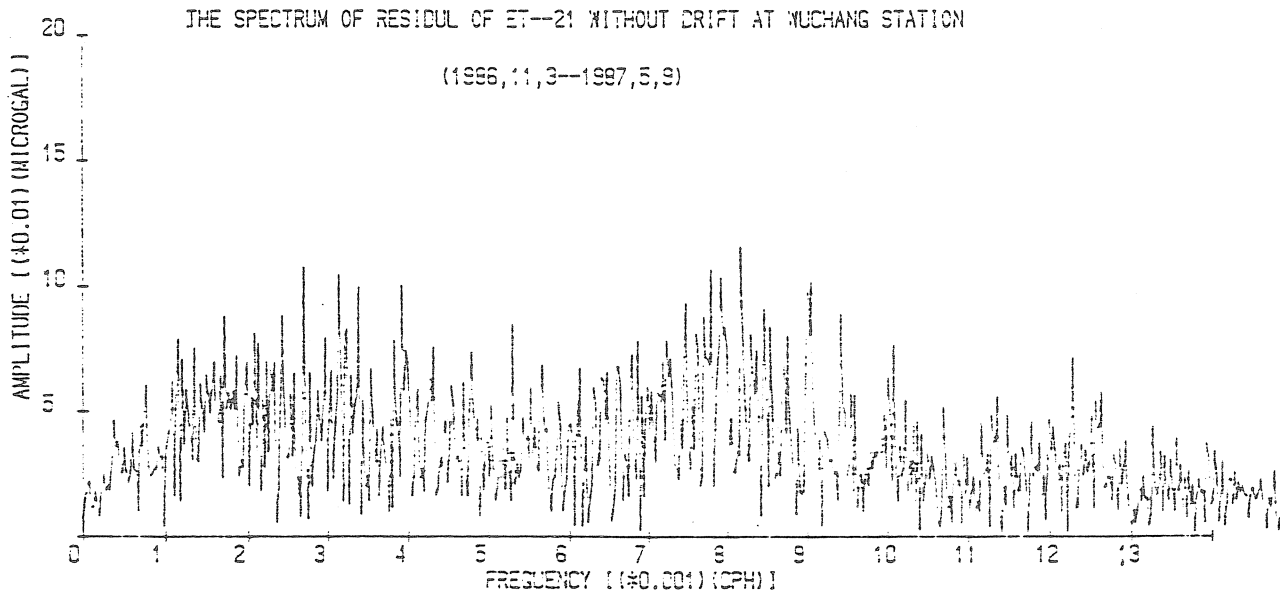


Fig 6(a).

