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1st Meeting of the

"Working Group on Data Processing in Tidal Research"

Bonn 14 th - 16 th march 1978.

At the 8th International Symposium on Earth Tides in Bonn, 19th - 24th September 1977, the President of the Permanent Commission on Earth Tides has appointed the "Working Group on Data Processing in Tidal Research". Furthermore, the Institut für Theoretische Geodäsie, Bonn, has been charged with the organisation of the 1st meeting in March 1978.

This meeting took place from March 14th to 16th, 1978 at the Institut für Theoretische Geodäsie, 5300 Bonn, Nussallee 17, Federal Republic of Germany.

Membership of the Working Group

as decided at 8 th International Symposium on Earth Tides by the Permanent Commission on Earth Tides.

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	BARTHA G.	(Hungary)
	CHOJNICKI T.	(Poland)
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	VENEDIKOV A.	(Bulgaria)
	WENZEL H.G.	(Germany)
	YARAMANCI U.	(Turkey)
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Program of the meeting

1. Opening session: BONATZ M.

2. Basic conditions for tidal analysis methods

BAKER, T.F. : A review of the objectives of tidal analysis

SCHÜLLER, K. : A unified presentation of the tidal force development with respect to spatial coordinate systems

WENZEL, H.-G. : A standard data set for comparison of tidal potential developments

BAKER, T.F. : Non equilibrium influences on the tidal signal

VENEDIKOV, A. : Stochastic models of tidal records

WENZEL, H.-G. : Estimation of accuracy for the earth tide analysis results. (paper published in BIM 76).

3. Review of tidal analysis methods

3.1. Least squares methods

YARAMANCI, U. : Tidal analysis methods and optimal linear system approximation

VENEDIKOV, A. : Upon the analysis of Earth tidal data

SCHÜLLER, K. : About the sensitivity of the Venedikov tidal parameter estimates to leakage effects

YARAMANCI, U. : Principles of the response method

YARAMANCI, U. : Principles of the T.I.F.A. method

SCHÜLLER, K. : Principles of the HYCON-Method

CHOJNICKI, T. : Supplementary precision estimation of results of tidal data adjustment

YARAMANCI, U. : Spectral interpretation of the error calculation in tidal analysis

BARTHA, G. : Remarks to the investigation of the residual curve

CHOJNICKI, T. : Some remarks on Venedikov method

3.2. Spectral and optimum filter methods

SUKHWANI, P. : Three different methods for taking in account the gaps in spectral analysis of Earth tide records

JENTZSCH, G. : Selective filtering method

JENTZSCH, G. : Maximum entropy method

4. Unification in the presentation of tidal data and analysis results

DUCARME, B. : Data Standardisation in tidal research

MELCHIOR, P. : Standardisation rules for the presentation of results in tidal research : "Earth Tide analysis abbreviated computer output"

CHOJNICKI, T. : Tidal data recording on the magnetic tape

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Data Prehandling

6. Closing Session

- discussion on the general results of the meeting
- recommendations and publications
- next meeting

A REVIEW OF THE OBJECTIVES OF TIDAL ANALYSIS

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(Paper presented to the Working Group on Data Processing in Tidal Research, Bonn, March 1978)

ABSTRACT

The objectives and required accuracy of tidal analysis procedures are examined. It is not sufficient to state that tidal analysis is required to be as accurate as possible. The accuracy required depends upon the physical problem under investigation and the constituents being considered. An understanding of the mechanism of the signal generation and its perturbations are essential.

1. INTRODUCTION

Over the last 15 years with the wider use of computers there has been a rapid development in the quantity and quality of the data analysis methods used in Earth Tides. In many ways tidal analysis has become a specialist discipline in its own right. However, there are clearly dangers in this since a detailed understanding of the physics of the signal generation and its associated perturbations is necessary in order to perform a satisfactory analysis. Many of the methods involve implicit assumptions based on the hypothetical concept of a pure body tide signal. Even in the centre of continents the ocean loading can be as large as 20% of the body tide signal. In a separate paper [3] it is shown that due to the dynamic response of an ocean basin, the tidal potential relationships are not always adequate for defining the non separable waves within a group.

A further problem arises from non-linear waves due to shallow water loading, the site geology or the instrument [3]. These non-linear waves can be important both in the main tidal bands and at higher frequencies.

Multichannel inputs must be considered if the analysis results are not to be biased by the effects of ambient temperature, thermoelastic tilt, rainfall, air pressure, changes in water table etc. Consideration of some of these inputs may be important for tidal gravity if we are to achieve the precision required for the ocean tide inverse problem [1] . For tilt and strain, it may be argued that sites, such as deep mines, should be used such that the effects of these other inputs are minimised. However it turns out that such sites generally have strain-tilt (strain-strain) coupling perturbations that are impossible to model [1] . It will probably be necessary therefore to use other techniques such as boreholes or shallow trenches [2] . In some cases the reduction in the coupling perturbations may be achieved at the expense of increased meteorological noise, thus increasing the importance of multichannel modelling.

Tidal parameters can only be used for physical interpretation if realistic error estimates are given. Again this can only be achieved if we have some understanding of the physics of the noise processes.

The many different methods of tidal analysis that have been developed often give significantly different results when applied to the same data and it is important to understand the fundamental assumptions that lead to these differences [6,7].

2. REQUIREMENTS

The required accuracy and the number of waves to be considered depend upon the physical problem under investigation and the magnitude of the associated noise. In oceanographic work it is common to use only 20 to 30 astronomical waves (plus several shallow water waves). These waves are corrected for the 9 and 19 year variations using tables of so called nodal factors [4] . Errors obviously arise from the many neglected small waves in each wave group which can lead to half yearly or yearly astronomical modulations of monthly analysis results. In the M_2 group, for example, the Cartwright-Tayler-Edden tables contain 19 waves and the neglected waves can give rise to a 1.3% modulation in M_2. In general, however, an analysis accuracy of 1 or 2% is sufficient for most purposes since there is a large amount of meteorological noise due to storm surges, for example and there are difficulties due to complex non-linear interactions in shallow water. For specialised research purposes, particularly for sites with lower noise or where long series of observations are available a more complete tidal development is normally considered.

2.1 Tilt and strain

There are very few, if any, measurements that have been made where it can be guaranteed that unknown strain-tilt (strain-strain) coupling perturbations are less than 5%. In most cases these perturbations are much greater. Clearly therefore an accuracy of 1 or 2% in the analysis is all that is required at the present time. Also, for geophysical investigations it should be noted that there are very few sea areas in the world where even M_2 is known to better than $\pm 5\%$ in amplitude and $\pm 5°$ in phase.

At this stage, therefore, it is unnecessary to use 363 astronomical waves as in Venedikov's least squares method or 505 waves as in Chojnicki's least squares method. 30 or so waves in an analysis are sufficient. It is probably more important to consider multichannel inputs and non-linear waves than to extend the number of astronomical waves.

If we wish to examine waves other than the 5 major waves (M_2, S_2, N_2, O_1 and K_1) then a fuller astronomical development is required (e.g. for examination of the core resonance). Since, however, cotidal maps are not available for the smaller waves, then ocean loading corrections cannot be made. The smaller waves are therefore difficult to interpret, although they may eventually prove useful for the ocean tide inverse problem. With the smaller waves the effects of noise are relatively more important and a spectral analysis of the observations (high pass filtered) and of the residuals is essential in order to assess the significance of the results.

2.2 Gravity

It has been clearly demonstrated that tidal gravity is relatively insensitive to the elastic properties of the Earth. The principal use of tidal gravity measurements is therefore for the ocean tide inverse problem. It has been shown [1] that due to the non-uniqueness and ill-conditioning of the tidal gravity inverse problem, all experimental errors must be reduced to a minimum. This, of course, includes errors arising from the data analysis.

In order to illustrate this point, let us consider a typical inverse loading problem. Consider a pair of stations operating for 6 months and suitably situated for examining the North Atlantic tides - a station in mid-France and a station in mid-Germany. It has been shown [1] that in order to reduce the non-uniqueness, it is essential to use the differential signal between pairs of stations. Loading calculations show that in this case we can expect a differential M_2 loading signal Δg of 1 microgal. Using typical internal errors from Venedikov analyses for Geodynamics (or Askania GS 15 or LaCoste G) gravity records of 6 months duration, we can expect root mean square errors of $\sim$ 0.04 microgals in the semi diurnal band and $\sim$ 0.06 microgals in the diurnal band (i.e. approximately 0.1% error in M_2 and 0.2% error in O_1).

We know the approximate differential loading Δg between the two stations for other constituents by using the amplitude relationships from tide gauges [3] . Then, using the above typical errors we obtain the following table for the errors in the observed differential gravity due to internal errors only:

	Δg (microgals)	% error in Δg
M_2	1	6%
S_2	0.33	17%
N_2	0.2	28%
O_1/K_1	0.04	200%

We need an uncertainty in Δg of less than $\pm$5% in order to find new oceanographic information. Thus we require:
Systematic errors + internal errors + analysis errors $\leqslant$ 0.1% of the amplitude of the tidal constituent.

Three important points should be made at this stage.

(1) These internal errors are typical for gravity measurements in Europe with Geodynamics, Askania GS 15 and LaCoste G

gravitymeters. For the LaCoste tidal gravitymeters and the superconducting gravitymeter the errors can be reduced typically by factors of 2 and 3 respectively. For tidal gravity profiles in some areas of the world, the errors are 5 to 10 times larger.

(2) Generally the above internal errors are smaller than the systematic errors, particularly those arising from poor intercalibrations of instruments (amplitude and phase).

(3) The large O_1 and K_1 % errors are due to the small amplitudes of the diurnal tides in the N. Atlantic. The % errors would be smaller for other ocean basins.

The major problems at the present time are instrumental and experimental. However, in the context of this meeting it should be noted that the differences between different analysis methods when applied to the same gravity data are of the order of 0.5% even for the larger constituents [9] . This means that instruments used in World gravity profiles must not only be all regularly intercompared at base stations but also, if different analysis procedures are used, then they also must be compatible to a very high accuracy.

An examination of analysis results by different methods shows that many of the differences arise from small differences in the theoretical tides used in the methods. Differences are often of the order of 0.3% and arise from differing geodetic and astronomical coefficients and the inclusion or exclusion of the ellipsoidal correction and the inertial correction. Since small changes of this order are inevitable in the future, we must be careful to ensure that it is absolutely clear what coefficient and corrections have been included. In this context, the differences between the methods are much smaller if the amplitudes in microgals are used rather than the δ factors. The results in microgals are more fundamental since they are the physical quantities recorded and differences in δ arise by normalizing using differing theoretical tides.

From the above it is clear that for the inverse problem we also need realistic estimates of the errors of the tidal constituents. Unfortunately the different methods also give widely differing error estimates. These are usually based on a white noise assumption, either across the whole spectrum or across individual tidal bands. It is essential that the residual spectrum is used to obtain more realistic error limits [8] . Similarly, it is important to consider the time variation of the tidal constituents using shifting analyses [5]. The residual spectrum and the time variation of the constituents not only give more realistic error limits but may also contain interesting physical information about the instrument, the Earth or the oceans [3] .

It is clear from these discussions that, at least for the immediate future, the analysis requirements for tidal gravity are an order of magnitude greater than for tilt or strain.

3. CONCLUDING REMARKS

Taking into account all the above requirements that are necessary for physical interpretation of the data, any future analysis program should include the following points in its procedure and presentation:
(where necessary notes for additional clarification of the requirements are included in parenthesis)

(1) The analysis procedure should be based on least squares [6,7].

(2) For harmonic methods, the full Cartwright-Tayler-Edden potential should be used. (A listing of the theoretical tides at the station should be given).

(3) More realistic error limits based on the residual spectrum should be given [8] .

(4) An indication of the time variation of the main tidal waves should be given. (This gives information on instrumental performance and the changes with time in the response of the Earth or oceans). [5].

(5) Non-linear waves should be included in the least squares analysis [3] .

(6) For non-separable waves, an attempt should be made to allow for departures from the equilibrium response to the potential [3] .

(7) Multichannel inputs should be included. (Experimental investigations are required in order to establish the physical mechanisms of meteorological and environmental perturbations).

(8) Full details of the numerical filters used should be given.

(9) In addition to δ or γ factors, the results should be given in fundamental physical units (microgals or milliseconds and phases). (The given amplitude should be for the main wave in the group).

(10) A complete description of the experiment should be given:-
station, instrument, dates, amplitude and phase calibration methods, azimuth and azimuth accuracy, experimental difficulties etc.

((a) It is impossible to give sufficient information in a short computer listing for satisfactory assessment and interpretation of the experiment. A full text is required.

(b) Phase and azimuth conventions should be clearly stated.)

(11) For tidal gravity, M_2 and O_1 results with dates and errors should be given for previous measurements with the same instrument at the fundamental base stations.

((a) For the inverse problem very accurate absolute calibrations of amplitude and phase are not required but an accurate relative calibration of all instruments is essential. The previous base station values allow the determination of the relative calibrations of the instruments and also provide a check on the variation of instrumental constants with time.

(b) One fundamental base station is required in each 'geographical area' and should be accurately 'tied' in to the other fundamental stations.

(c) If amplitude and phase corrections have already been included in the analysis in an attempt to obtain better absolute parameters then these corrections should be stated explicitly since it is inevitable that they will change as further experimental work on calibrations is performed.)

(12) Plots of the amplitude factors (δ,γ) and phases as functions of frequency (i.e. the response functions) with error bars should be provided [3] . (These plots assist in the identification of non-linear effects and other input channels).

(13) Plots should be given of the observed and the residual spectra. (For assessment of signal to noise ratio, physics of 'noise' processes and error limits on tidal constituents.

(14) Plots should be given of the observed and the residual time series. (For assessment of variation of signal quality and examination of non-stationary noise).

(15) A secondary print out of the results should be given (both amplitude and phase) but with the theoretical body tide removed. (For assessment of the magnitude of the load tide signal and/or the magnitude of coupling perturbations).

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Unified Presentation of the Tidal Force Development with Respect to Different Spatial Coordinate Systems.

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Summary: The tidal potential for a rigid earth and its associated forces are discussed in spatial cartesian , spherical and ellipsoidal coordinates. The properties of the different systems are evaluated and the transformation laws are presented by means of the tensor calculus. Special attention is devoted to the spatial ellipsoidal coordinate system because it represents the reference system with respect to the orientation of tidal instruments.

I. Formulation of the Problem

As the tidal force in a point P of a rigid model earth depends on the actual positions of moon and sun, the tidal potential may be written as a function of astronomical constants {a}, the coordinates b^i of the position vector $\vec{r}$ of P in an arbitrary coordinate system and the time variable t

$$V = V(b^1, b^2, b^3, \{a\}, t) \tag{1.1}$$

In this paper we shall discuss the representation of $\vec{r}$ in different coordinate systems and therefore the dependence of V on {a} and t is dropped for the moment, so

$$V = V(b^1, b^2, b^3) \tag{1.2}$$

From potential theory it is well known that the application of the Nabla-operator ∇ will lead to the components of the force associated with V. However, the components of the Nabla operator itself depend on the actual choice of the coordinate system. In the following the properties of the spatial cartesian , spherical and ellipsoidal coordinate systems are evaluated, their transformation laws into each other are presented, and the Nabla-operators of the different systems are worked out. As the orientations of tidal instruments coincident with sufficient accuracy with the ellipsoidal normal of the reference ellipsoid of revolution (I.U.G.G.1967), it is shown how to derive the resulting tidal force components into the desired directions.

II. Transformation Properties between Spatial Cartesian, Spherical and Ellipsoidal Coordinate Systems

As it is shown in a lot of geodetic textbooks (e.g. TORGE 1975), the following transformation equations are valid (see Fig.1)

a) cartesian (x^i) -- spherical (σ^i)

$$\begin{aligned} x^1 &= r\cos\beta\cos\lambda = \sigma^1\cos\sigma^2\cos\sigma^3 \\ x^2 &= r\cos\beta\sin\lambda = \sigma^1\cos\sigma^2\sin\sigma^3 \\ x^3 &= r\sin\beta = \sigma^1\sin\sigma^2 \end{aligned} \tag{2.1}$$

b) cartesian (x^i) -- ellipsoidal (η^i)

$$\begin{aligned} x^1 &= (N+h)\cos B\cos L = (N(\eta^2)+\eta^1)\cos\eta^2\cos\eta^3 \\ x^2 &= (N+h)\cos B\sin L = (N(\eta^2)+\eta^1)\cos\eta^2\sin\eta^3 \\ x^3 &= ((1-e^2)N+h)\sin B = ((1-e^2)N(\eta^2)+\eta^1)\sin\eta^2 \end{aligned} \tag{2.2}$$

c) spherical - ellipsoidal

$$\begin{aligned} r &= \sqrt{(N+h)^2\cos^2B + ((1-e^2)N+h)^2\sin^2B} \\ \beta &= \arctan\left\{\frac{(N(1-e^2)+h)\sin B}{(N+h)\cos B}\right\} \\ \lambda &= L \end{aligned} \tag{2.3}$$

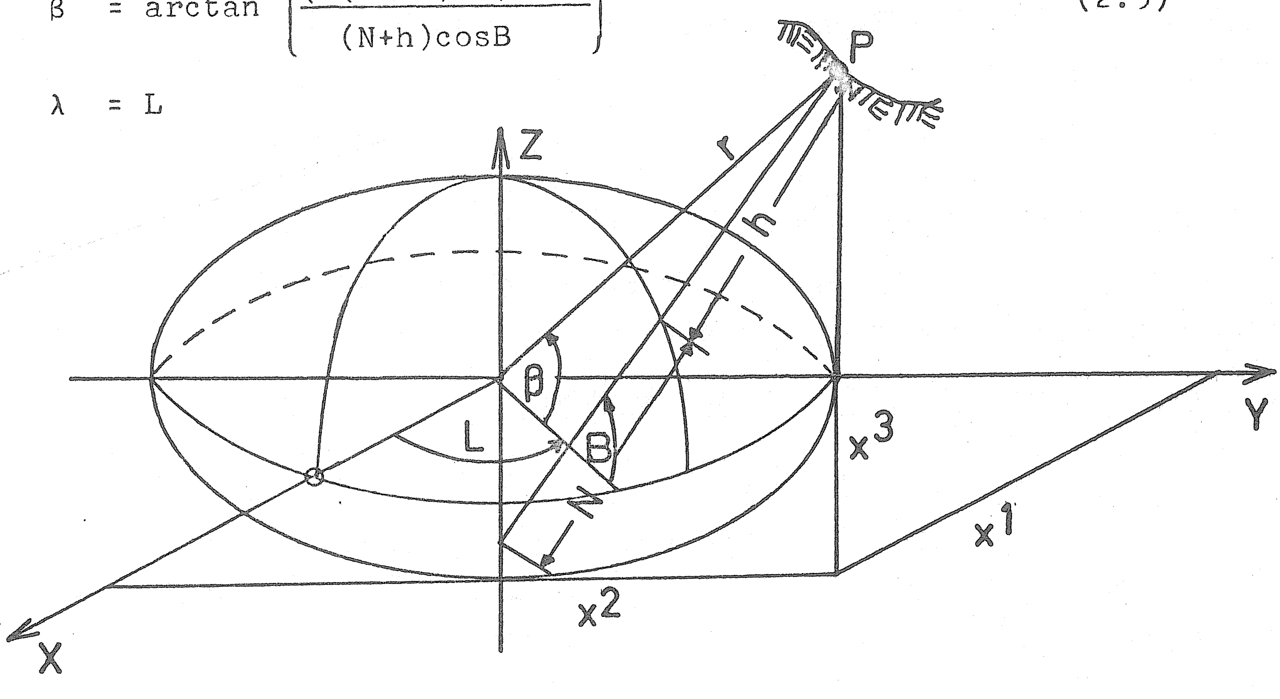

Fig.1: The position vector $\vec{r}$ in the different coordinate systems

Due to the relations (2.1) - (2.3), the properties of the three different systems may be described by their basis systems and their metric tensors. Let b^i (i=1,2,3) be the components of $\vec{r}$, the covariant system of basis vectors is defined by

$$\vec{b}_i = \frac{\partial \vec{r}}{\partial b^i} \qquad (2.4)$$

The associated covariant metric tensor is then given by the scalar product

$$b_{ij} = \vec{b}_i \circ \vec{b}_j \qquad (2.5)$$

with the matrix of covariant metric coefficients

$$(b_{ij}) = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \qquad (2.6)$$

Furthermore a contravariant basis $\vec{b}^i$ is defined as

$$\vec{b}^i = b^{ij} \vec{b}_j \qquad (2.7)$$

where the contravariant metric tensor b^{ij} is related to the covariant one by

$$b^{ij} \circ b_{jk} = \delta^i_k \qquad (2.8)$$

(δ^i_k — Kronecker symbol)

Hence the following equation holds for the matrix of contravariant metric coefficients:

$$(b^{ij}) = (b_{ij})^{-1} \qquad (2.9)$$

(Hereby we presuppose that $(b_{ij})^{-1}$ uniquely exists).

Applying (2.4) - (2.9) to the actual coordinate systems, the bases and metric coefficients are found, namely:

a) Spatial cartesian coordinate system

$$\vec{r} = x^1\vec{c}_1 + x^2\vec{c}_2 + x^3\vec{c}_3 \qquad (2.10a)$$

where c_i are orthogonal basis vectors of unity length by definition. Such a basis is called orthonormal. Hence the matrix (c_{ij}) of covariant metric coefficients is

$$(c_{ij}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.10b}$$

b) Spherical coordinate system

From (2.10a), (2,4), and (2.1) follows for the sperical basis

$$\vec{s}_K = \frac{\partial \vec{r}}{\partial x^i} \frac{\partial x^i}{\partial \sigma^k} = \frac{\partial x^i}{\partial \sigma^k} \vec{c}_i \tag{2.11a}$$

and the matrix of covariant metric coefficients will be:

$$(s_{ij}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\cos^2\beta \end{pmatrix} \tag{2.11b}$$

From (2.11b) it is clear that the spherical coordinate basis is orthogonal, however as (2.11b) shows, the basis vectors are not of unity length. Moreover their lengths are dependent from the actual position.

c) Spatial ellipsoidal coordinate system

From (2.10a), (2.4), (2.2) and (2.3) follows for the ellipsoidal basis:

$$\vec{e}_k = \frac{\partial \vec{r}}{\partial x^i} \frac{\partial x^i}{\partial \eta^k} = \frac{\partial x^i}{\partial \eta^k} \vec{c}_i = \frac{\partial \sigma^i}{\partial \eta^k} \vec{s}_i \tag{2.12a}$$

and the matrix of covariant metric coefficients is:

$$(e_{ij}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & (M+h)^2 & 0 \\ 0 & 0 & (N+h)^2\cos^2 B \end{pmatrix} \tag{2.12b}$$

with M as the normal radius of curvature in meridian direction. Again, this basis is orthogonal but with position-dependent lengths of the basis vectors.

Summarizing the result of this section, all coordinate systems under consideration have orthogonal basis vectors. This common property will become most important in the following.

III. Derivation of the Tidal Force Components in the Different Basis Systems

The Nabla operator ∇ with respect to an arbitrary covariant basis $\vec{b}_i$ is defined as (KLINGBEIL 1966):

$$\nabla = \vec{b}^i \cdot \frac{\partial}{\partial b^i} = b^{ij}\vec{b}_j \frac{\partial}{\partial b^i} \quad (3.1)$$

Usually the tidal force components are given in spherical coordinates. Hence, the spherical basis has to be introduced in (3.1) together with the contravariant metric coefficients, which follow from (2.9) and(2.12b):

$$\nabla_s = s^{ij}\vec{s}_j \frac{\partial}{\partial \sigma^i} \quad (3.2)$$

As the application of ∇ to V should give the force components it has to be related to basis vectors of unity length. With

$$\vec{s}_i{}^* = \frac{\vec{s}_i}{\sqrt{s_{(ii)}}} \quad (3.3)$$

(3.2) changes to

$$\nabla_s = \sqrt{s_{(jj)}} \cdot s^{ij}\vec{s}_j{}^* \frac{\partial}{\partial \sigma^i} \quad (3.4)$$

where the brackets (ii) indicate that no summation has to be done over these indices.

Similarly, the Nabla operator for spatial ellipsoidal coordinates will be obtained, namely:

$$\nabla_e = \sqrt{e_{(jj)}} \quad e^{ij}\vec{e}_j{}^* \frac{\partial}{\partial \eta^i} \quad (3.5)$$

Finally the Nabla operators of the different coordinate systems have to be applied to the potential V, leading to the tidal force $\vec{T}$:

a) in spherical coordinates:

$$\vec{T} = -\nabla_s\{V(\sigma^i)\} = -\sqrt{s_{(jj)}}\; s^{ij}\vec{s}_j{}^* \frac{\partial V(\sigma^1,\sigma^2,\sigma^3)}{\partial \sigma^i}$$
$$= -\left(\frac{\partial V}{\partial r}\vec{s}_1{}^* + \frac{1}{r}\frac{\partial V}{\partial \beta}\vec{s}_2{}^* + \frac{1}{r\cos\beta}\frac{\partial V}{\partial \lambda}\vec{s}_3{}^*\right) \quad (3.6)$$

b) spatial ellipsoidal coordinates:

$$\vec{T} = -\nabla_e\{V(\eta^i)\} = -\sqrt{e_{(jj)}}\, e^{ij} \vec{e}_j * \frac{\partial V(\eta^1,\eta^2,\eta^3)}{\partial \eta^i}$$

$$= -\sqrt{e_{(jj)}}\, e^{ij} \vec{e}_j * \frac{\partial V}{\partial \sigma^k} \frac{\partial \sigma^k}{\partial \eta^i} \qquad (3.7)$$

It is important to emphasize that (3.6),(3.7) describe the same tidal force vector $\vec{T}$, however the components refer to different bases. While the application of ∇_s results in tidal force components with respect to the geocentre, the application of ∇_e gives directly the tidal force components orientated to the ellipsoidal normal. Hence, the components of (3.6) must be rotated with an angle $\varepsilon = B-\beta$, which then leads to the results of WENZEL 1974. Attention has to be paid when using the tidal force components published in CHOJNICKI 1972, because they are not representing an orthogonal system. (Vertical component to the geocentre, horizontal ones to ellipsoidal directions).

IV. Detailed Evaluation of the Tidal Force Components with Respect to Spatial Ellipsoidal Coordinates

The tidal potential of 2nd and 3rd degree for a point P(B,L,h) in an ellipsoidal coordinate system at a time instant t may be presented as:

$$V = V(r(B,h),\, \beta(B,h),\, L) = G \sum_{i=2}^{3} \sum_{j=0}^{i} \left(\frac{r}{r_o}\right)^i M_{ij} N_{ij} \qquad (4.1)$$

where

B,L,h —— ellipsoidal latitude, longitude, altitude

r —— geocentric radius

r_o —— radius of a sphere of the same volume as the ellipsoid (a,e)

a,e —— ellipsoidal major axis, 1st excentricity

β —— geocentric latitude

$M_{ij} = M_{ij}(\beta)$ - functions of geocentric latitude

$N_{ij} = N_{ij}(L)$ - functions of ellipsoidal longitude

G —— Doodson constant

g —— gravity

The formulas for r and β are according to (2.3):

$$r = ((N+h)^2\cos^2 B + (N(1-e^2)+h)^2\sin^2 B)^{\frac{1}{2}} \qquad (4.2)$$

and

$$\beta = \arctan\left(\frac{(N(1-e^2)+h)\sin B}{(N+h)\cos B}\right) \qquad (4.3)$$

$$N = a(1-e^2\sin^2 B)^{-\frac{1}{2}}$$ —— normal radius of curvature in L-direction (4.4)

$$M = a(1-e^2)(1-e^2\sin^2 B)^{-\frac{3}{2}}$$ — normal radius of curvature in meridian direction (4.5)

As the ellipsoidal height h of a surface point is measured along its corresponding ellipsoidal normal, the differentiation of V with respect to h will directly provide the modulus of the vertical tidal force component (see (3.7)):

$$\Delta g = -\left(\frac{\partial V}{\partial r}\frac{\partial r}{\partial h} + \frac{\partial V}{\partial \beta}\frac{\partial \beta}{\partial h}\right)$$

$$= -\frac{G}{r_o}\sum_{i=2}^{3}\sum_{j=o}^{i}\left(\frac{r}{r_o}\right)^{i-1}\left(iM_{ij}\frac{\partial r}{\partial h} + r\frac{\partial M_{ij}}{\partial \beta}\frac{\partial \beta}{\partial h}\right)N_{ij} \qquad (4.6)$$

The required partial differentiations result in:

$$\frac{\partial r}{\partial h} = \frac{a^2 + Nh}{rN} \qquad (4.7)$$

$$\frac{\partial \beta}{\partial h} = \frac{e^2 N}{(N+h)^2}\tan B\cos^2\beta \qquad (4.8)$$

The $\frac{\partial M_{ij}}{\partial \beta}$ can easily be derived from the M_{ij} (WENZEL 1976) and have therefore been omitted here.