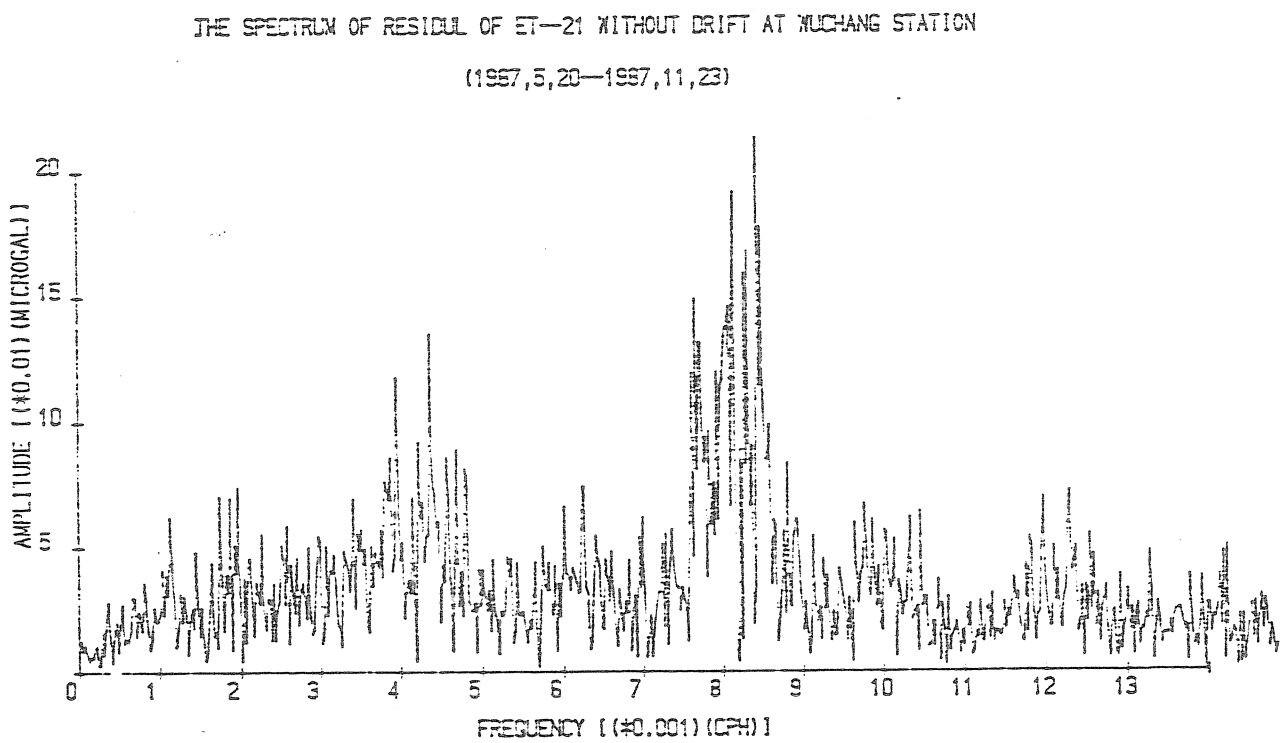


Fig 6(b).

TIDAL REGISTRATION WUHAN UNDERGROUND LABORATORY							
Date: 25-02-1986 - 02-08-1988							
ET16 HS Scale factor 0.0414 μ gal/millivolt							
No.	Tide	Delta	RMS	C (95%)	Kappa	RMS	C (95%)
1	SIQ1	1.2083	0.1029	0.2022	- 0.41	4.88	9.59
2	SIG1	1.1833	0.0216	0.0425	0.20	1.05	2.06
3	Q1	1.1731	0.0044	0.0087	- 0.35	0.22	0.43
4	O1	1.1673	0.0008	0.00177	- 0.50	0.04	0.08
5	M1	1.1660	0.0107	0.0210	- 0.15	0.52	1.03
6	P1	1.1442	0.0021	0.0042	- 0.62	0.11	0.21
7	K1	1.1414	0.0007	0.0013	- 0.62	0.03	0.06
8	J1	1.1470	0.0109	0.0214	- 1.00	0.54	1.07
9	OO1	1.1541	0.0128	0.0252	- 0.81	0.64	1.25
10	V1	1.1423	0.0680	0.1335	0.27	3.41	6.70
11	EPS2	1.1414	0.0516	0.1014	0.48	2.59	5.09
12	2N2	1.1694	0.0107	0.0211	- 0.59	0.53	1.03
13	N2	1.1698	0.0022	0.0043	- 0.54	0.11	0.21
14	M2	1.1641	0.0004	0.0008	- 0.50	0.02	0.04
15	L2	1.1537	0.0091	0.0178	- 0.53	0.45	0.89
16	S2	1.1643	0.0008	0.0016	- 0.18	0.04	0.08
17	K2	1.1595	0.0024	0.0046	- 0.23	0.12	0.23
18	ETA2	1.1551	0.0383	0.0752	- 0.90	1.90	3.73
19	M3	1.0645	0.0129	0.0253	- 0.15	0.69	1.36