The horizontal tidal force modulus in a point P(B,L,h) with respect to an arbitrary azimuth A is given by (see (3.7)):

$$\zeta^* = -\frac{\partial V}{\partial B}\frac{\cos A}{(M+h)} + \frac{\partial V}{\partial L}\frac{\sin A}{(N+h)\cos B} \qquad (4.9)$$

The corresponding deflection of the vertical is then given by

$$\zeta = \frac{\zeta^*}{g} = \xi \cos A + \eta \sin A \qquad (4.10)$$

with (see (3.7)):

$$\xi = -\frac{1}{g(M+h)} \frac{\partial V}{\partial B} \quad \text{——— North-South-component} \qquad (4.11)$$

and

$$\eta = -\frac{1}{g(N+h)\cos B} \frac{\partial V}{\partial L} \quad \text{——— East-West component} \qquad (4.12)$$

The remaining partial derivations $\frac{\partial V}{\partial B}$ and $\frac{\partial V}{\partial L}$ become:

$$\frac{\partial V}{\partial B} = \frac{\partial V}{\partial r}\frac{\partial r}{\partial B} + \frac{\partial V}{\partial \beta}\frac{\partial \beta}{\partial B} \qquad (4.13)$$

with

$$\frac{\partial r}{\partial B} = (N+h+e^2\sin^2 B(N(e^2-1)-(N+h)))\ \frac{dN}{rdB} + \frac{e^2 N\sin 2B}{2r}\ (e^2N-2(N+h)) \qquad (4.14)$$

$$\frac{dN}{dB} = \frac{N^3 e^2 \sin 2B}{2a^2} \qquad (4.15)$$

$$\frac{\partial \beta}{\partial B} = \frac{\cos^2\beta}{\cos^2 B}\left(1 - \frac{e^2 N}{N+h}\right) - \cos^2\beta \tan B\ \frac{e^2 h}{(N+h)^2}\ \frac{dN}{dB} \qquad (4.16)$$

and

$$\frac{\partial V}{\partial L} = \frac{\partial V}{\partial N_{ij}} \cdot \frac{\partial N_{ij}}{\partial L} \qquad (4.17)$$

Finally, we obtain for the NS- and EW component

$$\xi = -\frac{1}{g(M+h)}\left(\frac{\partial V}{\partial r}\frac{\partial r}{\partial B} + \frac{\partial V}{\partial \beta}\frac{\partial \beta}{\partial B}\right)$$

$$= -\frac{G}{g r_o (M+h)} \sum_{i=2}^{3} \sum_{j=o}^{i} \left(\frac{r}{r_o}\right)^{i-1} \left(iM_{ij}\frac{\partial r}{\partial B} + r\frac{\partial M_{ij}}{\partial \beta}\frac{\partial \beta}{\partial B}\right) N_{ij} \qquad (4.18)$$

and

$$\eta = -\frac{1}{g(N+h)\cos B}\frac{\partial V}{\partial L}$$

$$= -\frac{G}{g(N+h)\cos B} \sum_{i=2}^{3} \sum_{j=o}^{i} \left(\frac{r}{r_o}\right)^{i} M_{ij} \frac{\partial N_{ij}}{\partial L} \qquad (4.19)$$

Computational tests have been carried out in order to check both the presented derivations and the programs in use. There is complete agreement with respect to the tidal force development of WENZEL 1974 and the approach in spatial ellipsoidal coordinates presented here.

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Standard data sets for comparison of tidal potential developments and analysis methods *

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1. Introduction

The computation of theoretical tides from a harmonic tidal potential development is based on a large number of parameters and computational procedures. The analysis methods for earth tide data, most of them using a harmonic tidal potential development, use additional computational procedures which can systematically influence the computed tidal parameters. Therefore, a comparison of the different analysis methods today in use is necessary. For a first step in this direction, standard data sets of theoretical tides which are not based on a harmonic tidal potential development can be used for evaluation of systematic differences between the analysis methods.

2. The computation of theoretical tides from ephemeris of the moon and the sun

The tidal force due to the moon (sun) is defined as the difference of the attraction forces of the moon (sun) in the observation point and in the centre of the earth. By this definition, we can compute the tidal force directly from the masses of the moon and the sun and from the geocentric coordinates of the observation point, the moon and the sun (WENZEL 1976) without using a harmonic tidal potential development.

At first, we define the geocentric cartesian coordinate system (Fig. 1) as usual with the ZG-axis in the rotational axis of the earth, the XG-axis in the equator and in the meridian of Greenwich and the YG-axis perpendicular to XG and ZG axis.

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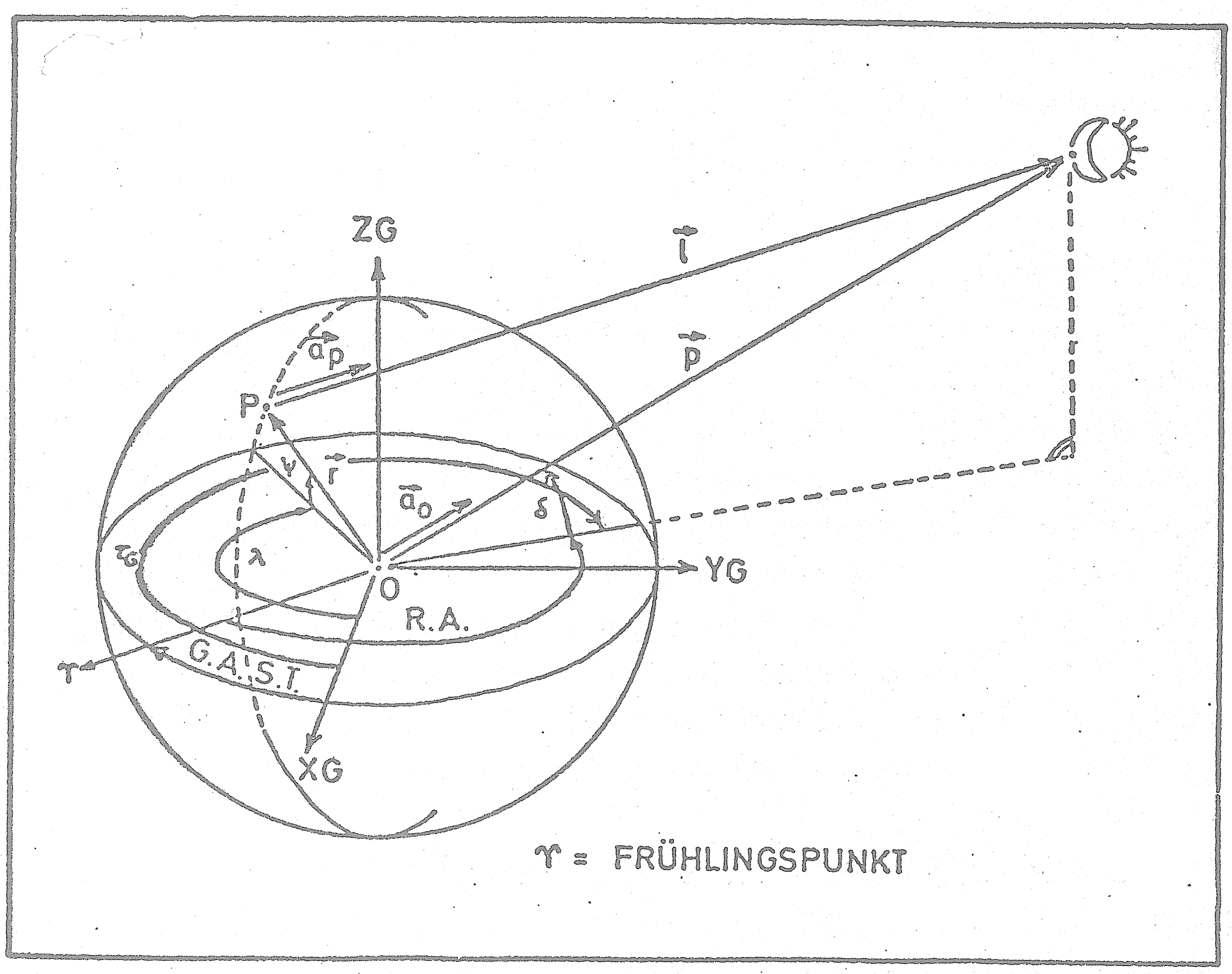

Fig. 1: Attraction forces in the geocentric coordinate system

The geocentric coordinates $\underline{r}$ of the observation point can be computed by

$$\underline{r} = \left\| \begin{matrix} r\cos\psi\cos\lambda \\ -r\cos\psi\sin\lambda \\ r\sin\psi \end{matrix} \right\| \quad (1)$$

with

r = geocentric distance of the observation point,
ψ = geocentric latitude of the observation point,
λ = ellipsoidic longitude of the observation point (positiv west of Greenwich).

The geocentric quantities r and ψ can be computed from the usual given ellipsoidic coordinates ϕ, λ and h of the observation point by

$$r = \sqrt{(N+h)^2 \cos^2\phi + (N(1-e^2)+h)^2 \sin^2\phi} \qquad (2)$$

$$\psi = \text{arc tan}\, \frac{(N(1-e^2)+h)\, \sin\phi}{(N+h)\, \cos\phi} \qquad (3)$$

with

$$N = \frac{a}{\sqrt{1-e^2 \sin^2\phi}} \qquad (4)$$

and

a = major half axis of the reference ellipsoid,
e^2 = 1. numeric excentricity of the reference ellipsoid.

The geocentric coordinates $\underline{p}$ of the moon and the sun can be computed from their ephemeris given in several astronomical almanacs (e.g. THE ASTRONOMICAL EPHEMERIS, THE AMERICAN EPHEMERIS) by

$$\underline{p} = \begin{Vmatrix} R\cos\delta\cos\tau_G \\ -R\cos\delta\sin\tau_G \\ R\sin\delta \end{Vmatrix} \qquad (5)$$

with

R = geocentric distance of the moon or sun,
δ = declination of the moon or sun,
τ_G = hour angle of the moon or sun in the meridian of Greenwich.

The quantities R and τ_G are not given directly in the almanac but can be computed from other given quantities. In the ASTRONOMICAL EPHEMERIS is given for 0^h ephemeris time (ET) of each day

α^*_M = apparent right ascension of the moon to 0.001^s,

δ^*_M = apparent declination of the moon to $0\overset{''}{.}01$,

π_M = equatorial parallax of the moon to $0\overset{''}{.}001 \cong 3\cdot 10^{-8} \sin\pi_M$,

α^*_S = apparent right ascension of the sun to 0.01^s,

δ^*_S = apparent declination of the sun to $0\overset{''}{.}1$,

R^*_S = geocentric distance of the sun in astronomic units (AU) to $1 \cdot 10^{-7}$ AU

and for 0^h universal time (UT) of each day the sideral time in the meridian of Greenwich (GAST).

The geocentric distance of the moon can be computed by

$$R_M = \frac{a}{\sin(\pi_M)} \quad , \tag{6}$$

the geocentric distance of the sun can be computed by

$$R_S = c_S \cdot R^*_S \quad . \tag{7}$$

The hour angle of the moon and the sun in the meridian of Greenwich can be computed by

$$\tau_G = GAST - \alpha. \tag{8}$$

For the computation of theoretical tides in hourly values the ephemeris of the moon and the sun and the Greenwich apparent sideral time have to be computed for each hour; this can easily be done by a LAGRANGE interpolation by which the difference of Ephemeris Time and Universal Time ($\Delta T = ET - UT$) can also be taken into account.

The accuracy of the interpolation procedure can be verified by comparison of the interpolated ephemeris of the moon with the values for 12^h ET tabulated in THE ASTRONOMICAL EPHEMERIS; using a five degree polynominal for the interpolation we found

$$m_{\alpha^*M} = \pm 1\overset{''}{.}73, \; m_{\delta^*M} = \pm 0\overset{''}{.}31; \quad m_{\pi M} = \pm 0\overset{''}{.}0007.$$

The ephemeris of the sun as well as the GAST are much smoother than the ephemeris of the moon; therefore interpolation errors are smaller for the sun

$$m_{\alpha^*S} \leq \pm 0\overset{''}{.}1, \; m_{\delta^*S} \leq \pm 0\overset{''}{.}1, \; m_{R^*S} \leq \pm 1 \cdot 10^{-7} \text{ AU}$$

$$m_{GAST} \leq 0.001^s.$$

The influence of the interpolation errors to the computation of the tidal force can be estimated (WENZEL 1976) to ±0.0015 μGal for the vertical component of the tidal force.

For the computation of tidal forces the ephemeris have to be corrected for aberrational effects in the same way as for optical observations, because the speed of gravitational forces equals the speed of light (MISNER et.al. 1973, FRITZE 1974). The tabulated (respectively interpolated) ephemeris are already corrected for the geocentric aberration, the correction for topocentric aberration ($< 0\overset{''}{.}32$) has not been applied because of the larger interpolation errors.

The topocentric coordinates $\underline{l}$ of the moon and the sun can be computed by

$$\underline{l} = \underline{p} - \underline{r} \quad , \tag{9}$$

the tidal attraction forces $\underline{f}_G$ can be computed from the difference of the attraction forces in the observation point and in the geocentre:

$$\underline{f}_G = \underline{a}_p - \underline{a}_o = k^2 m \left(\underline{l} \cdot \frac{1}{|l|^3} - \underline{p} \cdot \frac{1}{|\underline{p}|^3} \right) \tag{10}$$

with

k^2 = gravitational constant,

m = mass of the moon (sun).

The coordinate system of $\underline{f}_G$ is the geocentric coordinate system XG, YG, ZG; the tidal forces $\underline{f}_L$ in a local kartesian coordinate system XL, YL, ZL (orientation to the plump line and meridian of the observation point) can be computed by a rotation of $\underline{f}_G$:

$$\underline{f}_L = \underline{R}_2 \cdot \underline{f}_G \tag{11}$$

with

$$\underline{R}_2 = \left\| \begin{matrix} \sin\Phi\cos\Lambda, & -\sin\Phi\sin\Lambda, & -\cos\Phi \\ -\sin\Lambda \quad , & -\cos\Lambda \quad , & 0 \\ -\cos\Phi\cos\Lambda, & \cos\Phi\sin\Lambda, & -\sin\Phi \end{matrix} \right\| \tag{12}$$

Φ = astronomical latitude of the observation point,

Λ = astronomical longitude of the observation point.

For the practical computations, the astronomical latitude and longitude can be approximated by the ellipsoidic latitude ϕ and longitude λ.

3. Description of the standard data set

By the above described method three standard data sets with hourly values from 0^h UT january, 1^{st} 1972 to 23^h UT december 31^{th} 1972 have been computed for an observation point with the ellipsoidic coordinates $\phi = 52^{\circ}.387$N, $\lambda = 9^{\circ}.712$E and h = 50m (earth tide station Hannover).

STANDARD DATA SET No. 1: Vertical component of the tidal force in μGal

STANDARD DATA SET No. 2: North-south component of the tidal force in mseca, g = 9.812706 $msec^{-2}$

STANDARD DATA SET No. 3: East-west component of the tidal force in mseca, g = 9.812706 $msec^{-2}$

As a final check, the values of standard data set No. 1 have been compared with hourly values of the vertical component of the tidal force computed with a modified program of BROUCKE, ZUERN and SLICHTER 1972. This program uses a truncated series expansion of the ephemeris of the moon given by ECKERT, JONES and CLARK 1954 and a simple KEPLER ellipse for the earth orbit. The rms deviation between both computation methods was $\pm 5 \cdot 10^{-3}$ μGal probably due to the truncation of the series expansion in the BROUCKE, ZUERN and SLICHTER 1972 program.

The three standard data sets have been punched on cards in the international format for earth tide data with a resolution of 0.01 μGal for the vertical component and 0.01 mseca for the horizontal components. The standard data sets will be distributed on request from the INTERNATIONAL CENTRE FOR EARTH TIDES (ICET), 3. Avenue Circulaire, B1180 Bruxelles, Belgium, either on punched cards or on magnetic tape.

Table 1: Geodetic and astronomic constants

GRS 67 = Geodetic Reference system 1967, see MORITZ 1969

IAU 64 = International Astronomic Union 1964, see DEJAIFFE 1974

Original constants:

a	$= 637816000$ cm	major half axis of the reference ellipsoid GRS 67
e^2	$= 6.694605 \cdot 10^{-3}$	1. numeric excentricity of the reference ellipsoid GRS 67
k^2E	$= 398603 \cdot 10^{15} cm^3 sec^{-2}$	geocentric gravitation constant of GRS 67
$\sin\pi_M$	$= 3422\overset{''}{.}451$	sinus equatorial parallax of the moon IAU 64
π_S	$= 8\overset{''}{.}79405$	equatorial parallax of the sun IAU 64
M/E	= 1/81.30	mass of the moon divided by the mass of the earth IAU 64
S/E	= 332958	mass of the sun divided by the mass of the earth IAU 64

derivated constants:

k^2M	$= 4902.866 \cdot 10^{15} cm^3 sec^{-2}$	selenocentric gravitation constant
k^2S	$= 13.27180 \cdot 10^{25} cm^3 sec^{-2}$	heliocentric gravitation constant
c_M	$= 3.844000 \cdot 10^{10}$ cm	major half axis of the moons orbit
c_S	$= 1496.000 \cdot 10^{10}$ cm	major half axis of the earth orbit = Astronomic Unit

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NON-EQUILIBRIUM INFLUENCES ON THE TIDAL SIGNAL

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ABSTRACT

The dynamic response of the oceans to the tidal forces is manifested in the Earth tide signal through the ocean loading. For data lengths of less than one year this gives rise to problems in relating the non separable waves within a group.

Non-linear waves arising from shallow water loading (or instrumental and geological effects) can be important in the main tidal bands and at higher frequencies. Multichannel analysis may be necessary for taking into account perturbations due to meteorological and associated phenomena. The residuals after tidal analysis contain important information concerning both the noise mechanisms and the time variations of the tidal constituents.

1. INTRODUCTION

It is perhaps surprising that over the last 15 years the tidal analysis methods used in Earth tides and in ocean tides have to a great extent developed independently of one another. It is now clear that the two disciplines can learn from each other since natural differences in emphasis have arisen due to differences in the two phenomena. Here we discuss particular features of ocean tide analysis that can be usefully incorporated into Earth tide analysis programs.

The development of Earth tide analysis programs has traditionally been approached with the concept of the body tide (or equilibrium tide) in mind. Thus it is natural that the full Cartwright-Tayler-Edden potential is usually incorporated directly into the least squares analysis programs and that the computation is carried out from the beginning with the auxiliary unknowns δ or γ . In harmonic ocean tide analysis the potential is only used for a phase reference and the computation is usually carried out using the amplitudes in physical units [11]

In harmonic analysis the problem of relating waves which cannot be separated in the available data length is of fundamental importance. The Earth tide least squares analysis programs (Chojnicki and Venedikov) by incorporating the theoretical tides directly in the computation have implicitly assumed that the non-separable waves within a group are related to one another in the same way as they are in the tidal potential. In practice this assumption is valid for the body tide signal. For the oceans, however, the dynamic response of the ocean basin is important and the tidal potential relationships no longer apply. This problem has therefore been considered in ocean tide analysis programs but not in Earth tide programs. It is evidently important for the ocean loading portion of our signals.

The dynamic response of the oceans to the tidal forces leads to the concept of the response function relating an observed time series to the tidal potential [5] . It is found in practice that the ocean response to the astronomical input is a smooth function of frequency. The response method [5,10] uses the tidal potential in the time domain and this has naturally led to the consideration of other inputs in the time domain. With a few exceptions such multichannel inputs have been largely ignored in the analysis of Earth tides. However, it is important to monitor all perturbing phenomena (temperature, rainfall, air pressure, water table etc.) and to include these in the analysis of the observations wherever necessary.

Non-linear waves were introduced at an early stage of ocean tide analysis since the amplitudes of these waves are relatively large in shallow water areas. The hydrodynamic equations of motion become increasingly non-linear as the depth of water decreases. Non-linear waves have generally not been included in Earth tide analysis. They are important however in areas affected by shallow water loading and they may also be generated by non-linear instrumental responses and by non-linear effects in the site geology.

With the possible exception of the core resonance, the response function of the Earth to the astronomical input should be an even smoother function of frequency than the ocean response function. If the usual Earth tide analysis programs are used (Chojnicki and Venedikov), it is therefore an advantage to plot the amplitude factors (δ,γ) and phases of the waves, as functions of frequency (with realistic error bars). Any significant departures from a smooth curve may then indicate neglected effects due to non-linearities or the presence of other input channels.

There is also some evidence to suggest that the 'noise' in the vicinity of the tidal bands is generated by different mechanisms for continental and coastal Earth tide stations.

Some of the above problems are evidently of greater importance for Earth tide measurements adjacent to the ocean. It should be emphasised however that even in Bonn (800 kilometres from the Atlantic Ocean) calculations show that the M_2 loading from the N. Atlantic alone is 3%, 14%, 25%, 18% and 147% of the body tide for gravity, north tilt, east tilt, north-south strain and east-west strain respectively.

In the following sections examples will be given to illustrate many of the above points.

2. THE OCEAN RESPONSE

Table 1 shows the results of harmonic analysis of some long series of tide gauge data. The sites have been chosen to cover a wide area of the north west European shelf. It should be noted, however, that the results may be considerably different for other oceanic areas. The table compares the amplitude ratios and the phase differences of some of the waves in the semi-diurnal band with same quantities calculated from the tidal potential. It can be seen that within the semi-diurnal band the tidal potential relationships do not apply. It should also be noted that, although there are considerable differences from the potential relationships, these differences are, in general, constant over a wide geographical area. These relationships should therefore apply to the ocean loading portion of the Earth tide signal over a wide area of Europe.

TABLE 1

TIDE GAUGE SEMI-DIURNAL RELATIONSHIPS (LAGS POSITIVE)

	S_2/M_2	$(S_2^{\circ}-M_2^{\circ})$	L_2/M_2	$(L_2^{\circ}-M_2^{\circ})$	N_2/M_2	$(N_2^{\circ}-M_2^{\circ})$	$2N_2/M_2$	$(2N_2^{\circ}-M_2^{\circ})$
TIDAL POTENTIAL	0.465	(0°)	0.028	(0°)	0.191	(0°)	0.025	(0°)
NEWLYN (9 YRS - CELTIC SEA)	0.34	(44°)	0.04	(2°)	0.19	(-20°)	0.03	(-37°)
HILBRE (1 YR - IRISH SEA)	0.32	(43°)	0.05	(14°)	0.19	(-23°)	0.04	(-57°)
STORNOWAY (15 YRS - NW SCOTLAND)	0.39	(33°)	0.02	(34°)	0.20	(-22°)	0.03	(-40°)
ABERDEEN (9 YRS - NORTH SEA)	0.34	(38°)	0.03	(21°)	0.20	(-23°)	0.03	(-47°)
I. DOWSING (1 YR - NORTH SEA)	0.34	(46°)	0.04	(3°)	0.19	(-23°)	0.03	(-66°)
DIEPPE (2 YRS - CHANNEL)	0.33	(50°)	0.06	(-10°)	0.19	(-20°)	0.02	(-71°)

TABLE 2

TIDE GAUGE RELATIONSHIPS FOR CLOSE WAVES (LAGS POSITIVE)

	P_1/K_1	$(P_1^{\circ}-K_1^{\circ})$	K_2/S_2	$(K_2^{\circ}-S_2^{\circ})$	T_2/S_2	$(T_2^{\circ}-S_2^{\circ})$	$2N_2/\mu_2$	$(2N_2^{\circ}-\mu_2^{\circ})$
TIDAL POTENTIAL	0.331	(0°)	0.272	(0°)	0.059	(0°)	0.829	(0°)
NEWLYN (9 YRS - CELTIC SEA)	0.34	(-7°)	0.28	(-3°)	0.06	(-5°)	0.80	(-73°)
HILBRE (1 YR - IRISH SEA)	0.36	(-7°)	0.29	(-2°)	0.06	(-1°)	3.28	(215°)
STORNOWAY (15 YRS - NW SCOTLAND)	0.30	(-11°)	0.28	(-2°)	0.05	(-7°)	0.70	(17°)
ABERDEEN (9 YRS - NORTH SEA)	0.32	(-17°)	0.29	(-2°)	0.05	(-12°)	1.81	(19°)
I. DOWSING (1 YR - NORTH SEA)	0.27	(-15°)	0.28	(-3°)	0.05	(-17°)	2.32	(-108°)
DIEPPE (2 YRS - CHANNEL)	0.35	(-6°)	0.29	(-3°)	0.06	(-8°)	0.46	(229°)

Fortunately all the waves shown in table 1 are separable with one months data. In table 2 the relationships are shown for some constituents that require 6 months or 1 years data for separation. Due to the closer frequencies, the relationships correspond more closely to the tidal potential but the differences are still significant. Taking into account the increased effects of noise on the small waves, it can be seen that the differences are still reasonably uniform over a wide geographical area. However in the tables it can be seen that the (L_2, M_2), ($2N_2$,M_2) and ($2N_2$,μ_2) relationships show large anomalies at some locations. This problem is discussed in the next section.

If we have long enough series of Earth tide observations then the problem of relating the waves in tables 1 and 2 does not arise since they can all be separated. (In fact, we can then check that the relationships apply to the loading portion of the signal [1]). Normally, however, we have less than 1 years observations and some assumptions are necessary. The problem is of less importance for tidal gravity since the loading signal is typically 1 to 10% of the body tide signal. For tilt and strain even in the centre of continents it is found that the loading signal is often 20% of the body tide signal. For sites adjacent to the ocean, the loading is typically five to ten times greater than the body tide signal. Clearly in the latter case it is more appropriate to use the relations from table 2. The major difficulties arise where the two signals are of similar size. In that case one can either use directly the information from loading calculations or use a more empirical approach. For example, the relationships could be adjusted in order to ascertain if there is any significant reduction in the residuals. However, great care should be taken if it is found that relationships are required that are outside the limits given by the tidal potential and the tide gauges (relationships outside these limits are, however, possible for certain phase differences between the body and load tides).

Figure 1 shows the results of the harmonic analysis for Newlyn for the semi-diurnal band plotted as functions of frequency. The upper plot shows the ratio of the amplitude of each wave to the M_2 amplitude divided by the same ratio in the tidal potential. The lower plot shows the differences between the phase lag of each wave and the M_2 phase lag. Smooth curves can be drawn through the results for 4 of the waves. The results for μ_2 and L_2, however, are obviously anomalous. These anomalies are due to non-linear interactions at these frequencies (see below). There is also an anomaly in the vicinity of S_2. This indicates the presence of another input channel - the radiational tide [5] .

In figure 1 the dynamic response of the ocean is very clear particularly in phase. The plots are analogous to plots of the gravimetric factors δ (or diminishing factors γ) and phases as functions of frequency. For the body tide, of course, such plots would be essentially lines of zero slope (with the exception of the core resonance in the diurnal band). Plotting Earth tide observations in this form will show the presence of the loading signal, non-linearities and perturbations due to other input channels.

3. NON-LINEAR WAVES

In tables 1 and 2 and in figure 1 we have already seen the importance and the spatial variability of non-linear waves on the European shelf. Table 3 shows some of the astronomical waves in the semi-diurnal band that have non-linear waves at essentially the same frequency. The most important ones are indicated with asterisks. For example $2MS_2$ (arising from the non-linear interaction of S_2 and M_2) is often found to be 2 or 3 times larger than the astronomical μ_2.