TABLE 4 RESULTS FROM HYCON DATA PROCESSING FOR ET16

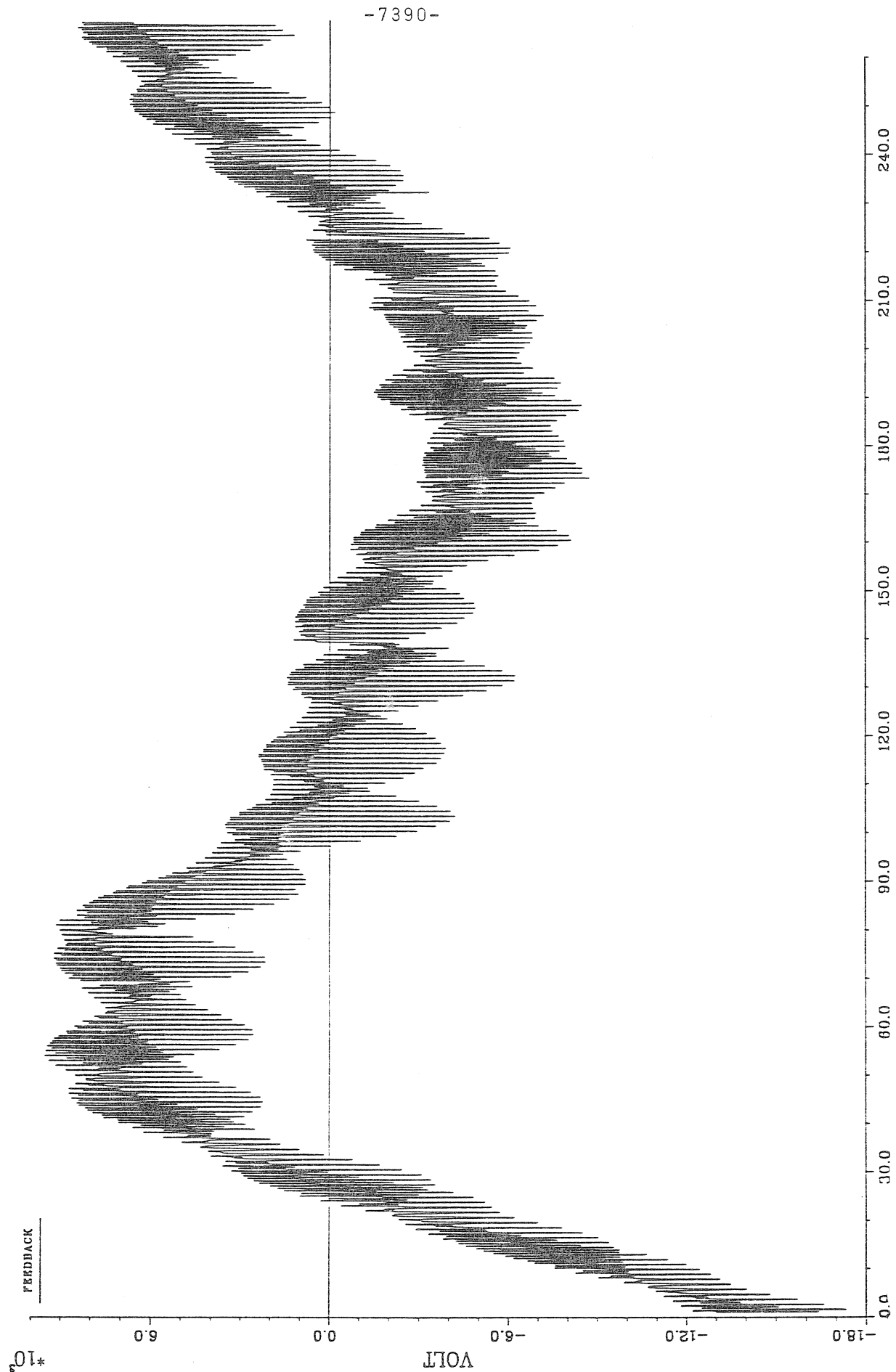


Fig. 7 The record of ET16 from 1987/09/07 to 1988/06/02

gravi-meter	epoch	O_1		K_1		M_2		S_2	
ET21	05/85-04/88	1.1780 ± 8	-0.43 ± 4	1.1549 ± 6	-0.59 ± 3	1.1747 ± 6	-0.50 ± 3	1.1751 ± 12	-0.20 ± 6
GWR TT70	01/86	1.1772 ± 15	-0.43 ± 7	1.1517 ± 11	-0.55 ± 5	1.1720 ± 6	-0.38 ± 3	1.1677 ± 12	-0.38 ± 6
Melchior et al.	09/79-09/81	1.1800	-0.84	1.1576	-0.87	1.1778	-0.56	1.176	0.14
Brussels system ET16	02/86-12/86	1.1673	-0.50	1.1414	-0.62	1.1641	-0.50	1.1643	0.18
	09/87-06/88	± 08	± 4	± 7	± 3	± 4	± 2	± 08	± 4
Calibration in F.R.G. ET16 (N)		1.1773		1.1512		1.1741		1.1743	
normalized to ET15 $f = 1.00858$									

TABLE 5 COMPARISON OBSERVATIONS FOR TIDAL GRAVITY AT WUCHANG, CHINA

		loading		loading corrected		Wahr's	Dehant's	Dehant's
wave	inst.	correction		values		model	model I	model II
	ET21	0.63	-12°	1.1544	-0.19			
	GWR TT70			1.1536	-0.20			
O_1	ET16 (N)			1.1537	-0.27	1.1564	1.1561	1.1555
	Melchior et al.			1.1573	-0.62			
	ET21	0.72	-37°	1.1605	-0.12			
	GWR TT70			1.1578	-0.00			
M_2	ET16 (N)			1.1599	-0.12	1.1565	1.1607	1.1593
	Melchior et al.			1.1674	-0.18			

Table 6 COMPARISON OF THE GRAVITY TIDE FACTORS WITH THEORETICAL MODELS

To compare the test result with a theoretical model, one should make the ocean loading correction for observation results first. Using Schwidersky's global cotidal maps and 4 Chinese coastal cotidal maps (M_2 , O_1 , K_1 , S_2), and the calculation methods of convolution (Farell, W.E., 1972) and convolution-spherical expansion (Hsu, H.T., 1982) we obtained the expected loading correction for 8 components, the results obtained from the two methods are the same. The result gained after loading correction is shown in Table 6. In addition the theoretical model value of Wuchang region is given too, where Wahr's model refers to a 1066A elastic model, while Dehant (Dehant, V. and Ducarme, B., 1978b) are calculated in terms of a re-definition of the gravity tidal factor for the same model. Dehant (Dehant, V. 1987a) takes the interior inelasticity of the earth into account based on Zschau's model. Obviously, our test results (Wuchang datum) agree quite well with Dehant's model in tidal factors. The variation of the factors for diurnal tidal waves separated from the long series registration with ET21 meter is shown in Fig. 8. We can see that two small waves ψ_1 and ϕ_1 describe clearly the dynamic effect of the liquid core of the earth.