TABLE 3 NON-LINEAR WAVES INTERFERING WITH ASTRONOMICAL WAVES

ASTRONOMICAL COMPONENT	SHALLOW WATER COMPONENT
$2N_2$	{ $2MK_2$ * { MNL_2
μ_2	$2MS_2$ *
N_2	$2ML_2$
ν_2	MLS_2
M_2	OK_2
λ_2	{ $2M\nu_2$ { $2NM_2$
L_2	$2MN_2$ *
K_2	$2M(2N_2)2$
S_2	$2M\mu_2$

As was shown in figure 1, neglect of these non-linear waves can cause difficulties in the physical interpretation of the results.

Table 4 shows the magnitude of some other important non-linear waves in relation to M_2. It can be seen that M_4 is significant (2-9% of M_2) over much of the shelf.

TABLE 4 THE MAGNITUDES OF SOME IMPORTANT NON-LINEAR WAVES

	M_4/M_2	M_3/M_2	MNS_2/M_2	OP_2/M_2
NEWLYN (9 YRS - CELTIC SEA)	0.06	0.006	0.007	0.002
HILBRE (1 YR - IRISH SEA)	0.07	0.010	0.004	0.007
STORNOWAY (15 YRS - NW SCOTLAND)	0.04	0.022	0.009	0.002
ABERDEEN (9 YRS - NORTH SEA)	0.02	0.008	0.003	0.005
I. DOWSING (1 YR - NORTH SEA)	0.02	0.004	0.005	0.006
DIEPPE (2 YRS - CHANNEL)	0.09	0.004	0.010	0.010

OP_2 (together with MKS_2) give an important modulation of M_2 in short analyses since they are only separable from M_2 with 6 months data. The table shows that OP_2 can be as large as 1% of M_2. There are also annual modulations of M_2 in shallow water for which the mechanism is still poorly understood. In the North Sea the amplitude of the two constituents introduced to allow for the annual modulation are as large as 1 to 2% of M_2 [6] . Together these 4 constituents can give, in some areas, a variation in M_2 in monthly analyses of up to 10% in range. Smaller seasonal variations in M_2 tilt have been observed at Llanrwst [2] .

It is important to include the non-linear terms for Earth tide measurements affected by shallow water loading (say, within 200 kilometres of the coastline). M_4 may be detectable over a wider area. Non-linear waves may also arise from a non-linear instrumental response or from a non-linear response of site inhomogeneities.

* N.B. The M_3/M_2 ratio is given for the total observed waves. The non-linear part of M_3 is very small.

The significance of non-linear waves should be assessed from the amplitude (or power) spectra of the observations and residuals and included in the least squares analysis wherever necessary. More complete tables of shallow water waves are given in [3,7] .

4. THE RESIDUAL SPECTRUM

Figures 2 and 3 show the residual spectra of data from the Askania tiltmeter at Llanrwst (tunnel azimuth) in summer (85 days, July - September 1974) and in winter (85 days, December 1974 - February 1975) respectively. The tides have been removed with a least squares analysis using 11 constituents in the diurnal band, 12 in the semi-diurnal, 4 in the three cycle per day band, 5 in the four cycle per day band and 5 in the six cycle per day band. It should be noted that the residuals are not reduced in the first three bands even if a Chojnicki least squares analysis with 377 waves is used [8] .

It is important to consider the residual spectrum after tidal analysis for two reasons. Firstly, the amplitude of the residuals should be used to provide error limits on the tidal constituents [9] . Evidently the usual assumptions based on white noise spectra do not apply. Secondly, the residuals may contain further interesting physical information.

It can be seen immediately from the figures that the noise is greater in winter (a factor of 2 larger in power at high frequencies increasing to a factor of 4 larger in the diurnal band). In the three cycle per day band a line appears in the winter residuals (0.1245 c.p.h.) more strongly than in the summer residuals. Similarly in the semi-diurnal band a line appears more strongly at 0.0815 c.p.h. in winter.

One very noticeable feature of the spectra are the 'cusps' in the main tidal bands. The power in the tidal bands is a factor of 10 greater than between the bands. The cusps are even more strongly noticeable in the residual spectra of Irish Sea tide gauges for the same time periods.

In the summer data the Askania semi-diurnal cusp amplitude is approximately 0.8% of the M_2 amplitude (N.B. amplitude ratio not power ratio). An identical analysis of the Liverpool tide gauge gives a cusp amplitude of 1.5% of the M_2 amplitude. For both the Askania and the Liverpool tide gauge the summer noise amplitude between the diurnal and semi-diurnal band is approximately 0.3% of the M_2 amplitude. It seems therefore that it may be possible to explain many of the features of the residual spectra in terms of Irish Sea loading. However, it should be noted that 'cusps' can also be produced by instrumental non-linearities, variations in calibration and timing and digitizing errors.

It is interesting to note that the residual power in summer between the diurnal and semi-diurnal bands ($\sim$4msec^2/cph) is similar to that found for continental Earth tide stations [4]. This raises an important question. It is found in practice that for both continental and coastal tilt measurements M_2 can be determined to within an uncertainty of about 1 or 2% from a single months analysis despite the latter signal being typically 5 times greater in magnitude than the body tide signal. One possible explanation is that the noise becomes larger as the load signal becomes larger. Thus, in addition to the site meteorological noise, there is also noise from the ocean in the loading signal, particularly in shallow water areas. The cusps may represent this increased noise in the tidal bands but further work is required before any definite conclusions can be reached.

From this brief discussion it can be seen that the residual spectrum may contain useful physical information and therefore should always be examined. A time series plot of the residuals can provide useful information on the time variation of the noise and may provide additional clues as to the noise mechanisms. Similarly, shifting tidal analyses can provide complementary information on the mechanisms causing time variations of the harmonic constituents.

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TABLE 1

TIDE GAUGE SEMI-DIURNAL RELATIONSHIPS (LAGS POSITIVE)

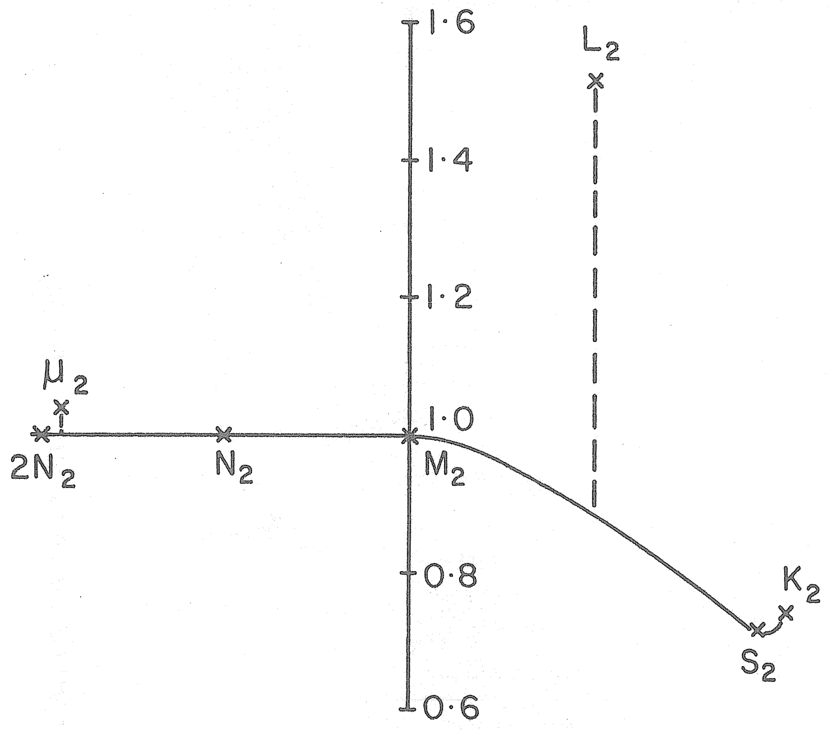
	S_2/M_2	($S_2^\circ - M_2^\circ$)	L_2/M_2	($L_2^\circ - M_2^\circ$)	N_2/M_2	($N_2^\circ - M_2^\circ$)	$2N_2/M_2$	($2N_2^\circ - M_2^\circ$)
TIDAL POTENTIAL	0.465	(0°)	0.028	(0°)	0.191	(0°)	0.025	(0°)
NEWLYN (9 YRS - CELTIC SEA)	0.34	(44°)	0.04	(2°)	0.19	(-20°)	0.03	(-37°)
HILBRE (1 YR - IRISH SEA)	0.32	(43°)	0.05	(14°)	0.19	(-23°)	0.04	(-57°)
STORNOWAY (15 YRS - NW SCOTLAND)	0.39	(33°)	0.02	(34°)	0.20	(-22°)	0.03	(-40°)
ABERDEEN (9 YRS - NORTH SEA)	0.34	(38°)	0.03	(21°)	0.20	(-23°)	0.03	(-47°)
I. DOWSING (1 YR - NORTH SEA)	0.34	(46°)	0.04	(3°)	0.19	(-23°)	0.03	(-66°)
DIEPPE (2 YRS - CHANNEL)	0.33	(50°)	0.06	(-10°)	0.19	(-20°)	0.02	(-71°)

TABLE 2

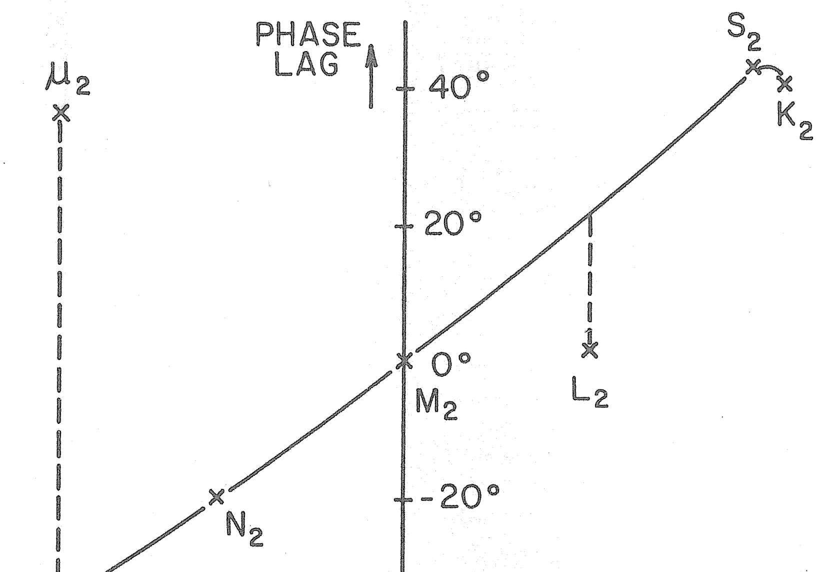
TIDE GAUGE RELATIONSHIPS FOR CLOSE WAVES (LAGS POSITIVE)

	P_1/K_1	$(\overset{\circ}{P}_1-\overset{\circ}{K}_1)$	K_2/S_2	$(\overset{\circ}{K}_2-\overset{\circ}{S}_2)$	T_2/S_2	$(\overset{\circ}{T}_2-\overset{\circ}{S}_2)$	$2N_2/\mu_2$	$(2\overset{\circ}{N}_2-\overset{\circ}{\mu}_2)$
TIDAL POTENTIAL	0.331	(0°)	0.272	(0°)	0.059	(0°)	0.829	(0°)
NEWLYN (9 YRS - CELTIC SEA)	0.34	(-7°)	0.28	(-3°)	0.06	(-5°)	0.80	(-73°)
HILBRE (1 YR - IRISH SEA)	0.36	(-7°)	0.29	(-2°)	0.06	(-1°)	3.28	(215°)
STORNOWAY (15 YRS - NW SCOTLAND)	0.30	(-11°)	0.28	(-2°)	0.05	(-7°)	0.70	(17°)
ABERDEEN (9 YRS - NORTH SEA)	0.32	(-17°)	0.29	(-2°)	0.05	(-12°)	1.81	(19°)
I. DOWSING (1 YR - NORTH SEA)	0.27	(-15°)	0.28	(-3°)	0.05	(-17°)	2.32	(-108°)
DIEPPE (2 YRS - CHANNEL)	0.35	(-6°)	0.29	(-3°)	0.06	(-8°)	0.46	(229°)

FIG. I. NEWLYN TIDE GAUGE (9 YEARS DATA)



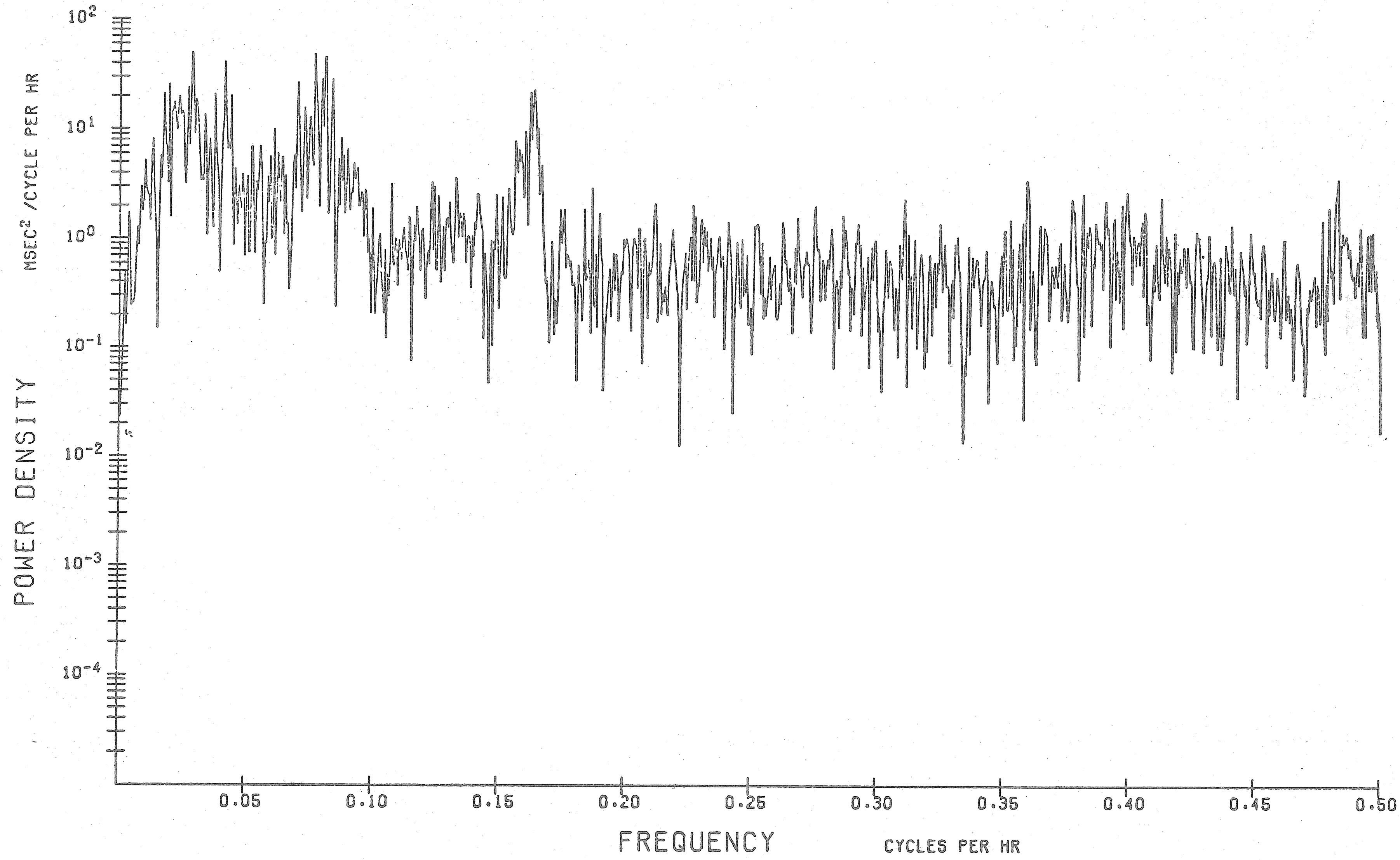
AMPLITUDE RESPONSE



PHASE RESPONSE

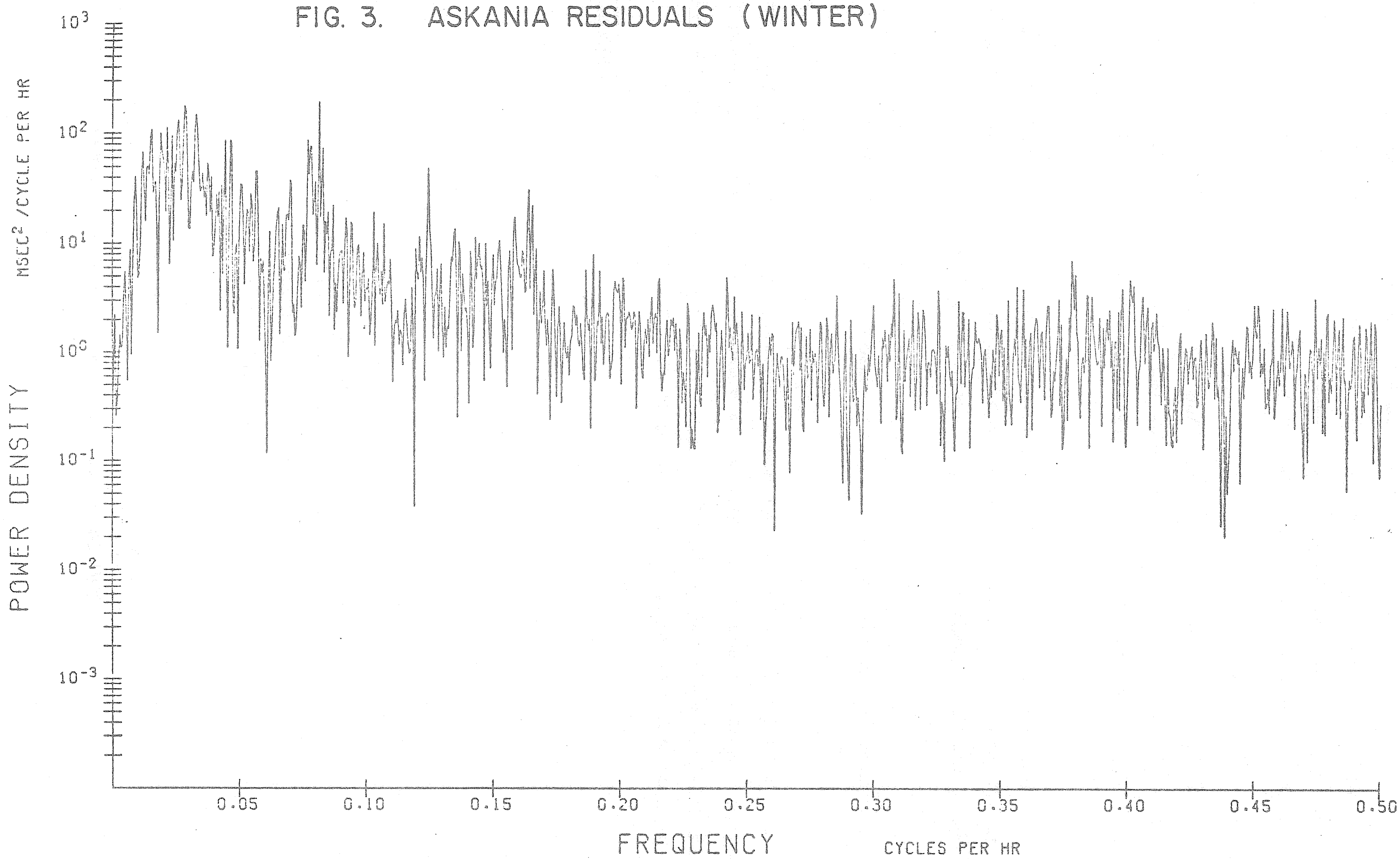
4609

FIG. 2. ASKANIA RESIDUALS (SUMMER)



4610

FIG. 3. ASKANIA RESIDUALS (WINTER)



STOCHASTIC MODELS OF THE EARTH TIDAL RECORDS

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I. INTRODUCTION

A method for the analysis of Earth tidal records is based on some conceptions about the properties of the noise and the drift of the records. We do not know well those properties hence our preference to one or another method is always somewhat subjective. Therefore there is an idea to test the methods through numerical models of the records.

Unfortunatly there is again a great risk for subjective conclusions if not a special strategy is choosen.

Let us have two more or less controvercial conceptions, say X and Y. Let A(i) and M(i) be the method and the model based on the conceptions i and let R(i/j) be the result from the application of A(i) on M(j) (i,j = X,Y).

It is easy to predict that R(X/X) will be better than R(Y/X) as well as that R(Y/Y) will be better than R(X/Y) . It is clear that from such models and results we do not get any objective informations about the advantages of any of the methods A (X) and A(Y).

In our opinion a model, M(Z) , may be useful for the comparison of A(X) and A(Y) if it is based on conceptions Z more pessimistic than both X and Y . This means that M(Z) should include all perturbations whose existance in the real records may be suspected.

In the following paragraphs we shall consider the simulation by means of a computer of some time series that may be used as components of different variants of M (Z).

The creation of such models is an application of the method of Monte Carlo. The aim of that method is the determination of the parameters of the distribution of a random variable using its simulated values.

In the present case the random variable is not the model but the estimation of a parameter and of its precision through the analysis of the model. If this point of view is accepted the problem of making decision on the basis of the analysis of models is not a trivial one. Here we shall not deal with that problem though it deserves a special attention.

II. SIMULATION OF STOCHASTIC VARIABLES AND PHENOMENA

We have a program SVEMOD for an IBM 370 computer which calculates a tidal curve and simulates series of stochastic components. The tidal component may be in any possible variant. Here we shall not consider it as the tide does not make any principal difficulty neither in the analysis nor in the modelling.

Each component is constructed individually, hour by hour, for a given interval of time and for a given epoch. If we wish to analyse afterwords a component separately it is written on a tape in the format of a real record. If a combination of some components is to be analysed their sum is written on a tape. It is also possible to make combinations of components previously written on a tape.

The simulation is carried out by determining the hourly ordinates of a component which we denote by $L_k(T)$, where $T = 1, 2, \ldots$ is the time in hours and k is an index of the component. In the determination of $L_k(T)$ we use some of the technics described in the following points.

II.1. Random numbers H .

A random number H is a sample value of a random variable with homogeneous distribution in the interval (0,1). In SVEMOD H is determined (generated) by a subroutine RANDAV given by IBM. Each calling of RANDAV gives on value H which is independent from all other values.

II.2. Normal numbers G.

A normal number $G = G(A,S)$ is a sample value of a variable with a normal distribution with a mean A and with a standard S. It is generated in a subroutine NORMAV through

$$G(A,S) = A + \sum_{i=1}^{36} (H_i - 1/2) \cdot S/\sqrt{3} \qquad (1)$$

on the basis of the Central Limit Theorem. Here H_i are 36 random numbers.

Each calling of NORMAV generates one value G which is independent from all other values.

When G is related to a given moment of time T we write G(T,A,S). The series G(T,0,S) (T = 1,2,...) is a realisation of a white noise of a record.

II.3. Stationary time series U.

The sequence U(T,S) (T = 1,2,...) is a sample of stationary time series with a normal distribution (0,S). In SVEMOD it is generated in the following manner.

We have a filter, F, with $N' = N + 1$ coefficients F_o, F_1, ... F_N. We calculate

$$U(T,S) = \sum_{t=0}^{N} F_t \, G(T+t,0,S) \,/\, R$$

$$= \sum_{t=0}^{N} f_t \, G(T+t,0,S) \quad , \qquad (2)$$

where

$$R^2 = \sum_{t=0}^{N} F_t^2 \qquad \text{and} \qquad f_t = F_t/R \qquad (3)$$

The autocorrelation function is

$$r(q) = \sum_{t=0}^{N-q} f_t \, f_{t+q} \qquad 0 \le q \le N$$

$$r(q) = 0 \qquad N' \le q \qquad (4)$$

The power spectrum as a function of the angular velocity is

$$P(w) = 2\,(\sum_{t=0}^{N} f_t \cos wt)^2 + 2\,(\sum_{t=0}^{N} f_t \sin wt)^2 \qquad (5)$$

II.4. Phenomena that occurs randomly with a given frequency.

We shall denote by $V(d)$ a random phenomena about which we know that it occurs at an average once per d hours, without a preference to any particular moment of time. We shall use an index $I(V,T,t)$ for which

$$I(V,T,t) = \begin{cases} 1 & \text{if } V \text{ has occured in the interval } (T-t,T) \\ 0 & \text{if } V \text{ has not occured in } (T-t,T) \end{cases} \qquad (6)$$

The probability for V to occur in an interval of t hours is given through the Poisson's distribution, i.e.

$$p(V,t) = 1 - e^{-x}, \quad x = t/d \qquad (7)$$

In order to simulate the occurence of V the random number H is transformed into a variable with Poisson's distribution in the following manner. We choose H. If $H > e^{-x}$, then $I(V,T,t) = 1$ and if $H < e^{-x}$, then $I(V,T,t) = 0$.

The technics described here may be found in the literature on the theory of probability,the time series and the method of Monte Carlo.

III. SOME STOCHASTIC COMPONENTS OF THE MODELS

Each of the components described in the following points is an option of SVEMOD. Their parameters are input data of SVEMOD and they may be varyed arbitrarily. The given numerical values are only an exemple. Their units are to be understood as 0.1 mkgls or 0.01 msec. The physical meening of the components as well as their combination in one modes may be subject to a discution.

III.1. White normal noise.

We set up simply

$$L_1(T) = G(T,0,S) \qquad (8)$$

From our point of view this case is of the least possible interested. The conclusion that may be drawn from the analysis of L_1 are perfectly predictable.

Parameters : $s = 2$

III.2. Stationary noise.

We set up

$$L_2(T) = U(T,S) \qquad (9)$$

This is a very important component which is probably very near to a realy existing component of the noise. Unfortunately we have a very poor knowledge about it and it seems to be very difficult to make a suitable choice for the generating filter F.

Here we suggest to relate L_1 to the meteorological perturbations whose energy is distributed in the spectrum on the diurnal and semidiurnal frequencies. For that purpose we may use

$$F_t = \cos w_o(t - N/2), \; w_o = 15°/\text{hour and } w_o = 30°/\text{hour} \qquad (1\ 0)$$

and $w_o N' = (2k+1)\,\pi/2$, $k = 1,2, \ldots$. For such a filter we have

$$R^2 = N'/2 + \frac{\sin N' w_o}{2 \sin w_o} ,$$

$$r(q) = \left(1 - \frac{q}{2 R^2}\right) \cos w_o q \quad \text{and.}$$

$$P(w) = \frac{4 S^2}{R^2} \left[\frac{\sin N'(w-w_o)/2}{2 \sin (w-w_o)/2} + \frac{\sin N'(w+w_o)/2}{2 \sin (w-w_o)/2} \right] \qquad (10)$$

$$P(w_o) = 2 S^2$$

In the spectrum P the maximum is at $w = w_o$. The band $w_o \pm \pi/N'$ is also amplified. In this way L_2 will simulate a meteorological noise.

A problem is the choise of N . Greater N means a narrower amplified band arround w_o i.e. more stable meteorological waves. If on the contrary N is smaller, the energy of the noise will be dissipated upon a larger band and thus we shall have less stable meteorological waves.

Parameters :

$w_o = 15°$, N' = 60, S = 5

$w_o = 30°$, N' = 60, S = 3

III.3. White noise with a diurnal variation of its level.

If we relate the white noise of a record to the microseismic noise of urbanistic origin we must accept that its standard is changing with the time, i.e. we have, instead of (8),

$$L_3(T) = G(T,0,S), \text{ where } S = S(t) , \qquad (11)$$

t being the time measured separately each day with origin $t = 0$ at midnight.

We shall accept that S(0) is the lowest value of S and that S(12) is the highest value of S . In SVEMOD we have the following two models for the variation of S :

$$S(t) = \begin{cases} S(0) , & 0 \leq t < 6 \quad \text{and} \quad 18 \leq t \leq 23 \\ S(12), & 6 \leq t < 18 \end{cases} \qquad (12)$$

and

$$S(t) = \frac{S(0) + S(12)}{2} + \frac{S(0) - S(12)}{2} \cdot \cos 15°t \qquad (24)$$

parameters : S(0) = 3, S(12) = 6

III.4. White noise with storms.