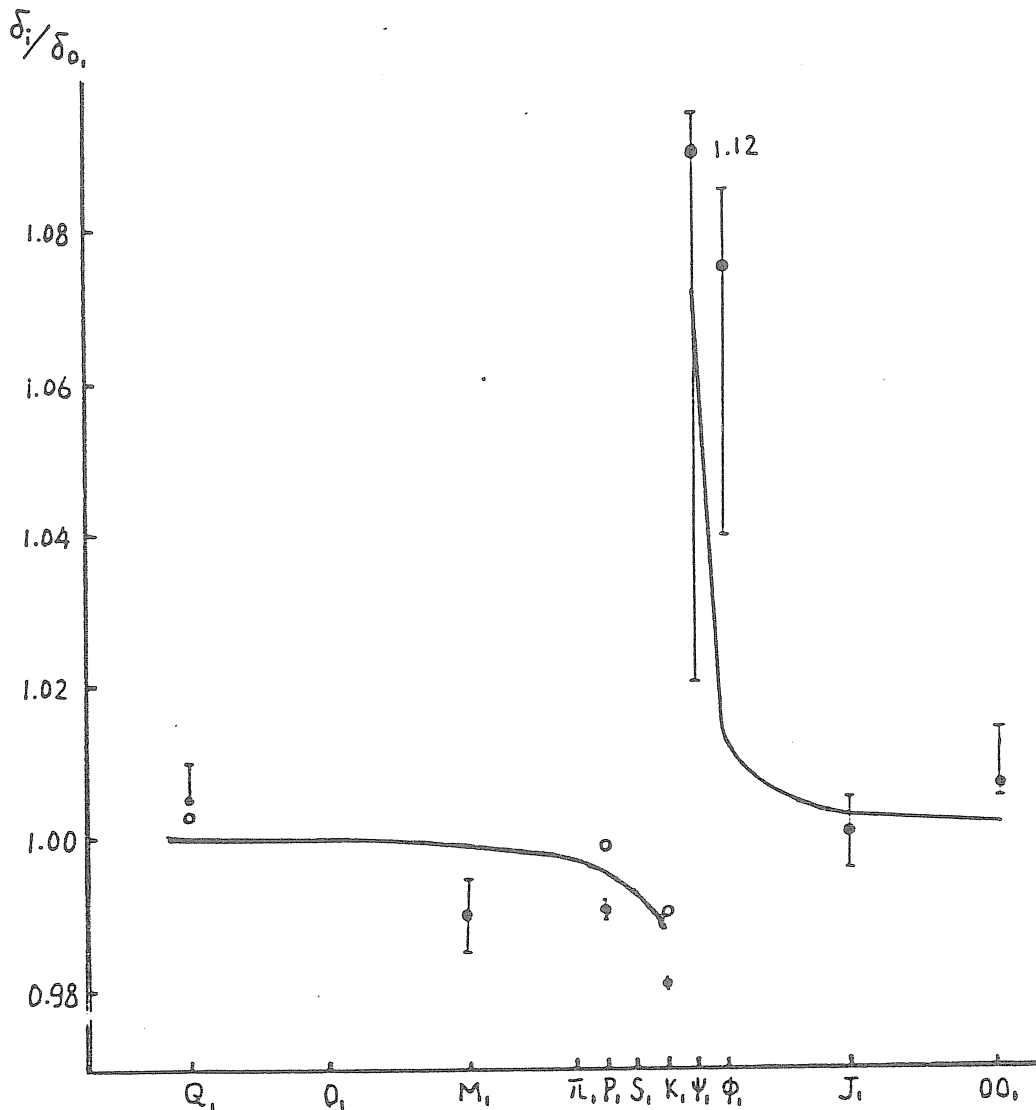


Fig. 8 Comparison of liquid core dynamical effects
 • observations, O observations corrected for loading
 — Dehant's model

4. DISCUSSION

The conversion of the ET-gravimeter into electrostatic feedback and the use of the digital acquisition system are effective for improving meter resolution and accuracy. According to the result of both ET16 and ET21, and comparing the mean-square error in each tidal band we find that the accuracy of the former is less than $1 \mu\text{gal}$, and better than that of the latter.

Scale calibration for gravimeters is an important problem. Investigation on ET21 meter shows that the manufacturers scale is 1% larger than that of the baseline value. To set up a global gravity tidal standard it is necessary to pay attention to this work. For early 1989 it is planned to calibrate ET16 on the Wuchang baseline and after the end of the observations in China on the line in Darmstadt. Also the transfer function of ET16 will be determined prior to the final analysis.

The established tidal datum based on Wuchang gravity baseline accords very well with Dehant's earth model, the mainwave parameters obtained are:

$$\delta_{M_2} = 1.1744 \qquad \Delta\varphi_{M_2} = -0.49$$

$$\delta_{O_1} = 1.1776 \qquad \Delta\varphi_{O_1} = -0.40$$

in comparison with the Bruxelles system there is a systematic discrepancy: - not only in amplitude but also in phase. These results are corroborated further by the observations of ET15 in Wuchang.

The observation results of ET16 and ET21 after scale reduction coincide well, their mutual differences are within 0.2% and $0^{\circ}05$, this indicates that the side by side comparison is successful, but because of the lack of absolute calibration for ET16, the existing 1% amplitude systematic discrepancy awaits to be examined. Further investigations of the instrumental parameters and environmental influences are under progress.

ACKNOWLEDGMENT

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