In each tidal record there are some intervals with wors quality - the recorded line becomes thicker, there are some visible oscillations with frequency higher than 1 cph. In analogy with the microseismic storms we may say that the white noise in our records has some storms.

It is possible roughly to estimate the mean number of the occurence of these storms without giving a prediction when they occur. That is why we may simulate the occurence of a storm as a phenomena V(d) discussed in II.4. Thus for each hour T = 1,2 .. we have to test whether V(d) has occured or not i.e. to determine whether I = I(V,T,1) = 1 or 0. Then we determine the model of the noise as

$$L_4(T) = G(T,0,S), \quad S = S(I) = \begin{cases} S_s & \text{if } I = 1 \\ S_q & \text{if } I = 0 \end{cases} \tag{25}$$

where S_s is the standard of the noise during the storm and S_q is the standard of the quiet noise.

There is a difference with the case discussed in II.4. If the storm has occured at a moment T it will last sometime, say t. We suppose that t has a known mean value and a known standard Δt and we determine it as

$$t = G(t_o, \Delta t) \tag{26}$$

Then for the interval (T,T+t) we accept $S = S_s$ without testing the occurence of a new storm.

Parameters :

$x = 1/d$, $d = 148$ hours = 7 days, $t_o = 72$ hours = 3 days,
$\Delta t = 24$ hours, $S_q = 2$, $S_s = 6$.

III.5. Variation of the sensibility (the calibration constant) - stationary noise.

It is very interesting to study what is the influence of the variation of the sensibility to the results of the analysis. It seems that this may be done only by the help of models.

There are at least two possible cases : (i) real natural variation of the sensibility of the measuring device, not taken into account, and (ii) a variation from one calibration to another due to the errors of the calibration. Here we shall give a model for the case (i). In the following point we shall discuss the case (ii).

Let $L_o(T)$ be the model of the tide. We set up

$$L_5(T) = L_o(T)\ (1 + U(T,S)) \tag{15}$$

i.e. the variation of the sensibility is a stationary noise. At the present we have no information about that noise and it is not clear how to choose the generating filter F . For the moment we suggest parameters :

$F_t = 1$, $t = 0,1, \ldots N$,

$N' = N+1 = 24$, $S = 0.005$

III.6. Errors in the calibration.

The calibrations are carried out at an average each t_o hours for instans each 240 hours = 10 days. This interval is not strictly observed and we may have a variation with a standard Δt hours.

Let T_i , $i = 1,2, \ldots$ are the epochs of the calibrations. In the model discussed here they are determined as

$$T_1 = 1 , \quad T_{i+1} - T_i = G(t_o, \Delta t) \tag{16}$$

Let the mean (true) value of the calibration constant be C and let the standard of the error of one calibration be S_c. For the calibration factor at the epoch T_i we choose the value

$$C_i = G(T_i, C, S_c) \tag{17}$$

The calibration factor for the epoch T which is between T_i and T_{i+1} is linearly interpolated

$$C(T) = C_i + \frac{C_{i+1} - C_i}{T_{i+1} - T_i} (T-T_i) \quad . \tag{18}$$

The model is

$$L_6 (T) = L_o(T) \cdot C(T) \tag{19}$$

Parameters :

$t_o = 240$ hours, $\Delta t = 72$ hours

$C = 1$, $S_c = 0.005$.

III.7. Instrumental drift as a polynomial with random interruptions of its derivatives.

This model of the drift is

$$L_7(T_i+t) = \sum_{k=0}^{r} A(T_i,k)\, t^k \quad , i = 1,2,\ldots \tag{20}$$

where $A(T_i,k)$ are constant coefficients in the interval (T_i, T_{i+1}). The moments T_i are choosen randomly as points in which one of the derivatives of L_7 has a random interruption. This is made in the following manner.

The coefficients $A(T_i,k)$ are considered as normal numbers with a mean $a(k)$ and a standard S_k. For the first hour of the model, $T_1 = 1$, we set up

$$A(T_1,k) = G(T_1, a(k), S_k) \quad , \quad k = 0,1, \dots r \qquad (21)$$

We suppose that the derivatives of $L_7 - L_7^{(k)}(T)$, including the case $k = 0$, i.e. the drift itself, are subject to occasional interruptions. The interruption of the k-th derivative is considered as a phenomena $V_k(d_k)$ (see II.4) which occurs at an average once per d_k hours. For each hour T we determine the index $I(V_k,T,1)$ and the value of the drift after (20) if $T = T_i + t$. If $I(V_k,T,1) = 0$ for all values of k we go to the next hour.

If for $t = t_1$ $I(V_k,T,1) = 1$ for $k = m$, we accept that the derivatives from the m-th to the r-th are interrupted. To realise this phenomena we determine the new coefficients

$$T_{i+1} = T_i + t_1$$

$$A(T_{i+1},k) = G(T_{i+1},a(k),S_k) \quad , \quad k = m, m+1, \dots r \qquad (22)$$

If $m > 0$ the coefficients $A(T,k)$ at $T = T_{i+1}$ are changed in such a manner that all derivatives of order lower than m , including the drift itself, remain continuous. When $m = 0$ the drift is also interrupted, all coefficients are changed occasionally after (21) or (22) and the new value of the drift at $T = T+1$ is

$$L_7(T_{i+1}) = A(T_{i+1},0) \qquad (23)$$

About this model we have in mind that the interruptions of the derivatives are not corrected in the analysis if $m > 0$. When $m = 0$ we have a simulation of a displacement which may be corrected with an error. If the displacement is determined as

$$L_7(T_i + t_1) - a(0) \qquad (24)$$

where the first term is the value of the drift at $T = T_i+1$ determined before the interruption and $a(0)$ is the mean value of the drift after the interruption then the standard of the error will be S_0.

Parameters : $r = 3$

k	$a(k)$	S_k	d_k (in days)
0	6000	20	15
1	1000	500	15
2	1000	500	15
3	1000	500	30

The magnitudes of S_k are expressed as if the unit of time was 15 days. Thus for instance the power $k = 2$ will change the drift in 15 days at an average of 500.

III.8. Meteorological tides.

It is clear, theoretically and practically, that in the real records there are a meteorological tide S1 and, may be less important, a meteorological tide S2. We may suppose that in these tides there are occasional changes in the amplitudes and the phases. For that reason we have the model

$$L_8(T) = \sum_{i=1,2} (x_i \cos w_i T + Y_i \sin w_i T) , \qquad (25)$$

$$w_1 = 15° \text{ and } w_2 = 30° ,$$

where X_i and Y_i are subject to occasional changes. We have the following two variants for X_i and Y_i.

III.8.1. Changes each day.

Let the means of X_i and Y_i are A_i and B_i and the standards be a_i and b_i. Then at 0 hours each day we determine

$$X_i = G(A_i,a_i) \text{ and } Y_i = G(B_i,b_i) \qquad (26)$$

Parameters :

$A_1 = 10$, $A_2 = 5$

$B_1 = B_2 = 0$ and $a_1 = \dots = b_2 = 2$

III.8.2. Occasional changes

We accept that X_i and Y_i change at an average once per d hours.

TIDAL ANALYSIS METHODS AND OPTIMAL LINEAR SYSTEM APPROXIMATION

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(Paper presented at the 1. Meeting of "Working Group on Data Processing in Tidal Research", Bonn, March 1978)

SUMMARY

In order to unify the theoretical basis of the tidal analysis methods the "Optimal Linear System Approximation" is used revealing the basic differences of the methods particularly those concerning the mathematical and geophysical assumptions about the response function of the Earth. Further the extensions of the basic modelling to non-linear, multichannel and time variant systems are discussed.

INTRODUCTION

Observation and analysis of signals arising as a result of tidal forces (e.g. in Oceans, solid Earth, Atmosphere etc.) is a very old and established research field. Ingenious analysis methods from pre-computer times are now very much improved and also the variety of the methods increased. Although most of these methods are very similar, the theoretical connections between them are rarely obvious, so that their performance is usually checked using test signals. Especially the physical assumptions and differences of approach for each individual method are often hidden and not directly comparable to other methods. This is generally due to the lack of a mathematical and physical framework. In an attempt to define such a framework the "Optimal Linear System Approximation" serves a very useful purpose, revealing the common basic principles and also differences, associated assumptions and approximations made in each individual method. Further this approach allows a systematic stepwise consideration of the very complex character of the physical Earth system (e.g. non-linearities, possible existence of other inputs than the tidal force and time variance). Also the very important problem concerning the errors (or confidence limits) of the results (diverging very widely from method to method) can be clarified within the applied framework and can be approached with more emphasis on the involved physical mechanisms. A generalising of the several (main) analysis methods therefore will improve the understanding of the analysis methods and also the understanding of the predominantly tidal phenomena investigated.

LINEAR SYSTEM AND ITS OPTIMIZATION

Generally the aim of the earth tidal analysis is to find the response function of the earth in order to compare it with the theoretical geophysical models (such as rigid Earth, elastic Earth, layered Earth, finite element model etc.). One can confirm such models or investigate the deviations of the results from those models and also the involved physical mechanisms (e.g. ocean loading, effects due to geology, topography, tectonics, meteorology). The output of the real system is the particular observed data $\tilde{o}(t)$ (e.g. tilt, gravity, strain etc.). The observed data is mainly caused by the tidal forces $\tilde{x}(t)$ due to the moon and sun. Therefore by a single input analysis, the tidal force at the particular station, azimuth etc. is considered. Due to the complexity of the real Earth, the term system is to be seen mainly as a mathematical "black box" description of the behaviour of the Earth to certain forces. In that sense it gives the relation between certain inputs and outputs and involves no real physical parameters such as density, elastic parameters etc. The interpretation of the analysis results is then the subject of extensive and careful investigations considering all the factors such as instrument, recording, geology, tectonic influences, the analysis method used and therefore imposed physical assumptions. (For the sake of simplicity, we assume that the response of the instrument is known, so that its effect can be removed at any stage of the analysis).

For the simplest form of the analysis the real physical Earth system can be approximated by a linear, stable, time invariant, single input (with only tidal forces) and non causal (which can be assumed without loss of any information for periodic inputs) model system (Fig.1). It is described by the system function $s(\tau)$ in time domain or by the complex response function $S(f)$ in frequency domain. $S(f)$ and $s(\tau)$ are related by the Fourier transform. With this the relation of the input $\tilde{x}(t)$ to an output $\tilde{p}(t)$ for the linear system described above is given by the convolution integral in the time domain

$$\tilde{p}(t)=\int_{-\infty}^{\infty} s(\tau)\,\tilde{x}(t-\tau)\,d\tau \tag{1a}$$

The Fourier transform of (1a) leads to a simple multiplication in frequency domain with $\tilde{P}(f)$ and $\tilde{X}(f)$ being the complex spectrums of $\tilde{p}(t)$ and $\tilde{x}(t)$ via Fourier transform.

$$\tilde{P}(f)= S(f)\,\tilde{X}(f) \tag{1b}$$

We note that the amplitude $|S(f)|$ and phase $\phi_s(f)$ of the response function are denoted often by δ and $\varkappa$ in case of gravity and by γ and α in case of tilt.

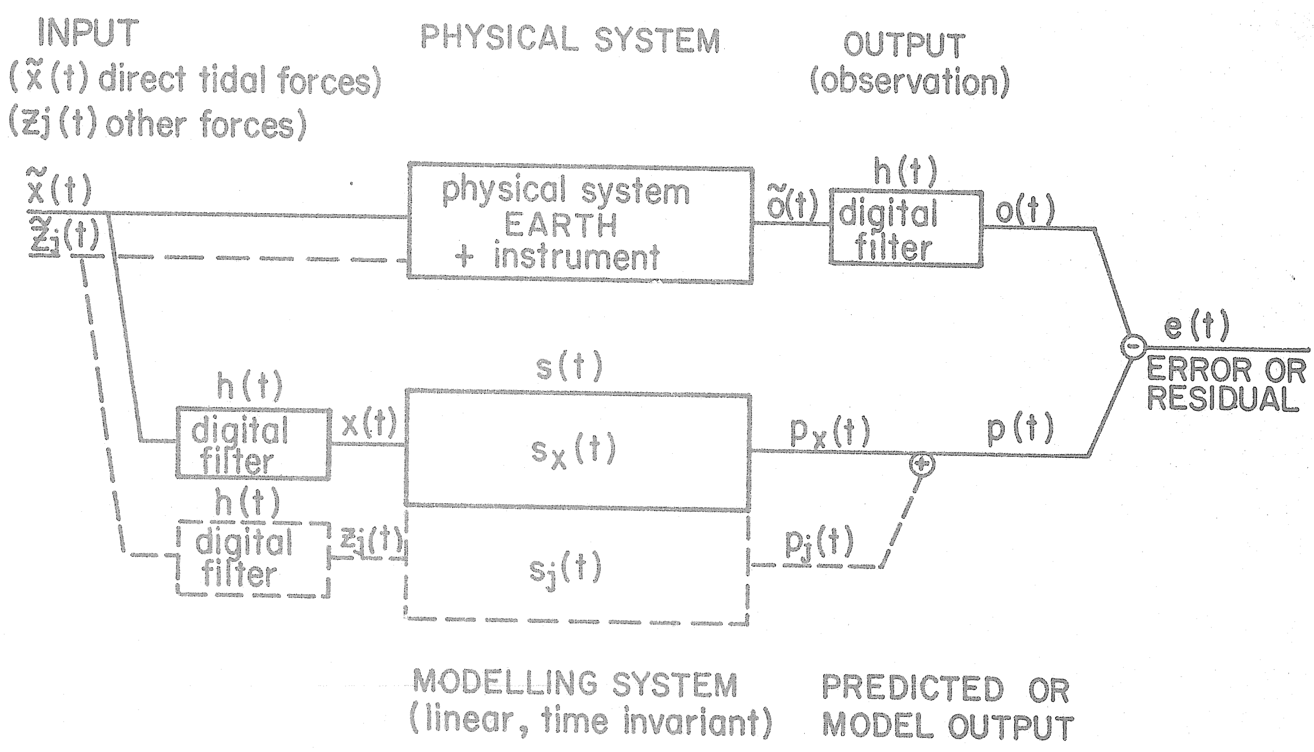

Fig 1: Principle of optimal linear system approximation with involved digital filtering and with an indicated multichannel input $z_j(t)$

The linear system has to approximate the Earth in an optimal sense, so that the residual or error $\tilde{e}(t)$ between the actual output $\tilde{o}(t)$ and the model output $\tilde{p}(t)$ is as small as possible. We have then for $\tilde{e}(t)$

$$\tilde{e}(t) = \tilde{o}(t) - \tilde{p}(t) \tag{2}$$

with the use of (1a) and (1b) we have

$$\tilde{e}(t) = \tilde{o}(t) - \int_{-\infty}^{\infty} s(\tau)\,\tilde{x}(t-\tau)\,d\tau \tag{3a}$$

$$\tilde{e}(t) = \tilde{o}(t) - \int_{-\infty}^{\infty} S(f)\,X(f)\,e^{i2\pi f t}\,df \tag{3b}$$

At this stage we consider the application of a digital filter $h(t)$ (or equivalently $H(f)$) to the observed data $\tilde{o}(t)$. From (2) it is clear that the same filter has to be applied to $\tilde{p}(t)$ as well, otherwise the comparison of the model output to the actual output is biased. Again from (2) the application of the filter automatically leads to filtered residuals. The application of the filter to $\tilde{p}(t)$ is the same as applying the filter to $\tilde{x}(t)$ since the filters are in most cases linear systems as well and the order of the applied linear systems on an input does not alter the final output. This is very convenient from a computational point of view and the filter is then applied in most cases to $\tilde{X}(f)$ in frequency domain. It is also apparent from the Fig.1, that for the described correct use of the filters the equations (1), (2) and (3) are still valid by replacing $\tilde{o}(t), \tilde{p}(t), \tilde{x}(t), \tilde{e}(t)$ by their filtered form $o(t), p(t), x(t), e(t)$. In the following, mainly the filtered form of the involved signals will be considered.

The optimization of the model system is made by minimizing the residual power (or mean square error). We use here a modified form (originally stated by Wiener 1949) of this least square criteria, in order to ensure the existence of the involved integral for any type of signal such as periodic aperiodic, random signals. We then have

$$\overline{e^2} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} e(t)^2\,dt = \text{Min!} \tag{4}$$

The derivation of the different analysis methods from above is given briefly in [9] and in more detail in [13]. Here we concentrate on two basic types of analysis namely the non harmonic and the harmonic approach and show some crucial differences concerning the involved assumptions.

NON HARMONIC APPROACH

The residual power in (4) can be written (for filtered signals) more explicitly by using (3a)

$$\overline{e^2} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left(o(t) - \int_{\infty}^{\infty} s(\tau)\, x(t-\tau)\, d\tau \right)^2 dt = \text{Min!} \tag{5}$$

$\overline{e^2}$ is then minimized with respect to the only unknown $s(\tau)$ using

$$\frac{\partial \overline{e^2}}{\partial s(\tau)} = 0 \quad \text{and} \quad \frac{\partial^2 \overline{e^2}}{\partial s(\tau)^2} > 0 \quad \text{for any } \tau \tag{6}$$

This yields

$$\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} 2 \cdot \left(o(t) - \int_{-\infty}^{\infty} s(\tau)\, x(t-\tau)\, d\tau \right) x(t-u)\, dt = 0 \tag{7}$$

Using the definitions of auto and cross correlation for time series [5] we form the following relations.

$$c_{ox}(\tau) = c_{xo}(-\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} o(t)\, x(t-u)\, dt \tag{8a}$$

$$c_{xx}(u-\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t-\tau)\, x(t-u)\, dt \tag{8b}$$

With this we have from (7) the well known Wiener-Hopf integral conditions for the optimized linear system [5].

$$c_{ox}(u) = \int_{-\infty}^{\infty} s(\tau)\, c_{xx}(u-\tau)\, d\tau \tag{9}$$

This condition integral can be written for finite and digital signals in matrix form and then solved for $s(\tau)$. The Fourier transform of $s(\tau)$ will then give $S(f)$. An important decision hereby is to be made about $s(\tau)$ concerning first its length (since $s(\tau)$ is an aperiodic function due to the stability property of the linear system) and then concerning the sampling rate $\Delta\tau$. (This is different from the sampling interval of the original data). Both decisions then define in terms of the response function $S(f)$ which band width it has to cover and which degree of minimal smoothness is imposed upon it [4][6][10]. Generally increasing the length of $s(\tau)$ leads to more allowed wiggles (oscillations) for $S(f)$ and increasing the sampling rate $\Delta\tau$ leads to a smaller band width over which $S(f)$ is defined. This non harmonic approach can be described as optimizing a continuous response function (with amplitude $|S(f)|$ and phase $\phi_s(f)$) over a defined spectral band width and with a defined minimum degree of smoothness (or maximum degree of wiggliness).

HARMONIC APPROACH

Because the tidal forces acting on the Earth are very well known from the astronomy and are almost perfectly periodic, it represents one of the exceptional cases in geophysics. The input has a very accurately given spectral form with well defined frequencies, amplitudes and phases [3][6]. In fact the time domain form of tidal input $x(t)$ can be found by synthesis of the given waves (e.g. N)

$$x(t)=\frac{1}{2}\sum_{n=-N}^{N} X(f_n)\, e^{i2\pi f_n t} \tag{10}$$

In this case one uses for $\overline{e^2}$ the expression from (3b) for filtered signals

$$\overline{e^2}=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\left(o(t)-\int_{-\infty}^{\infty} S(f)\,X(f)\,e^{i2\pi f t}\,df\right)^2 dt = \text{Min!} \tag{11}$$

Since $X(f)$ exists at some discrete tidal frequencies f_n, $S(f)$ also can be determined only at those frequencies. The integral over df in (11) is then to be replaced by a sum over n. The residual power $\overline{e^2}$ is then minimized with respect to the only unknown $S(f_n)$ using

$$\frac{\partial \overline{e^2}}{\partial S(f_n)}=0 \quad \text{and} \quad \frac{\partial^2 \overline{e^2}}{\partial S(f_n)^2}>0 \quad \text{for any } n \tag{12}$$

This yeilds

$$\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} 2\left(o(t)-\frac{1}{2}\sum_{n=-N}^{N} S(f_n)X(f_n)\,e^{i2\pi f_n t}\right)\left(\frac{1}{2}X(f_m)\,e^{i2\pi f_m t}\right)dt = 0 \qquad (13)$$

$$m=-N,\ldots,N$$

This equation system as a condition for an optimal response function $S(f_n)$ is in complex form so that its real and imaginary parts separately have to be zero. With the use of amplitude $|X(f_n)|$ and phase $\phi_x(f_n)$ of $X(f_n)$ and with the real part $a_s(f_n)$ and imaginary part $b_s(f_n)$ of $S(f_n)$ we get the following equation system (called normal equations)

$$\begin{aligned}\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \Big(& o(t)\,|X(f_m)| \,{\textstyle{\cos \atop \sin}}\,(2\pi f_m t+\phi_x(f_m)) \\ & -\sum_{n=1}^{N} a_s(f_n)|X(f_n)|\cos(2\pi f_n t+\phi_x(f_n))\,|X(f_m)|\,{\textstyle{\cos \atop \sin}}\,(2\pi f_m t+\phi_x(f_m)) \\ & + b_s(f_n)|X(f_n)|\sin(2\pi f_n t+\phi_x(f_n))\,|X(f_m)|\,{\textstyle{\cos \atop \sin}}\,(2\pi f_m t+\phi_x(f_m)) \Big)\,dt = 0 \end{aligned} \qquad (14)$$

$$m=1,2,\ldots,N$$

In practice (14) can be written in matrix form for finite and digital data and solved for $a_s(f_n)$ and $b_s(f_n)$ which are then combined to give the amplitude and phase of the response function. The solution of (14) runs into difficulties if the used tidal waves are very close in frequency and cannot be resolved with the existing length of data $o(t)$. In this case one is forced to use wave groups (such that the groups are separable with the given data length) formed from N waves into H groups each having G_h waves. For those groups the response is then usually assumed to be constant [3] , although other types of assumptions can be used as well, as is done in a slightly different variant of the harmonic method TIFA [7] ,[11] . The harmonic approach can be used in many variations concerning the number of waves and wave groups and also as for the non harmonic methods many different digital filters and sampling techniques can be used.

COMPARISONS

It should be noted at the beginning that the main conditions for the harmonic and non harmonic method namely the Wiener-Hopf integral (9) and the normal equations (14) are directly derivable from each other. This means that the minimization of $\overline{e^2}$ either in respect to $s(\tau)$ or in respect to $S(f_n)$ is not the major difference of the different approaches. Rather the use of the system describing function in time or frequency domain ($s(\tau)$ or $S(f_n)$) as the unknowns is important, since this influences the major assumptions being made.

Geophysical considerations and experience indicate a rather smooth and continuous response function for the Earth. This is taken fully into account within the formalisms of the non harmonic approach. Further, within this approach the question about the resolution of very close waves does not arise explicity. Because this is adjusted internally by using the system function in the time domain, not the individual frequencies but the whole is considered by allowing any shape for the response function over even very close waves. The response for these waves in harmonic methods would be derived with the rather unrealistic assumption of the constant response over the wave group. The result is then a point wise step function (over the wave groups at tidal frequencies) with undefined response function between. Fig.2 demonstrates this major difference. The observed data is 91 days long and comes from the LaCoste-Romberg tidal gravitymeter at Bidston. The Chojnicki method [3] was used as a harmonic approach and the Response method [6] was used for the non harmonic approach. Details about data and further results are given in [13] . Here we look closely into the involved assumptions about the response function and its demonstration. The step wise response from the harmonic method could of course be smoothed and extended between the waves by suitable weighting after the analysis but this is not as reliable as using the more relevant assumption at the very beginning of the analysis. One further advantage of the non harmonic approach is the use of the tidal input in the time domain. This can be derived from the position of the moon and sun and includes all the tidal waves present. The harmonic methods however are restricted to the use of only a finite number of waves.

One remarkable feature of the Fig.2 is the error limits. The harmonic methods assuming a constant response over a wave group, also quote one error limit for this group. This response error is mainly influenced by the response error for the frequency of the major wave. It is hard to accept that the response for the frequencies of the smaller waves have the same accuracy as the response at the frequency of the major wave. The non harmonic method allows, in the same manner as for the response function, the evaluation of a smooth and continuous error envelope [4] .

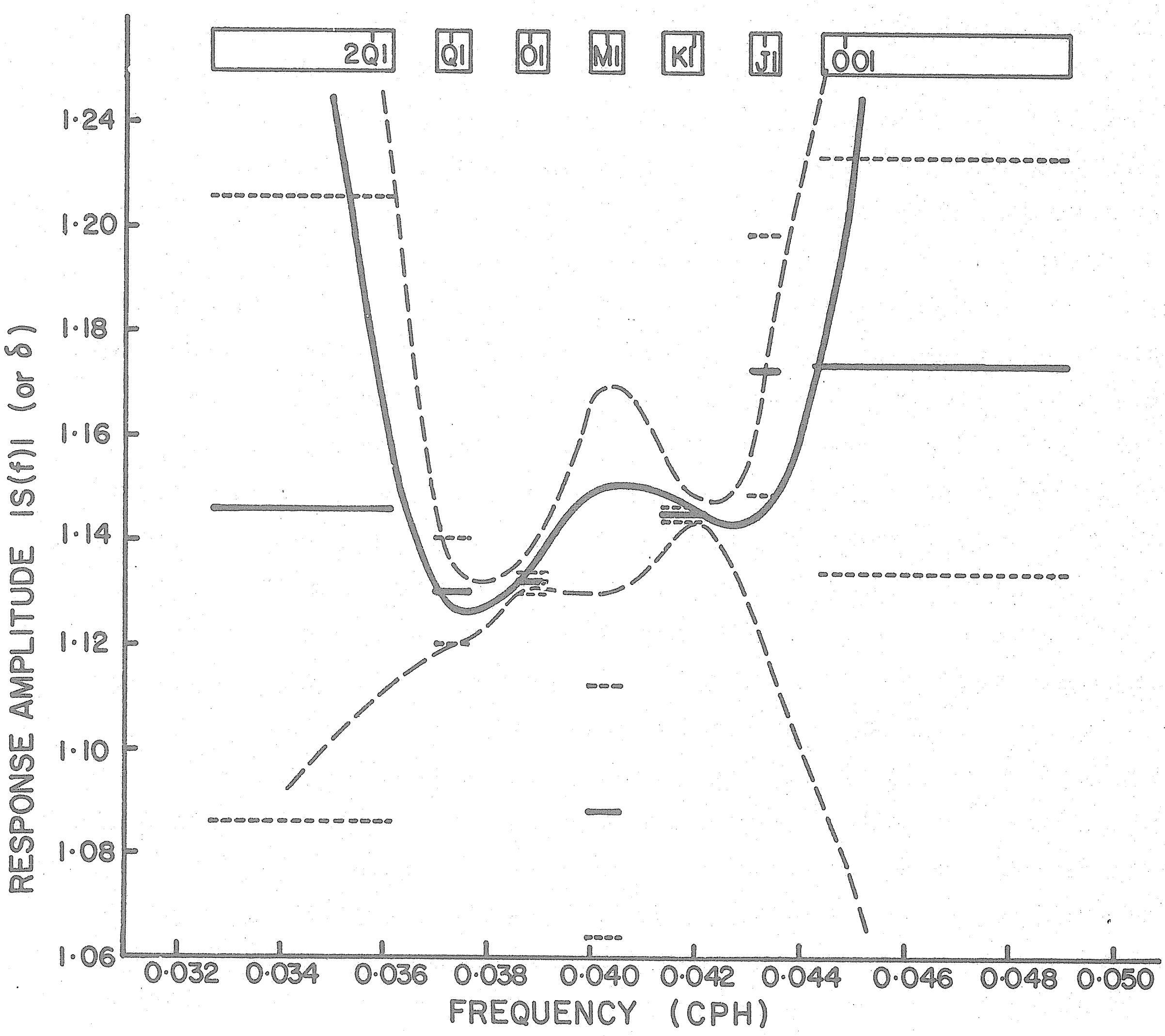

Fig. 2: Amplitude of the response function |S(f)| (or δ) in the diurnal band from a harmonic (Chojnicki method) and a non harmonic (Response method) approach.
Error bars given by dashed lines.
On top of the figure the width of the used wave groups and the major wave within a group for the harmonic method is given.
Data is from LaCoste tidal gravitymeter 15 at the station Bidston for 91 days starting at 11 March 1974, 0h.

The harmonic type of methods also have their advantages. If the response function is to be expected to have a resonance (such as for the core effect in the K1 wave group) this can be looked at (with long enough data) free from any prejudice about the smoothness of the response function. Further if any other input than tidal is present with the same or similar frequencies as some tidal waves and the sub response to this input is not properly modelled, this will bias the response to an anomalous level [2] . The non harmonic approach would strongly smooth out such anomalous parts of the response function. (For the response method this is valid in only a restricted sense since the use of the bilinear admittance and radiational input are used also for such effects [6][10]).

As shown, the basic background of the two fundamentally different approaches are the same so that a convergence of the results is expected for the longer signals. However this will not happen automatically unless the assumptions about the response function converge as well. However at the major waves the results agree quite well, although even this is below the desired level for some investigations such as the ocean tide inverse problem [1] .

EXTENSION OF THE MODEL AND SOME ASSOCIATED PROBLEMS

The possibility of other existing inputs and their effect on the response function may be crucial to the analysis, if these are not treated properly [2] . For an input with frequencies totally different to the frequencies in the tidal input, the situation is rather easy to handle. This input is either included into the analysis or the corresponding frequency band of the observed signal is separated with digital filters and investigated separately. In most cases, however, the additional input may have totally or partly the same frequencies as the tidal input (such as ocean loading or shallow water loading or even other inputs). In such cases where inputs are correlated among themselves, one cannot separate their individual effect on the output without introducing further assumptions or previous knowledge. In the most general form the multi input case can be introduced into the analysis with following extension of the model

$$P(f) = S_x(f) + \sum_{j=1}^{J} S_j(f)\, Z_j(f) \qquad (15)$$

or

$$p(t) = \int_{-\infty}^{\infty} s_x(\tau)\, x(t-\tau)\, d\tau + \sum_{j=1}^{J} \int_{-\infty}^{\infty} s_j(\tau)\, z_j(t-\tau)\, d\tau \qquad (16)$$

with $s_j(\tau)$ the sub system function and $z_j(t)$ the input of the j-th channel and $S_j(f)$ and $Z_j(f)$ their Fourier transforms respective by Fig.1 . One approach as used in [3] is the determination of a regression factor between the observed data and secondary input (such as air pressure with energy in some tidal and non tidal frequencies). It is equivalent to assuming a constant response function for the sub system over the entire frequency range available (0 - 0.5 cph for a data sampling rate of $\Delta t=1$h) and is therefore only a poor approximation of the real situation. Moreover this regression factor is more valid at lower frequencies, due to energy concentration there, for signals like air pressure and temperature. It also means a zero phase response at every frequency. This can be easily verified with (15) and (16), since a regression factor means a sub systems function $s_j(\tau)$ existing only at $\tau=0$.
Before introducing such inadequate assumptions, it seems to be sensible to make a straight forward analysis with only the tidal input (and also including eventually other non correlated inputs) and then to examine the response function and the residuals. The anomalies in the response function will indicate the existence of correlated inputs and the residuals will indicate other non correlated inputs. An example for this approach can be given from ocean tidal loading problems. In this case the output wave $P(f_n)$ is first determined from a single tidal input analysis. Then (15) is used by introducing body tide response $S_x(f_n)$ from a model (e.g. for the gravity the amplitude 1.16 and the phase 0^o) and $X(f_n)$ from the tidal input. Further one introduces for the other channel the input $Z_1(f_n)$ from ocean tidal maps and the sub response $S_1(f_n)$ from modelled Green functions. The unexplained part of $P(f_n)$ after the consideration of two input channels and sub systems is then investigated in order to correct the assumed parameters

of the two channels or eventually to reveal the existence of another additional channel. The inclusion of multichannels directly in to the analysis via (15) or (16) and the least square principle (2) and (4) leads to a formalism involving the cross correlations between the inputs themselves.

Extension of the model to non linear systems (due to the non linearities in the Earth or instrument) is theoretically a separate problem from multi inputs. Given the major input frequencies one can calculate by several models of non linearity the frequencies where certain extra energy might be expected. However those frequencies might be exactly on the frequencies of tidal or other inputs so one is back to a similar situation as in the multi input case. In this sense the problem of non linear systems and multi input systems converge in practice to the question of whether there are several causes at one frequency independent of their possible different origin. However, the actual possible origins will help the choice of assumptions made in order to solve the problem.

One further problem arises due to the non-linear waves (or similarly due to the other input channels) where only the involved frequencies are known but there is no explicit input at this frequency. Therefore the linear response function does not exist at such frequencies. Such waves can be picked up by the investigation of the residuals. But for a proper analysis these waves have to be considered in order to avoid leakage from these waves into other waves used in the analysis (note also that exactly for this reason, the tidal waves are introduced into the analysis together and not step wise after each other [13]). In this case with only information existing about the frequency one has to introduce as unknowns into the analysis $a_p(f)$ and $b_p(f)$, the real and imaginary part of the model output (leading in the end to the amplitude $|P(f)|$ and phase $\phi_p(f)$. This has been done in the TIFA method and some other classical tidal analysis methods. In fact those methods use directly $P(f)$ for the analysis instead of $S(f)\cdot X'(f)$ [11].

Finally, the time variance of the response function is presently investigated by applying the time invariant analysis to the sequences of the data with a suitable sequence length and a shifting interval [8] . The results however need very careful examination and interpretation. Often the real variation of the response function with the time is not detectable and the variation is instead caused by the wrong assumptions about the response function, neglected non linea-ities and inputs in the basic time invariant analysis.

FINAL REMARKS

It is very briefly demonstrated that the establishing of the common background of analysis methods using "Optimal Linear System Approximation" helps considerably to identify and to handle some major problems arising in the analysis of Earth tidal data. Further it gives the theoretical framework for the extension of the existing models, which might lead to further valuable geophysical information. This paper presents only a part of the work on time series and system analysis in progress which is soon to be presented in a complete form [13] .

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About the Sensitivity of the Venedikov Tidal Parameter Estimates to Leakage Effects

by

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Summary: It is shown that the estimates of the tidal parameters derived from the VENEDIKOV-Method (VENEDIKOV 1966a,b, 1968, 1977) and related variants (DUCARME 1975, WENZEL 1976) are generally more affected by leakage effects than other tidal analysis methods in practise (CHOJNICKI 1973, WENZEL 1976, SCHÜLLER 1975,1976, 1977a,b). This is due to inadequate sampling after filtering in combination with the non-ideal bandpass characteristics for the tidal domains. As it will be proved theoretically and numerically the VENEDIKOV estimates do not represent the energies of the individual tidal bands alone, but a conglomeration of energies over the whole passbands of the filters. Hence, the interpretation of tidal parameters derived from the VENEDIKOV-Method should take the leakage mechanism into account when looking for small effects like liquid core resonance,perturbation by ocean loading etc.

I. Tidal Parameter Estimation by the VENEDIKOV-Method.

Given a tidal record y(t) which is hourly sampled and (for simplicity) assumed without gaps. Furthermore, y(t) is a band-limited signal due to preceding smoothing, so that with

$$|Y(\omega)|^2 = 0 \text{ for } |\omega| \geq \omega_{\bar{\nu}} = \pi[\text{rad/h}] \qquad (1.0)$$

aliasing with respect to the hourly sampling rate is avoided. Defining a set of even and odd filters, each of length L = 21, i.e.

$$c_i(t) = c_i(-t) \quad \text{(even)} \qquad (1.1a)$$

$$s_i(t) = -s_i(-t) \quad \text{(odd)} \qquad (1.1b)$$

(i = 1 — diurnal(D)), (i = 2 — semidiurnal (SD)), (i = 3 — terdiurnal (TD)), and applying them to y(t), we find:

$$e_i(t) = \sum_{\tau=-l}^{l} c_i(\tau)y(t-\tau) \tag{1.2a}$$

and

$$o_i(t) = \sum_{\tau=-l}^{l} s_i(\tau)y(t-\tau) \tag{1.2b}$$

Denoting

$$C_i(\omega) = \sum_{t=-l}^{l} c_i(t)\cos\omega t \tag{1.3a}$$

and

$$S_i(\omega) = -\sum_{t=-l}^{l} s_i(t)\sin\omega t \tag{1.3b}$$

as the Fourier transforms of the time domain filters, the frequency domain representation of (1.2a,b) will be:

$$E_i(\omega) = \frac{1}{2}A_y(\omega)C_i(\omega)e^{i\varphi_y(\omega)} \tag{1.4a}$$

and

$$O_i(\omega) = \frac{1}{2}A_y(\omega)S_i(\omega)e^{i\left(\varphi_y(\omega)+\frac{\pi}{2}\right)} \tag{1.5a}$$

According to VENEDIKOV 1966b, 1977 the filtered observations (1.2a,b) are sampled with a rate equal to the filter length L in order to keep them algebraically uncorrelated with respect to (1.2a,b). Assuming an odd number n = 2m+1 of filtered ordinates and relating the time origin to the central time point, the following sets of error equations may be derived for each tidal domain (D,SD,TD):

$$\begin{aligned} e_i(T) + v_{e_i}(T) &= \sum_{j=1}^{u_i} \xi_{ij} \sum_{k=1}^{m_{ij}} C_i(\omega^*_{ijk})A^*(\omega^*_{ijk})\cos(\omega^*_{ijk}T+\varphi^*(\omega^*_{ijk})) \\ &\quad - \sum_{j=1}^{u_i} \eta_{ij} \sum_{k=1}^{m_{ij}} C_i(\omega^*_{ijk})A^*(\omega^*_{ijk})\sin(\omega^*_{ijk}T+\varphi^*(\omega^*_{ijk})) \\ &= \sum_{j=1}^{u_i} \xi_{ij}\alpha_{ij}(T) - \sum_{j=1}^{u_i} \eta_{ij}\beta_{ij}(T) \end{aligned} \tag{1.7 a}$$

and

$$o_i(T) + v_{o_i}(T) = -\sum_{j=1}^{u_i} \xi_{ij} \sum_{k=1}^{m_{ij}} S_i(\omega^*_{ijk})A^*(\omega^*_{ijk})\sin(\omega^*_{ijk}T+\varphi^*(\omega^*_{ijk}))$$

$$-\sum_{j=1}^{u_i} \eta_{ij} \sum_{k=1}^{m_{ij}} S(\omega^*_{ijk})A^*(\omega^*_{ijk})\cos(\omega^*_{ijk}T+\varphi^*(\omega^*_{ijk}))$$

$$= -\sum_{j=1}^{u_i} \xi_{ij}\bar{\beta}_{ij}(T) - \sum_{j=1}^{u_i} \eta_{ij}\bar{\alpha}_{ij}(T) \qquad (1.7b)$$

In (1.7a,b) ξ_{ij}, η_{ij} are the auxilary tidal unknowns, A^*, φ^*, ω^* are the harmonic elements of the CARTWRIGHT-EDDEN development, u_i is the number of wave groups, m_{ij} the number of harmonic constituents in the different wave groups, $T = 0, \pm 2L, \ldots, mL$ is the discrete time parameter.

Following the least squares rules, the tidal parameter estimates for each domain D, SD, TD will be derived from the associated systems of normal equations

$$\underline{N}_i \underline{x}_i = \underline{w}_i \qquad (1.8)$$

with

$$\underline{N}_i = \underline{N}_{e_i} + \underline{N}_{o_i} \quad \text{and} \quad \underline{w}_i = \underline{w}_{e_i} + \underline{w}_{o_i} \qquad (1.9)$$

Finally the solution for the auxilary unknowns will become:

$$\underline{x}_i = \underline{N}_i^{-1} \underline{w}_i \qquad (1.10)$$

leading to the tidal parameter estimates. The subsequent error calculation will not concern us here.

II. The Leakage Mechanism in the VENEDIKOV-Method.

Denoting the pure tidal signal by $y^*(t)$ and an arbitrary perturbation process by $z(t)$, the tidal observations may be written as:

$$y(t) = y^*(t) + z(t) \qquad (2.1)$$

(Condition (1.0) is assumed to be valid).

Then (1.2a,b) appear as:

$$e_i(t) = e_i^*(t) + z_{e_i}(t)$$

and $$o_i(t) = o_i^*(t) + z_{o_i}(t) \tag{2.2}$$

Setting up error equations of type (1.7a,b) and applying the least squares rule, the solutions for the auxilary unknowns result in:

$$\underline{x}_i = \underline{x}_i^* + \underline{\Delta x}_i = \underline{N}_i^{-1}\underline{w}_i^* + \underline{N}_i^{-1}\underline{\Delta w}_i, \tag{2.3}$$

where the perturbations $\underline{\Delta x}_i$ are caused by the $\underline{\Delta w}_i$ of the absolute vector. The perturbations $\underline{\Delta w}_i$ of the absolute vector are governed by the $z_{e_i}(t)$ and $z_{o_i}(t)$. Hence, $\underline{\Delta w}_i$ contains two different kinds of elements, that is:

$$\Delta w_{ij}^{\xi} = \sum_{T=-mL}^{mL} \{\alpha_{ij}(T) z_{e_i}(T) - \bar{\beta}_{ij}(T) z_{o_i}(T)\}$$

and $$\Delta w_{ij}^{\eta} = -\sum_{T=-mL}^{mL} \{\beta_{ij}(T) z_{e_i}(T) + \bar{\alpha}_{ij}(T) z_{o_i}(T)\} \tag{2.4}$$

As (2.4) in its time domain representation is hard to interpret with respect to signal and noise interactions, the frequency domain is chosen for understanding the influence of the perturbation process on the unknowns (SCHÜLLER 1976,1977a,b). For this purpose (2.4) may equivalently rewritten as:

$$\Delta w_{ij}^{\xi} = \sum_{t=-\infty}^{\infty} \{\alpha_{ij}(t) z_{e_i}(t) - \bar{\beta}_{ij}(t) z_{o_i}(t)\} d(t)$$

and $$\Delta w_{ij}^{\eta} = -\sum_{t=-\infty}^{\infty} \{\beta_{ij}(t) z_{e_i}(t) + \bar{\alpha}_{ij}(t) z_{o_i}(t)\} d(t) \tag{2.5}$$

The time function d(t) here is easily found to be:

$$d(t) = \begin{cases} \sum_{k=-m}^{m} \delta(t-kL) & \text{— for } |k| \leq m \\ 0 & \text{——— for } |k| > m \end{cases} \tag{2.6}$$

(δ — Kronecker symbol)

which is called rectangular window function in the time domain. Its normalized Fourier transform is then the 'spectral rectangular window', analytically expressed by (JENKINS & WATTS 1968) (Fig.1):

$$D(\omega) = \frac{\sin(\frac{n}{2}\omega L)}{n\sin(\frac{\omega L}{2})} \tag{2.7}$$

Applying now the convolution theorem of time series analysis to (2.5), we find (SCHÜLLER 1976,1977a,b):

$$\Delta w_{ij}^{\xi} = \frac{n}{2}\sum_{k=1}^{m_{ij}}\int_{\bar\omega=o}^{\omega_\nu} A^*(\omega^*_{ijk})A_z(\bar\omega)[C_i(\omega^*_{ijk})C_i(\bar\omega)+S_i(\omega^*_{ijk})S_i(\bar\omega)]\cdot$$
$$\cdot D(\bar\omega-\omega^*_{ijk})\cos(\varphi^*(\omega^*_{ijk})-\varphi_z(\bar\omega))d\bar\omega +$$
$$+\frac{n}{2}\sum_{k=1}^{m_{ij}}\int_{\bar\omega=o}^{\omega_\nu} A^*(\omega^*_{ijk})A_z(\bar\omega)[C_i(\omega^*_{ijk})C_i(\bar\omega)-S_i(\omega^*_{ijk})S_i(\bar\omega)]\cdot$$
$$\cdot D(\omega+\omega^*_{ijk})\cos(\varphi^*(\omega^*_{ijk})+\varphi_z(\bar\omega))d\bar\omega \tag{2.8}$$

with $A_z(\omega)$ and $\varphi_z(\omega)$ as amplitude and phase spectrum of z(t). The frequency domain representation of Δw_{ij}^{η} will be obtained, by replacing the cosine terms with minus sine ones.

Hence the order of magnitude of the perturbation elements Δw_{ij}^{η} and Δw_{ij}^{ξ}, that means the influence of the perturbation constituents are governed by the gains $C_i(\omega)$ and $S_i(\omega)$ of the even and odd filters and by the spectral rectangular window $D(\omega)$. In the following reference is given to the VENEDIKOV-Filters 1966b of 48^h length. Fig.2 shows the gains of the filters within the Nyquist-interval.

The spectral rectangular window, associated with the VENEDIKOV-Method has the following properties (see 2.7):

a) $D(\omega)$ is periodic with period $\frac{2\pi}{L} = 2\omega_L$, where ω_L is the Nyquist frequency associated with the sampling interval L of the filtered observations.

b) The main lobes are situated at $2\omega_L k$ (k=0,1,...).

c) The zeros are located at $\frac{2\pi}{nL}k$ (k=1,2...).

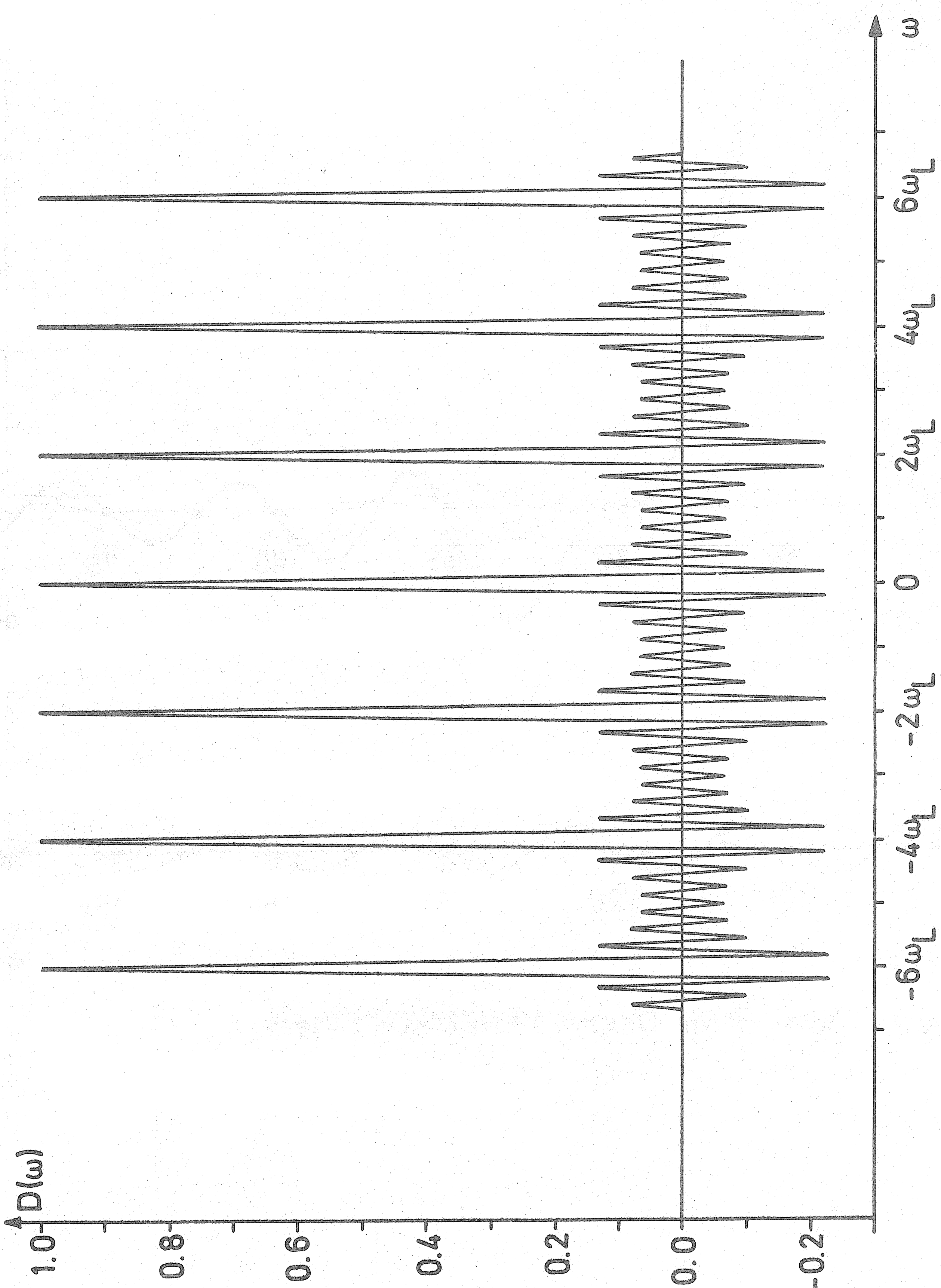

Fig. 1: Spectral rectangular window associated with the VENEDIKOV-method (n-odd, Nyquist frequency = ω_L)

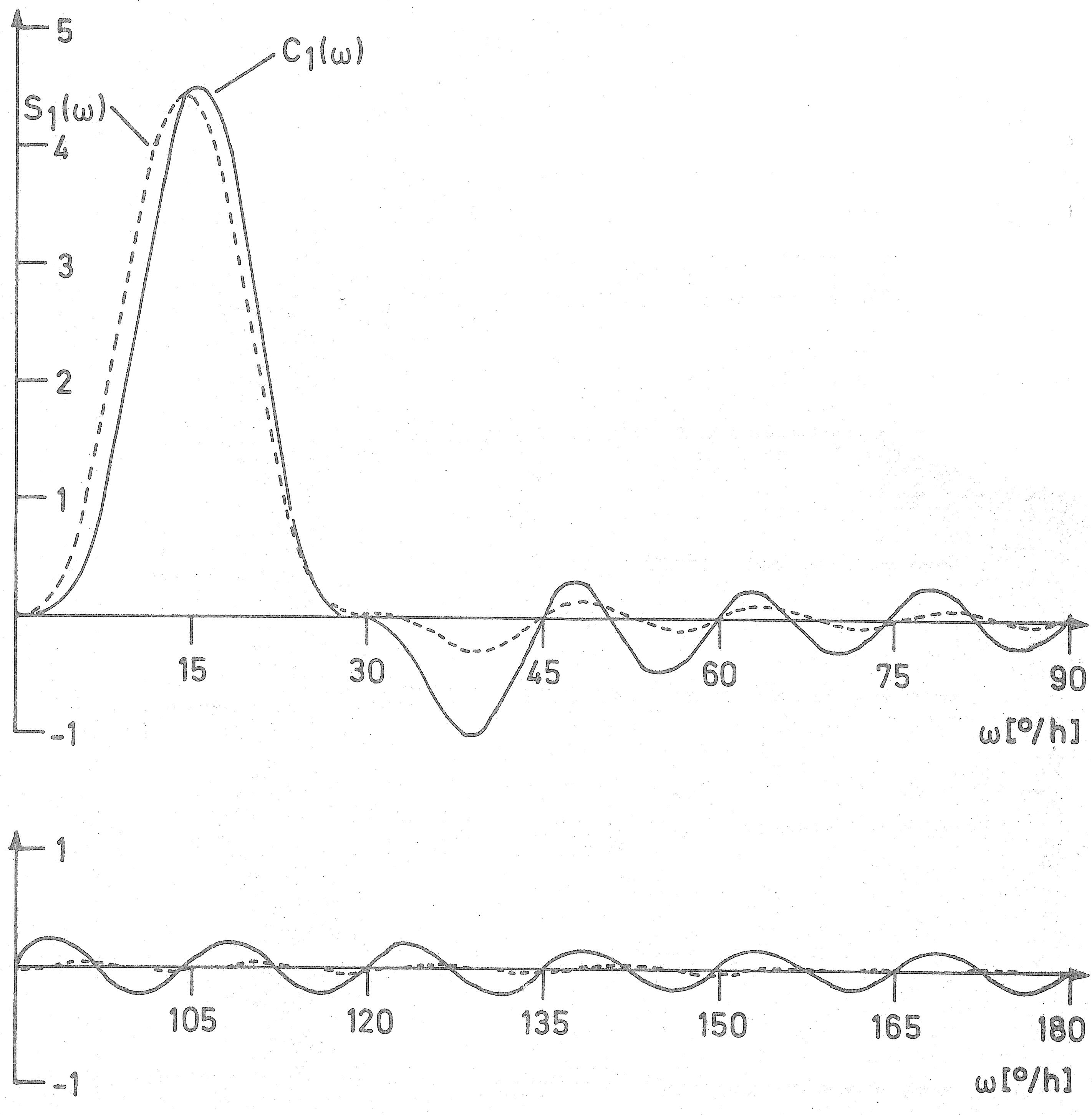

Fig.2 : Gains of the Diurnal VENEDIKOV Filters

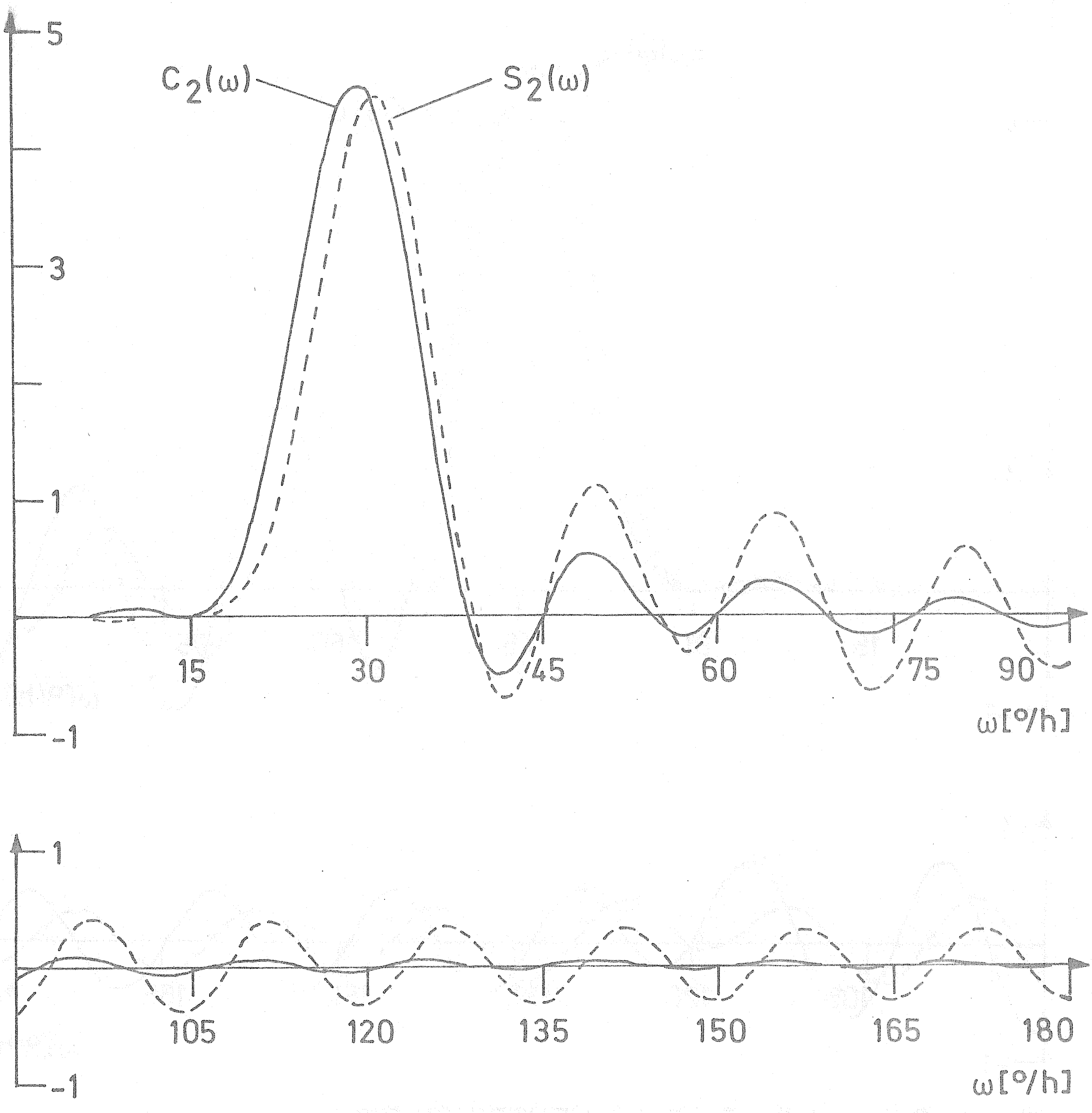

Fig. 2: Gains of the Semidiurnal VENEDIKOV Filters

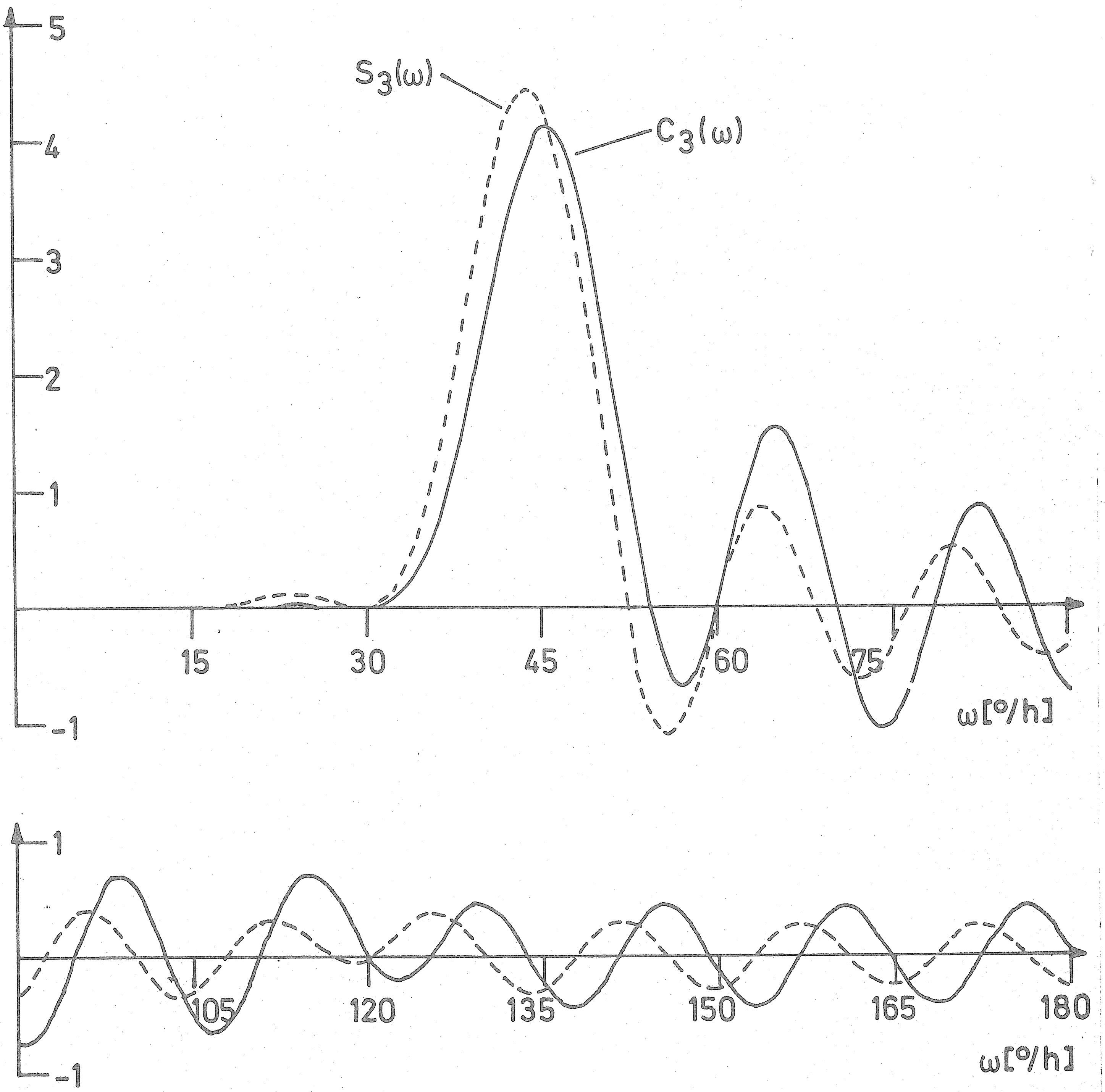

Fig. 2: Gains of the Terdiunal VENEDIKOV Filters

d) Within the k-th period $D(\omega)$ is
   - symmetric to $(2k-1)\omega_L$ when n is odd
   - antisymmetric to $(2k-1)\omega_L$, when n is even
   provided that no gaps are present.

Keeping the properties of the filters and the spectral window in mind, the following rules may be concluded from (2.8):

a) In those frequencies domains of the Nyquist interval where the $C_i(\omega)$ and $S_i(\omega)$ coincident, the second term of the right side of (2.8) is cancelling. However looking at the bands of the filters the first term of (2.8) cannot be cancelled by the filter properties. Hence, it is the spectral window which governs the influence of the passing perturbation constituents by its actual weight.

b) As the spectral window possesses zeros at $\omega_{o,k} = \frac{2\pi}{nL}k$, perturbation constituents which are located at frequencies being $\omega_{o,k}$ away from the tidal ones will contribute no perturbation energies to the Δw_{ij}^{ξ} and Δw_{ij}^{η}. All other frequencies do generally influence them.

c) As the spectral window has extrema equal to one at $\frac{2\pi}{L}k$, perturbation constituents which are $\frac{2\pi}{L}k$ away from the tidal frequencies, will contribute those energies which have passed the filters, without attenuation by the spectral window.

d) The influcnce of perturbation energies from outside the tidal bands is called leakage effect. Leakages, occurring from frequencies being more than ω_L away from the tidal ones are particular nominated: they are called 'aliasing effects'.

The conclusions a) - d) may be summarized to the following central statement:

The VENEDIKOV-estimates for the tidal parameters do not only represent the tidal energies inside the tidal bands when the observed tidal signal is superimposed by a perturbation process. Due to the unfavourable window function which is associated with the VENEDIKOV-Method in combination with the non ideal gains of the filters, perturbation energies from even distant frequencies can leak into the tidal bands, where they cause biases with respect to the tidal parameter estimates.

III. Numerical Demonstration of Leakage Effects Associated with the VENEDIKOV-METHOD

In order to demonstrate the theoretical derivations numerically, the following experiments were carried out by means of a 1 month series of theoretical tides.

(Bonn Station, $\varphi = 50.73^\circ$, $\lambda = 7.08^\circ E$, $h = 50m$).

As it is predicted by theory all energies of additional processes superimposing the tides are going to contribute maximum leakage, when they are concentrated at frequencies, being $\frac{2\pi}{L}k$ away from the tidal ones. Taking the familiar VENEDIKOV filters of length $L=48^h$ (VENEDIKOV 1966a,b), the Nyquist frequency ω_L with respect to that sampling is:

$$\omega_L = \frac{360^\circ}{2\cdot 48^h} = 3.75^\circ/h \qquad (3.0)$$

Hence, the maximum leakage frequencies $\omega_{\lambda_{i,k}}$ relative to the tidal frequency ω_i^* are:

$$\omega_{i,k} = \omega_i^* + 7.5\ k \qquad (|k| = 1,2...) \qquad (3.1)$$

Considering the tidal band O1 with $\omega_{O1} = 13.94^\circ/h$, we find

$$\omega_{O1,1} = 21.44^\circ/h \qquad (3.2)$$

and $\omega_{O1,-1} = 6.44^\circ/h$

to be frequencies from which maximum leakage must be expected with respect to the window. The gains of the even and odd filters are, normalized to O1:

$$\begin{array}{ll} C(21.44^\circ/h) = 0.32 & S(21.44^\circ/h) = 0.27 \\ C(\ 6.44^\circ/h) = 0.15 & S(\ 6.44^\circ/h) = 0.29 \end{array} \qquad (3.3)$$

Therefore two VENEDIKOV analyses were computed for the theoretical tides which have been separately superimposed by:

a) $z_1(t) = A\cos(21.44\ t + \varphi)$

b) $z_2(t) = A\cos(\ 6.44\ t + \varphi)$

where $A = 10\ \mu Gal$, $\varphi = 126{,}35^{\circ}$

The resulting leakage effect in the subsequent tidal parameters are shown in Table 1a and 1b for the main tidal bands:

Tidal Band	$\Delta\xi$	$\Delta\eta$	$\Delta\delta$	$\Delta\varkappa$
O1	0.1238	0.0734	-0.1208	-4.80
K1	-0.0003	-0.0006	0.0003	0.04
N2	-0.0364	-0.0076	0.0364	0.42
M2	0.0755	-0.0069	-0.0755	0.44
S2	0.0020	-0.0052	-0.0020	0.30

Table 1a: Leakage effects on the parameter estimates for a perturbation with 21.44°/h

Tidal Band	$\Delta\xi$	$\Delta\eta$	$\Delta\delta$	$\Delta\varkappa$
O1	-0.0586	0.0464	-0.0575	-2.83
K1	0.0006	0.0003	-0.0006	-0.02
N2	0.0005	0.0000	0.0005	0.00
M2	0.0002	0.0000	-0.0002	0.00
S2	-0.0001	0.0003	-0.0001	-0.01

Table 1b: Leakage effects on the parameter estimates for a perturbation with 6.44°/h

As it was predicted the perturbations at $21.44^o/h$ and $6.44^o/h$ respectively cause a bias in the tidal parameters of O1 due to leakage. Moreover, Table 1a shows also a tremendous effect in the M2 parameters. As we find

$$\omega_{M2,-1} \doteq 28.98 - 7.5^o/h = 21.48^o/h$$

also M2 is effected by nearly maximum leakage. As the gains of the semidiurnal filters are not zero at $21.48^o/h$ the influence of that perturbation on M2 cannot be avoided.

Table 1b is principly showing the same results. Again, O1 is detected to be disturbed by leakage from $6.44^o/h$ but as the gains of the diurnal filters are in all closer to zero in this frequency range (see (3.3)), the effect is smaller than from $21.44^o/h$. As the semidiurnal filter gains are very close to zero at $6.44^o/h$ there is practically no energy transfer to the semidiurnal tides.

Analogue experiments were also carried out for other tidal bands exhibiting comparable features and have therefore been omitted here. Although there may arise the objection that these experiments do not represent the actual perturbation processes they illustrate the leakage mechanisme associated with the VENEDIKOV-Method very well.

Conclusions:

It has been shown that the tidal parameter estimates of the VENEDIKOV-Method are effected by leakage effects, due to the inadequate sampling of the filtered observations when the tidal signal is superimposed by a perturbation process. Hence, interpretations of such analysis results have to be done with care.

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THE PRINCIPLES OF THE RESPONSE METHOD

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SUMMARY

The basic principles of the "Response Method" [7], [5] and its use for Earth Tides data are shown. Problems and criteria concerning the smoothness of the response function of the Earth and extensions of the method are discussed.

INTRODUCTION

Tidal analysis is a very advanced theoretical and practical branch of Oceanographic Sciences and much of the experience there is also useful in Earth Tide analysis. Therefore it was required for this meeting to review some of the analysis methods commonly used for the Ocean Tide analysis and to discuss the use of these methods,such as the Response method in this particular case,for Earth Tide data.

The Response method was first presented in 1966 [7] for Ocean Tides and its extension for Earth Tides is given in [5]. It is the most elaborate non harmonic analysis method for tides available. In this non harmonic approach the physical system of the Earth is described by a linear system using a representation with the system function $s(\tau)$ (weight function) in time domain. The Fourier transform of $s(\tau)$ leads to the system response function $S(f)$. As shown in [8] , the observed data $o(t)$ is described with a model output $p(t)$ which in turn is the convolution of the system function $s(\tau)$ with the tidal input $x(t)$ to the model

$$p(t) = \int_{-\infty}^{\infty} s(\tau)\, x(t-\tau)\, d\tau \qquad (1)$$

In order to find the best approximation of $s(\tau)$ to the real physical system Earth one has to minimise the residual variance (mean square error) $\overline{e^2}$. Because the residual function $e(t)$ is a measure of the difference between the observed output $o(t)$ of the real physical system and the output $p(t)$ of the model linear system [8].

$$e(t) = o(t) - p(t) \qquad (2)$$

The residual variance $\overline{e^2}$ is then given

$$\overline{e^2} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} e(t)^2 dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left(o(t) - \int_{-\infty}^{\infty} s(\tau) x(t-\tau) d\tau \right)^2 dt = \text{Min!} \qquad (3)$$

The minimization of $\overline{e^2}$ with respect to $s(\tau)$ leads to the well known Wiener-Hopf integral as a condition to be fulfilled by $s(\tau)$

$$c_{ox}(u) = \int_{-\infty}^{\infty} s(\tau)\, c_{xx}(u-\tau)\, d\tau \qquad \text{for all } u \qquad (4)$$

This equation is then solved for $s(\tau)$ with finite and digital data [8].

SPECIFIC FEATURES OF THE RESPONSE METHOD

In the Response method [7] the above outlined straight forward non harmonic approach is very much extended and enriched with the use of the tidal input $x(t)$ in its complex form involving spherical harmonics. We have then [appendix] for the tidal input $x(t)$

$$x(t) = \frac{1}{2} \sum_{n=2}^{N} \sum_{m=-n}^{n} x_n^m(t) = \frac{1}{2} \sum_{n=2}^{N} \sum_{m=-n}^{n} y_n^m(t) + i\, z_n^m(t) \qquad (5)$$

In this way the tidal input is used as a multichannel input to the system, so that the linear system has to be considered also with its sub systems $s_n^m(\tau)$. As with the input, the sub systems are also in complex form

$$s_n^m(\tau) = u_n^m(\tau) + i\, v_n^m(\tau) \qquad (6)$$

With both complex multi inputs $x_n^m(t)$ and corresponding complex sub systems $s_n^m(\tau)$ the model output $p(t)$ from (1) is given

$$p(t) = \frac{1}{2} \sum_{n=2}^{N} \sum_{m=-n}^{n} \int_{-\infty}^{\infty} s_n^m(\tau)\, x_n^m(t-\tau)^{*}\, d\tau \qquad (7)$$

(* denoting the complex conjugate). Using the real and imaginary parts for $s_n^m(\tau)$ and $x_n^m(t)$ from (5) and (6) and the symmetry properties of these for $-m$ or m (e.g. $x_n^{-m}(t)=(-1)^m x_n^m(t)^*$, $y_n^{-m}(t)=(-1)^m y_n^m(t)^*$, $z_n^{-m}=-(-1)^m z_n^m(t)^*$, similarly for $s_n^m(\tau)$) we get

$$p(t)=\sum_{n=2}^{N}\sum_{m=0}^{n}\int_{-\infty}^{\infty}\left(u_n^m(\tau)\,y_n^m(t-\tau)+v_n^m(\tau)\,z_n^m(t-\tau)\right)d\tau \qquad (8)$$

The model output $p(t)$ in (8) does not involve any complex quantities. For the convenience all involved parts of sub systems and sub inputs are indexed continuously yielding

$$p(t)=\sum_{j=1}^{J}\int_{-\infty}^{\infty} s_j(\tau)\,x_j(t-\tau)\,d\tau \qquad (9)$$

With (9) the minimizations of the residual variance $\overline{e^2}$ leads to the modified Wiener Hopf integral.

$$c_{ex_i}(u)=\sum_{j=1}^{J}\int_{-\infty}^{\infty} s_j(\tau)\,c_{x_ix_j}(u-\tau)\,d\tau \qquad \text{for all } u,\ i=1,2,\dots,J \qquad (10)$$

With the change to finite and digital data (10) can be written in matrix form

$$[R_i]=[M_{ij}][W_j] \qquad (11)$$

where R_i are the cross covariances of each input with the observed signal, M_{ij} are the cross covariances between the various inputs (for several values of u and τ) and W_j are the real and imaginary parts of the sub systems to be calculated. The solution of (11) is then given using the inverse matrix of $[M_{ij}]$

$$[W_j]=[M_{ij}]^{-1}[R_i] \qquad (12)$$

Before going into the details of (10), (11), (12) for finite and digital data and into the assumptions about $s_n^m(\tau)$, we outline the calculation of the response amplitude and phase for the sub systems. The complex response $S_n^m(f)$ of the sub system is given by the Fourier transform of the complex sub system function $s_n^m(\tau)$

$$S_n^m(f)=\int_{-\infty}^{\infty} s_n^m(\tau)\,e^{-i2\pi f\tau}\,d\tau \qquad (13)$$

With (6) and the real and imaginary parts of the exponential function we can identify in (13) the real and imaginary parts of the complex response function

$$S_n^m(f) = a_s^{m,n}(f) - i\, b_s^{m,n}(f) \tag{14}$$

$$a_s^{m,n}(f) = \int_{-\infty}^{\infty} \left(u_n^m(\tau) \cos(2\pi f \tau) + v_n^m(\tau) \sin(2\pi f \tau) \right) d\tau \tag{15a}$$

$$b_s^{m,n}(f) = \int_{-\infty}^{\infty} \left(u_n^m(\tau) \sin(2\pi f \tau) - v_n^m(\tau) \cos(2\pi f \tau) \right) d\tau \tag{15b}$$

The response amplitude $|S_n^m(f)|$ and the response phase $\phi_s^{m,n}(f)$ are then found by

$$|S_n^m(f)| = \sqrt{a_s^{m,n}(f)^2 + b_s^{m,n}(f)^2} \tag{16a}$$

$$\phi_s^{m,n}(f) = \operatorname{arctg}\left(- \frac{b_s^{m,n}(f)}{a_s^{m,n}(f)} \right) \tag{16b}$$

(The response amplitude and phase are often denoted for gravity by δ and $\varkappa$).

THE CHOICE OF THE RESPONSE

The Wiener Hopf equations in (10) as a condition to the real and imaginary parts of the sub systems lead for the digital and finite data to the matrix equation (11), which is then solved. There are two basic considerations in the step from (10) to (11). The first is the choice of the length of system function $s(\tau)$ (which is an aperiodic function due to the stability conditions for linear systems) denoted by $\pm\tau_{max}$ This reduces the integral limits from $\pm\infty$ to $\pm\tau_{max}$. Next in order to go over to a digital formula (that is to a sum instead of an integral) the continuous time variables u and τ are given in form $u_r = r\Delta u$ and $\tau_q = q\Delta\tau$, with $\Delta\tau$ the sampling rate of the system function and Δu the sampling rate of the correlation functions (and $\tau_{max} = q_{max}\Delta\tau + \Delta\tau$).

We then rewrite (10)

$$c_{ox_i}(u_r) = \sum_{j=1}^{J} \sum_{q=-q_{max}}^{q_{max}} s_j(\tau_q)\, c_{x_i x_j}(u_r - \tau_q) \qquad i=1,2,\dots,J \tag{17}$$

The length of the cross correlation $u_{max} = r_{max}\Delta u$ is therefore not the main criteria but is rather imposed by the choice of $\pm\tau_{max}$ so the basic decisions to be made are about the length and sampling of the system function $s(\tau)$. We will look into the influence on the response function $S(f)$ of those choices. Since the sub inputs of the tidal input cover only one major tidal band (e.g. diurnal, semi diurnal etc.) with the frequency width ΔF, we require that the continuous sub system response $S_n^m(f)$ extends over ΔF. Considering that the diurnal band extends approximately from 0.8 cpd to 1.1 cpd and the semi diurnal band from 1.75 cpd to 2.05 cpd, a band width ΔF of 0.3 cpd is appropriate. If $\Delta\tau$ is the sampling rate of $s_n^m(\tau)$ the Fourier transform of it has a Nyquist frequency or total bandwidth of $1/2\Delta\tau$ and will be periodic in frequency intervals $1/2\Delta\tau$. In [7] and also [5] a sampling rate of $\Delta\tau = 48$h gives a total band width for $s_n^m(\tau)$ of 0.25 cpd, close enough to $\Delta F = 0.3$cpd. One might be surprised by this choice because of the repetition of the response spectrum after 0.25 cpd, but due to $\Delta F = 0.3$ cpd it has a full independent band width over the tidal band, in which it has to represent the actual response (e.g. with $\Delta\tau = 96$h or equivalently a band width of 0.12 cph the response spectrum would repeat itself around 3 times over $\Delta F = 0.3$ cpd. This would impose an assumption that the response over e.g. first, second and third part within a tidal band are alike and would of course be unrealistic). Clearly the choice of $\Delta\tau = 1$h (in accordance to the data sampling) would allow us to represent the response spectrum over the whole Nyquist band of the signal, but this is unnecessary since the response outside the tidal band ΔF has no physical sense, because of the non existent input outside of this band. Certainly one could, and should, use (due to the points above) for $\Delta\tau$ anything from 48h down to 1h. The point to emphasise is that we deal with the question of sampling the system function and thereby the imposed band width. The problem of data sampling is quite separate from this problem.

Next is the choice of the length $\pm\tau_{max}$ of the system function. Any aperiodic function such as $s_n^m(\tau)$ with the length of $2\tau_{max}$ has a fundamental frequency interval (or resolution) of $F = 1/2\tau_{max}$, in the frequency domain. That is any change of the response function in the frequency domain with a spectral wave length (wiggle) smaller than $2F$ cannot be detected (analogously in the time domain according to the Nyquist theorem with a sampling rate Δt, the smallest wave period detectable is $2\Delta t$). Again [7] in practice no evidence was found for a smaller wiggle length then $1/6$ cpd $\approx$ 0.16 cpd. This holds even more so for Earth Tides since the response of the Earth is generally smoother than the response of the oceans. In the actual program at IOS one can go up to $q_{max} = \pm 2$. Considering the sampling rate of $\Delta\tau = 48$h one would have for this case 5 values at ±96h, ±48h, Oh. The values at ±144h and outside are then zero, so that the effective allowed length of the system function is just about 288h (12 days) leading to a $2F = 0.1666$ cpd. That means any wiggle of the response function with a spectral wave length smaller than 0.167 cpd will be smoothed out. Another way of looking at it is to say that we would have a maximum of about two full wiggles allowed over a tidal band with a band width $\Delta F = 0.3$ cpd. However those are the maximum limits that could happen. If these maximum limits occur in the analysis one could repeat the analysis allowing more wiggliness. This has then to be done with care in order not to introduce artificial wiggles. As we have shown the length and sampling of the system function imposes assumptions about the response function and has to be considered also in the interpretation of the results.

EXTENSIONS OF THE MODEL IN THE RESPONSE METHOD

Considering the equation (9) for the model output (or predicted output) one can increase the number of the inputs and also accordingly the number of sub systems for other influencing phenomena. The Wiener-Hopf integral in (10) would also be modified with further correlations of additional channels with the observed signal, various tidal input channels and other input channels. In this manner one can introduce any possible input to the system such as temperature, pressure etc. The cross correlations between the various inputs are then taken care of within the analysis. In fact, in the original presentation of the Response method [7] a so called radiational input is introduced (varying with the cosine of zenith angle in day time and being 0 at night time) to represent generally the non gravitational inputs. However in cases where records of other inputs are already available one might get more information by including them directly.

Also non linearities in the response of the system under investigation are considered in the Response method. For this purpose a suitable artificial input is produced by multiplying pair wise the existing tidal sub inputs (bilinear inputs). To each of those inputs a bilinear admitance $s(\tau,\tau')$ is assigned. With this the model output from (9) has the additional output

$$p_N(t) = \sum_{j=1}^{J}\sum_{k=1}^{J} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s_{jk}(\tau,\tau')\; x_j(t-\tau)\; x_k(t-\tau')\; d\tau\; d\tau' \qquad (18)$$

The associated bilinear response functions are found via the two dimensional Fourier transform of (18)

$$S_{jk}(f,f') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s_{jk}(\tau,\tau')\; e^{-i2\pi f\tau}\; e^{-i2\pi f'\tau'}\; d\tau\; d\tau' \qquad (19)$$

Multiplying the linear inputs produces in the bilinear input a large number of non linear frequencies which would be directly used in a harmonic analysis. However due to the use of the tidal input in the time domain (directly derived from ephemerides) the number of waves are not limited in the Response method both for the astronomical tidal waves and the non linear waves arising from them. We note that also trilinear input and responses are used in the Response method in the same manner as for the bilinear case.

FINAL REMARKS

The response method as a non harmonic method is widely used in the analysis of ocean tides. Its advantages or disadvantages (especially in comparison to classical harmonic methods) and the great variety of ways to use it, are extensively described in the publications given in the references. The assumptions made about the response function compared to those made in the harmonic methods are investigated in [8] . In general the realistic features of the response function being continuous and smooth are successfully used in this non harmonic approach. Therefore its increased use in Earth Tides will certainly increase the geophysical information and also contribute more to the understanding of analysis methods in general.

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APPENDIX

The following is a very brief account of the derivation of the tidal input for Earth tides [5][7]. The tidal potential V is given (at a test point $P(r,\theta,\lambda)$ resulting from two masses M_i)

$$V = \sum_{i=1}^{2} \frac{GM_i}{R_i} \sum_{n=2}^{\infty} \left(\frac{r}{R_i}\right)^n P_n(\cos\alpha_i) \qquad \text{(A1)}$$

i stands to indicate the origin of the potential due to the moon or sun ($i = 1$ or 2). G is the gravitational constant. r is the distance from the Earth's centre to P. R_i is the distance from the Earth's center to M_i. α_i is the zenith angle of the particular M_i. $P_n(\cos\alpha_i)$ are Legendere polynomials and are given in terms of complex spherical harmonics Y_n^m [7].

$$P_n(\cos\alpha_i) = \frac{4\pi}{2n+1} \sum_{m=-n}^{n} Y_n^m(Z_i, L_i)\; Y_n^m(\theta,\lambda)^{*} \qquad \text{(A2)}$$

θ,λ are geocentric coordinates of the observation point. Z_i, L_i are the geocentric coordinates of the i-th body (moon or sun). Z_i and L_i (or α_i) and also R_i are functions of the time, describing the position of the i-th body totally. $*$ denotes the complex conjugate. Spherical harmonics Y_n^m are expressed with Legendere functions P_n^m

$$Y_n^m(\theta,\lambda) = (-1)^m \sqrt{\frac{2n+1}{4\pi}} \sqrt{\frac{(n-m)!}{(n+m)!}}\; P_n^m(\cos\theta)\, e^{im\lambda} \qquad \text{(A3)}$$

and normalization

$$\int \left|Y_n^m(\theta,\lambda)\right|^2 \sin\theta \, d\theta \, d\lambda = 1 \qquad \text{(A4)}$$

The vertical and two horizontal components of the tidal acceleration (tidal input) are on the earth's surface ($r = a$)

$$A_r = -\frac{\partial V}{\partial r} \qquad A_\theta = \frac{1}{a} \cdot \frac{\partial V}{\partial \theta} \qquad A_\lambda = \frac{1}{a\cos\theta} \cdot \frac{\partial V}{\partial \lambda} \qquad \text{(A5)}$$

The tidal potential V in (A1) is complex with the use of (A2) and (A3), (A4) so is the tidal acceleration in (A5). We therefore denote the acceleration in the station and direction of observations with $x(t)=y(t)+iz(t)$. With the consideration of spherical harmonics we have then for the tidal input at a station and direction (with a finite number of terms N)

$$x(t)=\frac{1}{2}\sum_{n=2}^{N}\sum_{m=-n}^{n} x_n^m(t)=\frac{1}{2}\sum_{n=2}^{N}\sum_{m=-n}^{n} y_n^m(t)+iz_n^m(t) \qquad \text{(A6)}$$

"THE PRINCIPLES OF THE TIFA METHOD"

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SUMMARY

The basic principles of the TIFA method (Tidal Institute Flexible Analysis) and its use for Earth tide data are presented. The connections of the method to other harmonic analysis methods are shown. Attention is especially given to the derivation of the response function and the meaning of relation factors in terms of the involved assumptions about the response function.

INTRODUCTION

TIFA is a harmonic tidal analysis method along with many other existing classical harmonic methods. Although it is used very successfully for Ocean Tide data it is not widely known for Earth Tides and therefore it was required for this meeting to review the TIFA method and to discuss some of the important points by applying this method to Earth Tide data.

The idea to express the tides as a sum of cosine waves with the known frequencies (from the astronomy) and then to determine the amplitudes and phases through a least squares adjustment of this sum to the observed data goes back as far as Lord Kelvin and Sir George Darwin 1867. Many methods were devised with this principle of the least squares, some of them very ingenious in overcoming the problems of the pre-computer age [2] . Computers gradually allowed the use of the basic principle in its most extensive and general form so that programs such as TIFA could be devised [4], [3].

The harmonic model of observed tides $o(t)$ is given with N constituents by $p(t)$

$$p(t)=\sum_{n=1}^{N} |P(f_n)| \cos(2\pi f_n t + \phi_p(f_n)) \qquad (1)$$

or with complex spectrum $P(f_n)$

$$p(t)=\frac{1}{2}\sum_{n=-N}^{N} P(f_n)\, e^{i2\pi f_n t} \tag{2}$$

$p(t)$ obviously approximates the observed data $o(t)$ only up to a certain degree. The arising differences are called the residuals $e(t)$

$$e(t)= o(t)-p(t) \tag{3}$$

Another way of looking at this is to say that the observed signal $o(t)$ constitutes a real signal $p(t)$ and "noise" $e(t)$ However the "noise" here is not only in the physical sense but is also very much dependent on the model and is therefore a measure of the modelling. Consequently the amplitudes $|P(f_n)|$ and the phases $\phi_p(f_n)$ are found by minimizing the residual power $\overline{e^2}$. That is

$$\overline{e^2} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} e(t)^2\,dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\Big(o(t)-\frac{1}{2}\sum_{n=-N}^{N} P(f_n)\, e^{i2\pi f_n t}\Big)^2 dt = \text{Min!} \tag{4}$$

For the sake of simplicitly the mean value of observed data $o(t)$ is assumed to be removed so that $P(f_0)$ also can be set equal to zero. We emphasise at this stage that the $p(t)$ or $P(f_n)$ can be regarded as the output of a linear system model with the input of tidal forces $X(f_n)$ and response function $S(f_n)$ [5]. The minimization of $\overline{e^2}$ in (4) is done using

$$\frac{\partial e^2}{\partial P(f_n)} = 0 \quad \text{and} \quad \frac{\partial^2 e^2}{\partial P(f_n)^2} > 0 \quad \text{for all } n \tag{5}$$

and leads with the introduction of the real and imaginary parts $a_p(f_n)$ and $b_p(f_n)$ of $P(f_n)= a_p(f_n)-i\,b_p(f_n)$ to the set of normal equations

$$\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} o(t)\,{\cos \atop \sin}(2\pi f_m t)\,dt$$
$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\Big(\sum_{n=1}^{N} a_p(f_n)\cos(2\pi f_n t)\,{\cos \atop \sin}(2\pi f_m t) + b_p(f_n)\sin(2\pi f_n t)\,{\cos \atop \sin}(2\pi f_m t)\Big)\,dt \tag{6}$$
$$m = 1,2,\dots,N$$

Introducing finite and digital signals we can write (6) in matrix form

$$[Y_i] = [X_{ij}]\,[P_j] \tag{7}$$

where Y_i are the product sums of the observed data with $\frac{\cos}{\sin}(2\pi f_m t)$. P_j are the unknowns of $a_p(f_n)$ and $b_p(f_n)$ and X_{ij} are the product sums of the cosine and sine components with frequencies f_n and f_m. The solution of (7) is found by calculating the inverse matrix $[X_{ij}]^{-1}$

$$[P_j] = [X_{ij}]^{-1}\,[Y_i] \tag{8}$$

We note that in (6), (7), (8) the solution of the problem would be reduced to a simple Fourier series representation of observed data, if the used frequencies are integer multiples of the fundamental frequency (i.e. 1/data length). This in practice is very rare except for the case of approximately one year length of data where basic tidal frequencies are integer multiples of the fundamental frequency.

THE RELATION FACTORS AND THEIR INTERPRETATION IN TERMS OF THE RESPONSE

In an actual analysis often the data length is too short to resolve very close tidal frequencies because the matrix equations in (7) and (8) are then ill conditioned. One therefore has to work rather with wave groups instead of individual waves. In this case H wave groups arise (group index h) each having G_h close waves (the wave index within a group is g). Further in each group all the waves with the frequency f_{hg} (g-th wave of h-th group) are related to the major wave with the frequency $f_{h\bar{g}}$ using the relation factor ε_{hg} and λ_{hg}

$$|P(f_{hg})| = \varepsilon_{hg}\,|P(f_{h\bar{g}})| \quad \text{for amplitudes} \tag{9a}$$

$$\phi_p(f_{hg}) = \phi_p(f_{h\bar{g}}) + \lambda_{hg} \quad \text{for phases} \tag{9b}$$

or as follows in terms of real and imaginary parts

$$a_p(f_{hg}) = a_p(f_{h\bar{g}})\,\varepsilon_{hg}\cos\lambda_{hg} - b_p(f_{h\bar{g}})\,\varepsilon_{hg}\sin\lambda_{hg} \tag{10a}$$

$$b_p(f_{hg}) = a_p(f_{h\bar{g}})\ \varepsilon_{hg} \sin\lambda_{hg} + b_p(f_{h\bar{g}})\ \varepsilon_{hg} \cos\lambda_{hg} \qquad (10b)$$

Proceeding in this way the 2N unknowns of $a_p(f_n)$ and $b_p(f_n)$ in (6) are reduced to 2H unknowns of $a_p(f_{h\bar{g}})$ and $b_p(f_{h\bar{g}})$ leading to the modified normal equations

$$\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} o(t) \sum_{g'=1}^{G_h} \varepsilon_{h'g'} \frac{\cos}{\sin}(2\pi f_{h'g'}t + \lambda_{h'g'})$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{h=1}^{H} a_p(f_{h\bar{g}}) \sum_{g=1}^{G_h} \varepsilon_{hg} \cos(2\pi f_{hg}t - \lambda_{hg}) \sum_{g'=1}^{G_h} \varepsilon_{h'g'} \frac{\cos}{\sin}(2\pi f_{h'g'}t + \lambda_{h'g'})$$

$$+ b_p(f_{h\bar{g}}) \sum_{g=1}^{G_h} \varepsilon_{hg} \sin(2\pi f_{hg}t - \lambda_{hg}) \sum_{g'=1}^{G_h} \varepsilon_{h'g'} \frac{\cos}{\sin}(2\pi f_{h'g'}t + \lambda_{h'g'})$$

$$h' = 1, 2, \ldots, H \qquad (11)$$

This normal equation system (11) for wave groups is solved in the same manner as the equations in (6) for finite and digital data. The results for the individual waves are then formed using (10a) and (10b) or better using (9a) and (9b). The introduction of the relation factors ε_{hg} and λ_{hg} helps to overcome the problem of unresolvable waves but the additional information required in order to provide the relation factors is difficult to obtain. One reliable way of getting the relation factors is by analysing a long data set (e.g. one year) from the same station or a nearby station. Also considerations from geophysical models may contribute to this information. With the lack of such knowledge however one has still the simple choice of the assumption that the waves in a group are related in the same way as they are in the tidal input. To see the implications of such a procedure upon the response function we write the model tides $P(f_n)$ as an output of a linear model as mentioned earlier. (Also [5]).

$$P(f_n) = S(f_n)\, X(f_n) \qquad (12)$$

with $X(f_n)$ being the complex spectrum of the tidal input (at the same station, direction etc. as the observations) and $S(f_n)$ being the complex spectrum of the Earth's response. With amplitudes and phases from (12) we may rewrite the relations from (9a) and (9b) as follows

$$\varepsilon_{hg} = \frac{|P(f_{hg})|}{|P(f_{h\bar{g}})|} = \frac{|S(f_{hg})|}{|S(f_{h\bar{g}})|} \cdot \frac{|X(f_{hg})|}{|X(f_{h\bar{g}})|} \tag{13a}$$

$$\begin{aligned}\lambda_{hg} &= \phi_p(f_{hg}) - \phi_p(f_{h\bar{g}}) \\ &= \phi_s(f_{hg}) - \phi_s(f_{h\bar{g}}) + \phi_x(f_{hg}) - \phi_x(f_{h\bar{g}})\end{aligned} \tag{13b}$$

Now if we assume the response amplitude and phase to be constant over each individual wave group h the relation factors in (13a) and (13b) reduce to

$$\varepsilon_{hg} = \frac{|X(f_{hg})|}{|X(f_{h\bar{g}})|} \quad \text{and} \quad \lambda_{hg} = \phi_x(f_{hg}) - \phi_x(f_{h\bar{g}}) \tag{14}$$

Thus using the relation factors directly from the tidal input is equivalent to the assumption of constant response over the wave group. In fact this implicit assumption about the response for the classical harmonic method TIFA is used in harmonic methods for Earth tides such as Chojnicki, Venedikov, HYCON etc. [5] in a direct way, since in these methods the response $S(f_n)$ is used as unknowns (by using the knowledge of tidal input $X(f_n)$ and inserting linear system model equation (12) in to the formalism of TIFA). Incidentally for the mentioned methods using the response of the Earth $S(f_n)$ as unknowns, it would be better to use the terminology of "harmonic response" methods, since harmonic methods traditionally refer to methods like TIFA using directly the model of the tides $P(f_n)$ [4] ,[3] , [2] .

Considering the connection of the relation factors to the assumptions about the response, one can of course also use in the TIFA method any non equilibrium response assumption [1] over a band through (13a) and (13b) and check its validity by investigating the residual signal. Further, it is also very easy to introduce into the TIFA method waves with frequencies which are not present in the actual tidal input, that is waves with frequencies for which the response function cannot be determined (12). Such waves may be in the observed signal due to the shallow water loading or due to the non linearities in the system Earth or the instrument. Further there might be other possible inputs for which no explicit observation and information are available except the knowledge of the frequencies from theoretical consideration (e.g. high frequency components of temperature or air pressure not coinciding with a known tidal frequency). For that matter one also can include the cases where even the actual tidal input is zero for a tidal band (e.g. semi diurnal tides at the poles or the north south diurnal tilt at 45^{o} North) but there is a considerable observed signal due to the ocean loading.

RESPONSE FUNCTION FROM THE TIFA METHOD

The TIFA method has the advantage working with the fundamental elements of the signal namely with the amplitudes $|P(f_n)|$ which of course have the same physical units as the observed signal. However if one wants to have the response function and the amplitudes $|P(f_n)|$ and phases $\phi_p(f_n)$ are determined, one can insert them with the tidal input $X(f_n)$ (for the same station, direction etc., from tables for the harmonic development of tidal forces) in (12) and so calculate the response $S(f_n)$. In doing so one has to take extra care if any other response assumption (or equivalently relation factors for tidal input) than the constant response over wave groups is used in the preliminary analysis.

Generally in the TIFA method, few actual tidal input waves are used (e.g. 15 main waves and 5 related waves for a 3 month long signal). Obviously the number of the waves has to be increased for observations of good signal/noise ratio. One generally would include every wave with a bigger amplitude than the background noise level, again using the residuals as a guide. However, whichever way one uses a harmonic method, the number of the included waves have to be finite. Therefore one possible way is to subject time domain version of the tidal input itself (for the same station, direction and especially time interval and length as observed data) to exactly the same TIFA analysis as the observed data and use those results for tidal input to get the response function. This is one of the possible ways to decrease the effect due to the insufficient number of waves used (particularly in the routine runs for sea tide data).

ADDITIONAL REMARKS

We should note that for most of the analysis methods with the classical harmonic approach, including TIFA, the tidal waves arising from the 18.6 and 8.85 yearly variations in the tidal input are taken care of with the so called nodal factors [2]. Thus a time variation of the amplitudes $|P(f_n)|$ and phases $\phi_p(f_n)$ over 18.6 years are simulated by correcting the amplitudes and phases sequentially with these nodal factors. These are generally also derived from the tidal potential. Further the phase $\phi_p(f_n)$ is used in relation to the tidal input where

$$\phi_p(f_n) = v_n - g_n \tag{15}$$

Here v_n is the phase of n-th constituent of tidal input at Greenwich for $t=0$ and is given (λ being the longitude of the station, positive to the West) by

$$v_n = \phi_x(f_n) + k\lambda \qquad k = \begin{cases} 0 \text{ for long period} \\ 1 \text{ for diurnal} \\ 2 \text{ for semi diurnal} \\ 3 \text{ for diurnal} \end{cases} \tag{16}$$

With the introduction of (15) in TIFA, the actual calculated phase is g_n. We can verify with the use of (15) (16) and also the phase part of (12) that

$$g_n = -\phi_s(f_n) + k\lambda \tag{17}$$

With this special use of phases in TIFA (due to the desire to compare phases at different parts of the world) one is led directly to the response phase. We note further that the actual program uses an iterative method for the solution of the normal equations allowing a considerable saving on computer time [4] and no significant effect whatsoever in the results.

Finally, one can conclude that the classical harmonic method, used in different variations, serves a good purpose especially considering that one can use some of the properties of it for the "harmonic response" methods in earth tides such as the way of including non linear waves or even any wave with non tidal frequency. Also the direct use of TIFA with succeeding derivation of the response function as described above allows comparisons to any harmonic or non harmonic response method, with more emphasis on the involved assumptions about the response function, rather than simply numerical comparisons.

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Principles of the HYCON-Method

by

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The objectives of the HYCON-Method (HYBRID LEAST SQUARES FREQUENCY DOMAIN CONVOLUTION Method) aim at a detailed analysis of tidal observations. The main purpose is to reveal time variations of the related quantities in order to provide these analysis results for further investigations of physical phenomena. As the conceptional features of HYCON together with its mathematical formulation has recently been reported (SCHÜLLER 1976, 1977a,b) this paper shall represent a summary of the basic principles of the method.

1. HYCON is based on the least squares principle but generalised to allow modifications in the frequency domain convolution properties involved. This is done by introducing the Hanning window function into the least squares procedure in order to reduce biases due to leakage effects (SCHÜLLER 1976,1977a).

2. The mathematical models for the Earth's responses to the astronomical tidal forces include the Cartwright-Tayler-Edden development (CTED) completed by some Doodson constituents (505 in all). As it was proposed by Dr. Baker (BAKER 1978b) departures from the equilibrium response to the potential due to ocean loading and liquid core resonance can be modelled instead of constant bandwise responses. Moreover nonlinear tidal constituents can be introduced into the mathematical model provided that they can be separated from the 505 CTED constituents. Energies at low frequencies can optionally be removed by suitable highpass filters.

3. The most important feature of HYCON is the evaluation of socalled time variant parameter functions for the tidal parameters as well as for the amplitudes and phases of nonlinear tidal constituents. This is done by a special sliding analysis procedure (SCHÜLLER 1976, 1977b). By taking into account the relations between the amount of shifting of a basic interval and the properties of the window function (SCHÜLLER 1977b) biases due to aliasing are avoided what is extremely important for a unique interpretation of the variations of the parameters with time. Fourier spectra of the parameter functions are calculated to isolate the energy concentrations at certain frequencies. By means of those significant parameter function constituents together with their lag-lead patterns the physical reasons for occurring time variations may be explained (SCHÜLLER 1977b).

4. HYCON can also provide global tidal parameters by taking the expectations over the sample values of the parameter functions. The associated error propagation is then based on the residual parameter functions, that means on the deviations of the sample parameters themselves. It ends up with confidence intervals for the global tidal parameters and the amplitudes and phases of the nonlinear tidal constituents independently for each frequency band and taking into account the equivalent degrees of freedom associated with each parameter function.

5. Time variant spectra of the observation residuals are provided as well as the overall residual spectrum in order to detect persistent signals of nontidal origin. Furthermore all nonlinear tidal constituents not having been introduced into the least squares algorithm may be derived from the residual spectra.

6. Concerning the information given in the output of HYCON all the requirements listed in BAKER 1978a are taken into

account except the multi channal input problem which is planned to be investigated in the near future.

A FORTRAN program of HYCON is available which can be obtained from the author. It is not only restricted to the performance of HYCON but allows a standard least squares tidal analysis as well.

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SUPPLEMENTARY PRECISION ESTIMATION OF RESULTS OF TIDAL DATA ADJUSTMENT.

by

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The spectral analysis of the so called tidal residuals, i.e. the correction set v after adjustment by a least squares method (Chojnicki, 1972) shows that the noise is different in various frequency bands. It causes a deformation in the estimation of the adjustment result precision because the use of the classical least squares technique to obtain this estimation assumes a homogenous noise. The estimation deformation appears, first of all, as precision overestimation in the bands of longperiod and diurnal waves. This was indicated several times in many papers (Wenzel, 1976, 1977; Lecolazet, 1977; Venedikov, 1977).

It was therefore necessary to make a supplementary estimation of precision of adjustment results obtained by the classical method. One way for this estimation has been presented by Wenzel (1977). In the latest version (15 H) of the program we used however another way which seems to be easier for programming and to give more information. The principle of this estimation is given here.

Values of the v correction after adjustment that is of the so called tidal residual can be presented as a function of time

$$v_i = f(t_i) \qquad \text{or} \qquad v_i = f_i$$

which is given in a time segment $\Delta T = t_n - t_1$ by n discrete values in equal time intervals $\Delta t = t_i - t_{i-1}$:

$$\Delta t = \frac{\Delta T}{n-1} \quad .$$

The mean square error of observation after adjustment m_o according to these notations will be :

$$m_o = \sqrt{\frac{\sum_{i=1}^{n} f_i^2}{n-r}} \quad \text{or} \quad m_o = w\sqrt{\frac{\sum_{i=1}^{n} f_i^2}{n-1}} \tag{1}$$

where r is the number of unknowns determined from the adjustment, and $w = \sqrt{\frac{n-1}{n-r}}$.
After transformation :

$$m_o = w\sqrt{\frac{\sum_{i=1}^{n} f_i^2 \frac{t_n - t_1}{n-1}}{t_n - t_1}} = w\sqrt{\frac{\sum_{i=1}^{n} f_i^2 \Delta t}{\Delta T}} .$$

We can write approximately :

$$\sum_{i=1}^{n} f_i^2 \Delta t \cong \int_{t_1}^{t_n} f^2(t)dt ,$$

and then

$$m_o \cong w\sqrt{\frac{1}{\Delta T}\int_{t_1}^{t_n} f^2(t)dt} . \tag{2}$$

Let us assume now, that the function $f(t)$ has a finite development in Fourier series :

$$f(t) = \frac{1}{\sqrt{2}} a_o + \sum_{i=1}^{k} a_i \sin(\omega_i t + b_i), \text{ where } \omega_i - \omega_{i-1} = \text{const.}$$

Using such a form of the function for establishing tidal residual and on basis of the Parseval equality, if K is great enough we obtain :

$$\frac{1}{\Delta T}\int_{t_1}^{t_n} f^2(t)dt \cong \frac{1}{2}\sum_{i=0}^{k} a_i^2 \tag{3}$$

Finally, the formula for the mean square error m_o according to the data obtained from the tidal residual development in Fourier series has the form :

$$m_o = \frac{w}{\sqrt{2}}\sqrt{\sum_{i=0}^{k} a_i^2} \tag{4}$$

If the development of the tidal residual in Fourier series presents a homogenous noise, that is if all amplitudes are of the same order, the formula (4) can be simplified for a frequency band ($p \rightarrow q$) as follows :

$$m_o = \frac{w}{\sqrt{2}} \sqrt{\frac{k+1}{q-p+1} \sum_{i=p}^{q} a_i^2} = M_{o(p,q)} , \quad q-p < k . \qquad (5)$$

The equality $m_o = M_{o(p,q)}$ will be fulfilled for the homogenous noise only.

If tidal residual development presents a non homogenous noise, the formula (5) can be also used to calculate an imaginary mean square error $M_{o(p,q)} \neq m_o$ whose adjustment would give the residual with homogenous noise equal to the noise in the investigated frequency band (p,q). The mean square error calculated in such a way may be used for the estimation of precision of the tidal wave parameters determined during adjustment in selected frequency bands using formulae of the classical least squares technique :

$$M_{x(p,q)} = M_{o(p,q)} \sqrt{Q_{xx}}$$

that is

$$M_{x(p,q)} = M_{o(p,q)} \frac{m_x}{m_o} , \qquad (6)$$

where m_x is the mean square error of the unknown x after adjustment calculated by the use of the classical least squares technique.

Calculation of the mean square error $M_{o(p,q)}$ according to the formula (5) is possible only for the so called observation block, that is a data set made at equal time intervals (for example 1 hour) without interruptions. The analysis of observation series with gaps, that is series consisting of some number B of observation blocks is most common, and because of this it is necessary to calculate $M_{oB(p,q)}$ for the whole series on the basis of $M_{oj(p,q)}$ $j = 1, \ldots B$ calculated according to the formula (5) for particular blocks.

From the definition of the mean square error for the whole series it results :

$$m_o^2 = \frac{\sum_{j=1}^{B} \sum_{i=1}^{n_j} f_i^2}{\sum_{j=1}^{B} n_j - r} \qquad (7)$$

Inserting (1) to (7) and denoting $N = \sum_{j=1}^{B} n_j$, we obtain :

$$m_o^2 = \frac{1}{N-r} \sum_{j=1}^{B} m_{oj}^2 \; (n_j - r)$$

Similarly:

$$M_{oB(p,q)} = \sqrt{\frac{1}{N-r} \sum_{j=1}^{B} M_{oj(p,\, q)}^2 \; (n_j - r)} \qquad (8)$$

In the latest version (15 H) of the programme for tidal observations analysis by the classical method the above procedure for the precision estimation of adjustment results was introduced. Errors of the calculated unknowns and the mean square errors in bands of longperiod, diurnal, semidiurnal and terdiurnal waves are given in a table of results. For example, in the enclosed table the adjustment results and their accuracy estimation of several observation analysis by the classical and Venedikov methods are given.

I would like to add that the mentioned new version of the programme gives the possibility to use any filter (symmetrical and odd) to determine the drift or to eliminate intermediate residual in double adjustment. It has been made in order to eliminate the Pertsev filter from our method and replacing it by a more optimum filter without limitations caused by the calculation technique of the period of the Pertsev filter construction. The problem of the filter optimization in order to calculate the drift and presentation of filters we use is presented in preliminary version in Malkowski, 1978.

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Series	Meth. (Filter)	m_o	$M_{o(1)}$	$M_{o(2)}$	$M_{o(3)}$
OBSERVED (Pecny)	V		4.84	1.62	1.26
	C (71A)	1.68	3.77	1.54	1.31
	C (71B)	1.28	3.42	1.58	1.26
MCDELED	V		1.98	0.95	1.03
	C (37)	1.37	1.85	0.93	0.99

a white noise assumption, either across the whole spectrum or across individual tidal bands. It is essential that the residual spectrum is used to obtain more realistic error limits [8]. Similarly, it is important to consider the time variation of the tidal constituents using shifting analyses [5]. The residual spectrum and the time variation of the constituents not only give more realistic error limits but may also contain interesting physical information about the instrument, the Earth or the oceans [3].

It is clear from these discussions that, at least for the immediate future, the analysis requirements for tidal gravity are an order of magnitude greater than for tilt or strain.

3. CONCLUDING REMARKS

Taking into account all the above requirements that are necessary for physical interpretation of the data, any future analysis program should include the following points in its procedure and presentation:
(where necessary notes for additional clarification of the requirements are included in parenthesis)

(1) The analysis procedure should be based on least squares [6,7].
(2) For harmonic methods, the full Cartwright-Tayler-Edden potential should be used. (A listing of the theoretical tides at the station should be given).
(3) More realistic error limits based on the residual spectrum should be given [8].
(4) An indication of the time variation of the main tidal waves should be given. (This gives information on instrumental performance and the changes with time in the response of the Earth or oceans). [5].
(5) Non-linear waves should be included in the least squares analysis [3].
(6) For non-separable waves, an attempt should be made to allow for departures from the equilibrium response to the potential [3].
(7) Multichannel inputs should be included. (Experimental investigations are required in order to establish the physical mechanisms of meteorological and environmental perturbations).

(8) Full details of the numerical filters used should be given.

(9) In addition to δ or γ factors, the results should be given in fundamental physical units (microgals or milliseconds and phases). (The given amplitude should be for the main wave in the group).

(10) A complete description of the experiment should be given:-
station, instrument, dates, amplitude and phase calibration methods, azimuth and azimuth accuracy, experimental difficulties etc.

((a) It is impossible to give sufficient information in a short computer listing for satisfactory assessment and interpretation of the experiment. A full text is required.

(b) Phase and azimuth conventions should be clearly stated.)

(11) For tidal gravity, M_2 and O_1 results with dates and errors should be given for previous measurements with the same instrument at the fundamental base stations.

((a) For the inverse problem very accurate absolute calibrations of amplitude and phase are not required but an accurate relative calibration of all instruments is essential. The previous base station values allow the determination of the relative calibrations of the instruments and also provide a check on the variation of instrumental constants with time.

(b) One fundamental base station is required in each 'geographical area' and should be accurately 'tied' in to the other fundamental stations.

(c) If amplitude and phase corrections have already been included in the analysis in an attempt to obtain better absolute parameters then these corrections should be stated explicitly since it is inevitable that they will change as further experimental work on calibrations is performed.)

formulae (4a) and (4b) will lead to the amplitude and the phase errors of the model output revealing interesting aspects of the errors used in different methods. This easy calculation shows e.g. for the Chojnicki method [3] that the errors $\Delta|P(f_n)|$ are constant for any wave (with insignificant numerical differences). The same happens in the Venedikov method [9] for each individual tidal band (since in this case the tidal bands are analysed separately). Thus the error spectrum $\Delta P(f_n)$ behaves like a white noise spectrum with constant amplitude for the whole frequency range used in the analysis. This is due to the assumption that the residual $e(t)$ is a time series of white noise and as such its spectrum is fully determined by its variance $\overline{e^2}$. This special form for the errors $\Delta|P(f_n)|$ and $\Delta\phi_p(f_n)$ is often not realised and calculating them explicitly serves from a practical point of view a good purpose in recognising the actual character of the errors in different methods. However, theoretical handling of the problem gives more valuable insight into the error calculation and reveals that the examples above are inadequate special cases of a more general principle.

RESIDUAL SPECTRUM AND ERRORS

With the framework for all the analysis methods it can be shown that the residual function is not correlated (not coherent) with the tidal input $x(t)$ and consequently also with the model output $p(t)$ [11], [12].

$$C_{xe}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e(t+\tau)\, dt = 0 \qquad (5)$$

Considering that the tidal input $x(t)$ is composed of periodic components and has a line spectrum $X(f_n)$ the formula (5) means that the residual function $e(t)$ does not contain any periodic components with tidal frequencies. This statement is valid for most practical cases with digital and finite signals. It can be verified with an extra harmonic analysis of the residuals using the same tidal frequencies as in the primary analysis. Exceptions might occur in the analysis of short data. In these cases one is forced in harmonic methods to assume constant or other type of response for very close waves [12]. If these assumptions do not reflect the actual situation [2] then the tidal components in observed data are not properly modelled by $p(t)$. Their unmodelled part will appear in the residuals as periodic components with tidal frequencies in which case one has to look for better response assumptions. The opposite can happen too, so that due to the short length of the signal, the background random noise has not enough samples and therefore behaves rather in a deterministic way over the available signal length. It will then be included into the modelled tidal waves leading to dips (anticusps) in the residual spectrum over tidal waves or tidal bands. This might even happen for longer signals since the spectrum of a finite sample of random (e.g. white) noise does not converge necessarily to a constant spectrum with increasing length of the sample as shown in [5, pp 211].

Since the residual is not periodic at least in tidal frequencies (and other frequencies used in the analysis) it has to be random (incoherent). This takes also into account that individual aperiodic signals (e.g. due to rainfall or storm surge loading) behave as random signals in the long run and therefore as random signals in the spectrum. The spectrum, of course, as an integrated form of information gives at a particular frequency only an average of contributions from all individual aperiodic signals [7] . (This incidentally demonstrates again that notonly the spectrum but also the actual time series of the residuals should be investigated since the information about the non-stationarities in the residuals such as the aperiodic signals with varying types, lengths, occurrence times, number and also frequency content cannot be detected in the spectrum. As a compromise, however, one could make a sequential spectral analysis.) Also from a physical point of view, the residuals as a random disturbance influence the amplitudes and the phases of the model output, i.e. the optimal estimates for the tidal components in the observed data. We then have (for small errors) with Fourier amplitude spectrum $|E(f)|$ of the finite residual time series [13]

$$\Delta|P(f)| = |E(f)| \tag{6a}$$

$$\Delta\phi_P(f) = \frac{1}{|P(f)|}|E(f)| \tag{6b}$$

(We note that $\Delta\phi_p(f)$ has to be multiplied by $360^o/2\pi$ to correct it to degrees). Actually since $e(t)$ is random (after removal of all the mentioned effects), its spectral representation should be rather given with the power density spectrum $C_{ee}(f)$ [5],[7].

$$C_{ee}(f) \simeq \int_{-\infty}^{\infty} c_{ee}(\tau)\cos(2\pi f\tau)\,d\tau \qquad (7)$$

$c_{ee}(\tau)$ is the autocorrelation function of the residuals $e(t)$

$$c_{ee}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} e(t)\,e(t+\tau)\,dt \qquad (8)$$

However, for finite signals often the following estimation of $C_{ee}(f)$ from the amplitude spectrum $|E(f)|$ is used [5]

$$C_{ee}(f) = \frac{|E(f)|^2\,T}{2} \qquad (9)$$

(T is the length of the residual signal) (6a) and (6b) is normally given from this estimation for simplicity. Further problems related to the calculation of power density spectrums are given in many text books e.g. [5] .

WHITE NOISE RESIDUAL SPECTRUM AND ERRORS

If the residuals $e(t)$ are in fact or are assumed (in most cases rather wrongly) to be white noise, the auto correlation of $e(t)$ will exist only at $\tau = 0$ with the value $\overline{e^2}$ as it can be verified with (8) and (1). The power density spectrum is then constant and (7) gives

$$C_{ee}(f) = \overline{e^2} \qquad (10)$$

Further with the use of (9) we have finally for the errors $\Delta|P(f)|$ and $\Delta\phi_p(f)$ in this special case of white noise residuals

$$\Delta|P(f)| = \sqrt{\frac{\overline{e^2}}{T/2}} \qquad (11a)$$

$$\Delta\phi_p(f) = \frac{1}{|P(f)|}\cdot\sqrt{\frac{\overline{e^2}}{T/2}} \qquad (11b)$$

This special case of error estimation is also exactly given in [4], although the way of the derivation there is considerably different to ours. Using (11) with (4) i.e. relating $\Delta P(f)$ to the error of the response $\Delta S(f)$ and further using the quoted values in computer listings for the residual variance (or rather the square root of it) in the analyses one can verify that (11) is in fact used (allowing for insignificant differences) in all the methods assuming the residuals to be white noise (for tidal bands separately or together) [3],[6],[9].

Apart from the unjustified white noise assumption for the residuals, the value of the residual variance $\overline{e^2}$ used for the calculation can be wrong also. That is if any digital filtering is applied to the observed data but not the tidal input [3] the residuals are then distorted and a filter correction is necessary [10] [12]. In this case the results for $P(f)$ or for $S(f)$ have to be corrected as well. If a filter is applied both to the observed data and the tidal input, [9] e.g. if they are amplified and phase shifted this will not influence the result for the response $S(f)$, but the residuals are amplified and phase shifted as well. Although the phase shift does not influence $\overline{e^2}$ the gained amplitudes do influence $\overline{e^2}$ and are to be corrected. Further, if the hourly data after filtering are sampled e.g. with 48-hours [9], so will be the residuals. It is obvious that the variance of the 48 hourly sampled signal will be generally different from the variance of the 1-hourly sampled signal. Another point causing slight differences is the normalization of the residual variance $\overline{e^2}$ Sometimes $\overline{e^2}$ is not in agreement with (1b), since it is not normed to $\frac{1}{T} = \frac{1}{N\Delta t}$ as in (1b). (N is total number of the samples and Δt the sampling interval). The normalization often used is $\frac{1}{N-2r}$ (or similar) with $2r$ being the number of the unknowns (i.e. the real and complex part of the response) used in the analysis. This type of normalization could be justified from a statistical point of view if the analysis was applied to time independent samples but not for our signals. However, in most cases in practice $2r$ is very small compared to N, so that this difference is secondary to the other possible errors on $\overline{e^2}$ as mentioned above.

FINAL REMARKS

It is shown that most of the used error calculation approaches assume the residual to be white noise and use therefore the residual variance $\overline{e^2}$ directly for the errors. Apart from possible errors in the determination of $\overline{e^2}$, the major problem is the white noise assumption for the residuals, which does not hold. On the contrary the residuals are highly complicated containing all sorts of components such as periodic signals, aperiodic signals, various types of random signals, even non stationary phenomena leading to an equally complicated residual spectrum [2] and error estimation. For this and for other reasons presented here, we think that the error estimation of the results are better based on the real character of the residuals and their spectrum (6), (4). This involves careful investigation of the residuals and can certainly be managed with the powerful computers widely available today. It is obvious that such an approach leads not only to more realistic error limits but also to more geophysical information and understanding of the phenomena in the observed signal.

Finally we note that the considerations here are to a great extent for a standard analysis performed on the whole of the data, assuming time invariance (or stationarity) for the system response and also its error. The time variant analysis and error estimation are investigated elsewhere and are to be presented soon [13].

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REMARKS ON THE INVESTIGATION OF RESIDUAL CURVES OF EARTH TIDES RECORD

by

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The residual curves of the earth tides records are composed of the following parts :

$$R(t) = L(t) + P(t) + D(t) \tag{1}$$

where L(t) is a "residual" lunisolar effect not contained by the functional model; the second part P(t) is a real non-lunisolar effect; the third part D(t) is the instrumental drift.

Although at certain methods for the evaluation of the earth tides records - for example Fourier and stochastic spectral analysis - no kind of functional model of earth tides is applied in the first step, but for calculating the earth tides factors and the residual curve it is necessary to use some kind of functional model describing the earth tides. At the most frequently applied process, based on least squares method, the functional model of the earth tides is introduced immediately into the calculation. So, we can speak about "the residual" lunisolar part in the residual curves, independently of the evaluation method applied.

The real effect P(t) contains the secondary tides effects and the local movements. In the measurements with horizontal pendulums, the local movements can dominate, and these movements can be the subject of the interest for example in the investigations of the local geodynamics.

The instrumental drift D(t) is determined by construction of the measuring instrument. The drift is an answer of the instrument to the non-real effects. The relative change between the gravitational vector and the surface point where the instrument is placed, is called real effect and all the rest - for example barometric effects and temperature change - are called non-real effects.

As a consequence the residual curve contains informations, therefore its investigation is a very important question. Some attempts were made in this respect, nevertheless we can not speak about a generally accepted in every detail worked out process. In this paper I should like to contribute with some ideas to the development of such a process.

The earth tides records, mainly got by horizontal pendulums have very often gaps, and we can trace these gaps back to technical problems which can not eliminated in the practice. Therefore, the evaluation method has to solve the problem of linking the data series. For this purpose different processes are used in the different methods. In case of the method of spectral analysis the predictive method is applied; at the methods based on the least square technique the gaps are built into the functional model. Of course, in the case of time series with gaps the residual curve will also be a time series with gaps. So, our starting situation is the following : we have a time series with gaps without previous informations, and we should like to evaluate this data series in order to separate the three parts mentioned previously.

Let our conditions be :

1) The curve contains a polynomial part (in the first approximation this is a linear form) wich describes the local movement and a part of the instrumental drift.

2) The other part of the curve can be described with periodical functions.

These conditions do not mean a strong limitation. To eliminate the first part a numerical derivation is made, that is a differential filter is applied :

$$R^{*}(t) = R\left(t + \frac{\Delta t}{2}\right) - R\left(t - \frac{\Delta t}{2}\right) \qquad (2)$$

The amplitude spectrum of this filter is shown in Figure 1. In this figure we can see the noise level of the original earth tide records. This figure is after Wenzel's work. In such a way the linear change is transformed into a constant shift, which will not disturbe the investigation of the periodical part. We can not use directly Fourier analysis for determining the amplitude spectrum of the periodical part, because the time series have gaps. We can make some predictive replacements, but we have not previous informations. So, it seems more usefull to apply some series of narrow ban filter.

These filters are constructed on the basis of least squares method and fitted to the time series. The way of filter construction is the following. Let be

$$R^{*}(t) = a \sin(\omega t + \ell\omega) \qquad (3)$$

from which by usually transformation :

$$R^{*}(t) = (a \cos \ell\omega) \sin \omega t + (a \sin \ell\omega) \cos \omega t = \xi \sin \omega t + \eta \cos \omega t$$

$$\xi^2 + \eta^2 = a^2$$

$$\operatorname{arctg} \frac{\eta}{\xi} = \ell\omega \qquad (4)$$

In matrix form :

$$\underset{(n1)}{\overline{R^{*}}} = \underset{(n,2)}{\underline{\underline{A}}} \; \underset{(2,1)}{\begin{matrix}\xi \\ \eta\end{matrix}} \qquad \underline{\underline{A}} = \begin{bmatrix} \sin \omega t_1 & \cos \omega t_1 \\ \sin \omega t_2 & \cos \omega t_2 \\ \vdots & \vdots \\ \sin \omega t_n & \cos \omega t_n \end{bmatrix} \qquad (5)$$

In this process there is no such a condition which requires time series without gaps.

Next step follows from the principle of the least squares technique :

$$\underset{(2,1)}{\begin{bmatrix}\xi \\ \eta\end{bmatrix}} = (\underset{(2,n)}{\underline{\underline{A}}^T} \; \underset{(n,2)}{\underline{\underline{A}}})^{-1} \; \underset{(2,n)}{\underline{\underline{A}}^T} \; \underset{(n,1)}{\underline{R}^{*}} = \underset{(2,n)}{\underline{\underline{F}}(\omega)} \; \underset{(n,1)}{\underline{R}^{*}} \qquad (6)$$

where $F(\omega)$ contains two filter series fitted the size of the time series with gaps $\underline{B}^{*}(t)$. The amplitude transfer function of $\underline{\underline{F}}(\omega)$ where determined for the different types of time series. We can see the schemes of these on Figure 2.

1) Time series of different lengths without gaps

2) Time series of equal lengths with different regular gaps, number of gaps is equal in each series

3) Regular time series of differential lengths with gaps of equal lengths, number of gaps is different in the series

4) Time series of equal lengths with gaps of irregular lengths.

5) Time series of different lengths with gaps of regular lengths.

6) Time series of different lengths with gaps of irregular lengths.

The number of all data in the different series was the same and filter frequency too. The spectra got we can see in Figure 3.1 - 3.6.

In the case by increasing the length of the time series, the selectivity of the filter was increased. In the second case the increase of the length of the gaps worsened the characteristic but not to such an extent than was caused by increasing of the number of the gaps in the third case. From the fourth and fifth case it is clear that a regular series with irregular gaps causes a worsen characteristic than an irregular series with regular gaps. The best transfer function can be got from an irregular series with irregular gaps. In the practice the last case is the most frequent.

Now let us investigate some residual curves of the earth tides records. For this purpose two parallel measurements made by horizontal pendulums type of TE and VM in Sopron and Graz were chosen. We recorded the EW component in parallel measurements. In addition to a longer EW period and a NS period has been investigated. The measurements were made in the frame of the scientific cooperation of the Technische Universität Graz, and the Geodetic and Geophysical Research Institute Sopron. The evaluation of the records and the calculation of the residual curves were made in Sopron with a small computer of type Hewlett-Packard 2100A. A detailed description of the evaluation method can be found in my previous paper in Acta Geodetica, Geophysica et Montanistica, Number 3, Tomus 8. The earth tides factors of the measurements are collected in Table 1.

Let us first investigate the derivated curve of the parallel records in Figure 4.1, 4.2, 5.1, 5.2. We can see a constant shift on the station Graz at both pendulums TE and VM. So, we can speak with great probability about a linear local tilt. This shift can not be seen on the station of Sopron. The spectra on the Figure 6.1, 6.2, 7.1, 7.2 seem to be white noises, significant identities cannot be proved.

Investigate the spectrum of the derivated residual curve of a long period in EW component in Figure 8., the spectrum is composed from four parts

1) long periodical part 0 - 10 °/hour

2) residual of earth tides 11 - 35 °/hour

3) waves of one third day period 35 - 50 °/hour

4) waves of one fourth day period 50 - 70 °/hour

In the foregoings EW component records were investigated, which theoretically cannot contain the long period part of earth tides. So, the long period parts were not of lunisolar origin. At the NS component record in Figure 9. the low frequency Mf wave appears since it was not contained in the functional model.

Summing up finally : our method seems to be usefull for the preliminary investigation of the residual curve, and this may be a good start for further detailed analyses.

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THREE DIFFERENT METHODS FOR TAKING IN ACCOUNT THE GAPS IN SPECTRAL ANALYSIS OF EARTH TIDES RECORDS

by

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ABSTRACT

A method of spectral analysis of Earth Tides records in which there could be gaps was presented in the 8th International Symposium on Earth Tides. In this meeting some improvements introduced in this method, together with another two different approachs to the same objective are given.

First method

The main features of this method are given in (1). If we have a function "f_k" defined in the set of abscissas (0,n-1), but we have lost the values that the function take in some subsets (gaps) of (0,n-1), then we define another function "f'_k" equal to "f_k" outside the gaps and equal to zero in the gaps. Now we define the gap function "u_k", equal to one outside the gaps and to zero in the gaps. This means that $f'_k = f_k\, u_k$ and Fourier transforming

$$F'_j = F_j \otimes U_j \quad \text{(where } \otimes \text{ means convolution)}$$

Now, let us recall an intermediate result of the method of analysis given in reference (1), a system of equations marked there with number (4) :

$$R_j = \sum_{g=1}^{G(i)} x_g\, Y_{jg} + E_j \qquad (4)$$

$$j = j_{min}(i), \ldots, j_{max}(i) \qquad i = 1, 2, 3$$

where : R_j is the Fourier transform of the readings

x_g (unknowns of the system of equations) is the complex amplitude factor for the group "g" ($x_g = A_g\, e^{iP_g}$)

Y_{jg} are the sum of the Fourier series of all waves of the group "g" (theoretical amplitude and phase of each wave are taken in account here)

G(i) is the number of wave groups of i-diurnal waves

E_j are the error associated to R_j

($j_{min}(i), \ldots, j_{max}(i)$) is the interval of harmonics in which is appreciable the amplitude of i-diurnal waves (see reference (1))

i = 1 for diurnal, = 2 for semidiurnal, = 3 for terdiurnal waves.

But, when there are gaps in the readings we cannot make use of (4) for obtaining the amplitude factor (A_g) and the phase difference (P_g) of each group because we do not know "R_j". Instead of "R_j" what we know is $R'_j = R_j \otimes U_j$. Then, we convolve both members of (4) with the transform of the convolution function, U_j, and due to the linearity of the convolution the following is obtained

$$R_j \circledast U_j = \sum_{g=1}^{g(i)} x_g Y_{jg} \circledast U_j + E_j \circledast U_j$$

this, with the notation used above, can be expressed in this way

$$R'_j = \sum_{g=1}^{G(i)} x_g Y'_{jg} + E'_j \qquad (6)$$

we see that before introducing in the system the compound wave Y_{jg}, we must convolve it with the gap function. But, in the program for performing this method in a computer, at the stage it was in (1), we only computed for Y_{jg} the harmonics in the interval $(j_{min}(i), \ldots, j_{max}(i))$ and, therefore for U_j were only needed the harmonics form $j=0$ to $j = j_{max} - j_{min}$. Then, the accuracy of $Y'_{jg} = Y_{jg} \circledast U_j$ is diminished and decreases still more as we approach to the ends of the interval of harmonics, because there we dispose of still less harmonics of Y_{jg} to compute the convolution. Anyway, this method gives good results (see the output of the computer in reference (1)) as long as the number of missed records is very small when compared with the total number of records.

When there are more readings lost (let us say 5 % or more) it can be seen that the accuracy of the method diminishes. This happens because now "U_j" decreases too slowly as "j" increases. The situation is improved when more harmonics of "U_j" are computed and seeing from which "j" the amplitude of U_j is less that one hundredth of U_0 (or any other appropriate fraction). Accordingly to this value of "j" more harmonics of Y_{jg} are calculated (following a algorithm designed by us that can be seen in the subroutine FOUDIS of the program). Now, the convolution of both, Y'_{jg}, for the same interval of harmonics as before, is more accurate and results are better. The remainder of the method can be seen in (1).

Second method

We are going to see now another approach for taking in account the gaps. The difference with the previous method is in the way of calculating the matrix Y'_{jg} of the system of equations (6).

Each column of Y'_{jg} is the sum, for all the waves of the group, of the Fourier transforms of the individual waves in the time domain with zeros assigned to the time intervals corresponding to gaps. Let us compute this

Fourier transform and for this let us consider an individual wave "y_q" with amplitude "A", phase "F" (for the time of the first reading) and number of cycles "c" for the whole period of the records, including the gaps. Then, its angular velocity change between two consecutive readings is $2\pi c/n$, being "n" the sum of the number of readings plus the lost ones. We dispose of records for the following "s" subsets of (0,n-1) : $(0,k_1),(k_2,k_3), \ldots, (k_{2s-2},n-1)$. Therefore the individual wave in the time domain is

$$y_q(k) = A \cos(2\pi ck/n + F) \quad \text{for} \quad k = (0,k_1), \ldots, (k_{2s-2},n-1)$$

$$y_q(k) = 0 \quad \text{for} \quad k = (k_1+1,\ k_2-1),\ (k_3+1,\ k_4-1),\ \ldots$$

in other words $y_q(k) = 0$ in the gaps. Now, for computing the Fourier transform of this function we do not need an FFT algorithm, because it can be done analytically :

$$Y_q(j) = \frac{1}{n} \sum_{k=0}^{n-1} y_q(k) \exp(-i2\pi jk/n) =$$

$$= \frac{1}{n} \sum_{p=1}^{s} \sum_{k=k_p}^{k'_p} A \cos(F + 2\pi ck/n) \exp(-i\, 2\pi jk/n)$$

calling $D = \pi/n$; expanding $\cos(2Dck + F) =$

$$= (\exp(i(2Dck+F)) + \exp(-i(2Dck+F)))/2$$

taking in account that the sum from k_p to k'_p will consist in the sum of two geometric progressions, after some operations the following result will be obtained

$$Y_q(j) = \frac{A\, e^{iF}}{2ni} \sum_{p=1}^{s} \frac{e^{iD(c-j)(2k'_p+1)} - e^{iD(c-j)(2k_p-1)}}{2 \sin(D(c-j))} +$$

$$+ \frac{A\, e^{-iF}}{2n(-i)} \sum_{p=1}^{s} \frac{e^{-iD(c+j)(2k'_p+1)} - e^{-iD(c+j)(2k_p-1)}}{2 \sin(D(c+j))} \qquad (7)$$

When programming the method it is interesting to note that not all the sines and cosines that appear in the last equation need to be calculated for each wave "q", for each harmonic "j" and for each subset "p". Time required for the execution of the program in the computer is shorted following this procedure : the complex numbers that appear in the numerator are split in products of this type :

$$e^{iDc(2k'_p+1)} \; e^{-iDj(2k'_p+1)}$$

the last term of the product is computed for each harmonic and each subset and is stored during the whole time of resolution of (6). The first term of the product is computed for each wave and for each subset and is stored temporally, while calculating the corresponding harmonics. The terms of (7) of the type exp(iDj) are computed only once for each harmonic and are stored during the resolution of (6) and the terms of the type exp (iF) and exp(icD) need to be calculated only once for each wave.

The operations done for computing $Y_j(q)$ are repeated for each wave of the group, thus obtaining Y'_{jg}. Then, same procedure as in reference (1) is used for working out the system of equations (6).

Third method

Let us see another way of analyzing Earth Tides records in which there are gaps. First of all, let us express the system (6) in matrix notation :

$$R' = Y' X + E \qquad (8)$$

In the three methods, for solving this system of equations we multiply both members of (8) by Y^+, transposed and conjugate of Y' : $Y^+R' = Y^+YX$. Let us call $B = Y^+R'$ which is a column matrix of G(i) elements and $S = Y^{+Y}$ which is a hermitical matrix of G(i) x G(i) elements. Then the system of normal equations is B = Sx.

The difference of the actual method when compared with the another two given above, is that now matrices "B" and "S" are calculated directly from the time domain and not from the Fourier domain as was done previously. Let us see some details of this calculation. Each reading r'_k is after drift correction

$$r'_k = \sum_{g=1}^{G} A^n \sum_{j=n_g}^{g+1^{-1}} A_j \cos (c_j DK + P_j + P_g) \qquad (9)$$

$$k = 0,1, \ldots, n-1 \qquad D = 2\pi/n$$

being "n" the number of reccrds we have plus the missed records. In the gaps r'_k = o. The only unknowns in (9) are the amplitude factor Ag and the phase difference "Pg" for each group of waves "g". The last equation is the same as

$$r'_k = \frac{1}{2} \sum_{g=1}^{G} (A_g e^{iPg} \sum_{j=n_g}^{n_{g+1}-1} A_j e^{i}(P_j + c_j Dk) +$$

$$+ A_g e^{-iPg} \sum_{j=n_g}^{n_{g+1}-1} A_j e^{-i}(P_j + c_j Dk))$$

let us call

$$x_g = Ae^{iP}g$$

$$z_{kg} = \sum_{j=n_g}^{n_{g+1}-1} A_j e^{i} (P_j + c_j Dk)$$

Let us include the factor 1/2 in r_k : $r_k \to 2r_k$.

Then

$$r'_k = \sum_{g=1}^{G} (x_g z_{kg} + x^*_g z^*_{kg})$$

with z_{kg} = o when "k" belongs to a gap.

The last equation in matrix notation r' = ZX; multipling both members by Z^+ : $Z^+ r' = Z^+ ZX$

$$Z^+Z = \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}$$

where A, B are submatrices of G*G elements. Each of these last ones are

$$A_{mq} = \sum_k Z^*_{km} Z_{kq}$$

$$B_{mq} = \sum_k Z^*_{km} Z^*_{kq}$$

$$A_{mq} = \sum_k \left(\sum_j A_j e^{-i(P_j + c_j Dk)}\right)\left(\sum_d A_d e^{i(P_d + c_d Dk)}\right) =$$

$$= \sum_j \sum_d \left(A_j A_d e^{i(P_d - P_j)} \sum_k e^{i\,Dk(c_d - c_j)}\right)$$

$$B_{mq} = \sum_j \sum_d \left(A_j A_d e^{-i(P_d + P_j)} \sum_k e^{-iDk(c_d + c_j)}\right)$$

$j = n_m, \ldots, n_{m+1}-1 \qquad d = n_q, \ldots, n_{q+1}-1$

$m = 1, \ldots, G \qquad q = 1, \ldots, G$

The matrix Z^+r' and X are a column matrices of 2G elements $Z^+r' = \begin{pmatrix} C \\ C^* \end{pmatrix}$ being C and E a matrices of G elements

$$X = \begin{pmatrix} E \\ E^* \end{pmatrix}$$

$$C_m = \sum_k Z^*_{km} r'_k \qquad (10)$$

$$Z^+r' = \begin{pmatrix} C \\ C^* \end{pmatrix} = \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} E \\ E^* \end{pmatrix} = Z^+ZX \qquad (11)$$

But, in the expression obtained for B_{mq}, when evaluating the factor corresponding to the sum for all "K" it is seen that it is practically zero as compared to the terms of A_{mq}. This can be realized also in the frequency domain where the sum will now be for all harmonics of two functions whose cross power spectra is practically zero. Then, the submatrix $B \simeq 0$.

In the submatrix "A", for those elements A_{mq} where "m" correspond to a certain i-diurnal wave group and "q" correspond to another i'-diurnal wave group, for that same argument as in the previous paragraph $A_{mq} \simeq 0$. Then, only those elements A_{mq} for which "m" and "q" correspond to the same i-diurnal character A_{mq} may be different from zero.

From the above, system of equations (11) is now

$$C = AE$$

and can be worked out separately for diurnal, semidiurnal and terdiurnal waves.

For obtaining the elements A_{mq} it is interesting to note that the sum for all "K" can be evaluated analytically, for subsets of "k" between two gaps. Using the same notation

$$\sum_{k=k_p}^{k'_p} e^{iDk(c_d - c_j)} = \frac{e^{iD(c_d - c_j)(K'_p + 1)} - e^{iD(c_d - c_j)k_p}}{e^{iD(c_d - c_j)} - 1} \quad (12)$$

Some considerations as in method 2 are taken in account for storing intermediate results and saving time for the program in the computer. For elements of the diagonal A_{mm} only half operations are needed because they are real and here it may occur that d=j, then L'Hopital rule is used for calculating (12). The elements C_m of (10) :

$$C_m = \sum_k Z^*_{km} r'_k = \sum_k (\sum_j A_j e^{-i(P_j + c_j Dk)}) r'_k =$$

$$= \sum_j A_j e^{-iP_j} (\sum_k r'_k e^{-ic_j Dk})$$

where the sum inside the parethesis is evaluated with the help of the Gertz algorithm (reference (2)). The remainder of the method is the same in reference (1).

Comparison of the three methods

For comparing the methods three factors are taken in account : quality of the results, memory needed for allocating the program in the computer and time spent in the execution.

Method 3 is the one which gives worst results, but takes shorter time for the execution. The mean square errors of the results given by the first two methods are more or less the same (but less than the third method). Execution of method 2 is faster than method 1 when there is a small number of gaps. But, when there is a large number of gaps (although the same total time of gaps as previously) method 1 is faster. The memory needed is more or less the same for the three methods when the overlay option is used.

References

(1) P. Sukhwani and R. Vieira, 1977, "Spectral analysis of Earth Tides", paper presented in the 8th International Symposium on Earth Tides.

(2) G. Goertzel, 1958, "An algorithm for the evaluation of finite trigonometric series", Am. Math. Month. 65, 34.

THE ANALYSIS OF TIDAL CONSTITUENTS BY SELECTIVE FILTERING*)

by
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Abstract:
The computation of amplitude factor γ and phase difference φ leads, in the case of records with high noise level, to contradictory results if the model adjustment, deterministic or statistical, is not given. This is especially the case with records by tiltmeters (horizontal or vertical). Therefore, a purely stochastic model seems preferable and thus the application of statistical computing methods (co-variance spectral analysis and optimum filtering).

The method developed can be described as spike-filtering of the tidal frequencies as follows: over the cross co-variants between a test oscillation of the tidal frequency to be analyzed (as "filter operator") and the time series of the measured data and theoretical tides, relative regression coefficients and phase differences are being determined. A comparison of these reveals the required tidal parameters. This procedure meets the requirements of statistical computing methods. The errors are given by standard deviation and confidence limits.

The results for low-noise records (gravimeter) are in very good agreement with the results gained on the basis of a deterministic model; for tiltmeter records, results are more coherent which means parameters that are less noise-dependent.

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*) This paper has been presented at the 8th Int. Symp. on Earth Tides, Bonn 1977; therefore only an abstract is given here.

THE MAXIMUM ENTROPY SPECTRAL METHOD
- A SHORT REVIEW -

by
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1. The concept of maximum entropy

The common methods of power spectral analysis make rather unrealistic assumptions about the extension of the data: The periodogram assumes a periodic extension, whereas the autocorrelation approach assumes zero extension. Especially in the case of very short records (compared to signal period) this is of great disadvantage.

Especially for this case the maximum entropy method (MEM) has been developed: This type of analysis makes no assumptions about the data outside the interval. It may be discribed as a data adaptive method that adapts itself to the sample of the process under study in such a way that the spectral estimate displays maximum entropy or maximum information content for the sample, while still fully agreeing with the available data.

In general the term "maximum entropy" may be interpreted as maximum degrees of freedom of the sample after applying the prediction operator determined by this method.

2. Short outline of the method

The maximum entropy method uses WIENER optimum filter theory to design a prediction filter which will whiten the input time series. From the whitened output power and the response of the prediction filter it is possible to compute the input power spectrum. This leads to an estimate with a very high resolution since the method uses the available lags in the autocorrelation function without smoothing and makes a non-zero estimate or prediction of the autocorrelation function beyond those which can be calculated directly from the data. In this way the amount of information is maximized from the available time series.

Here only a short discription is given; for details see e.g. KANASEWICH (1973) or ULRYCH & BISHOP (1975).

If the prediction distance is one unit of sampled time, the optimum filter theory of WIENER (1949) leads to the matrix equation for the prediction operator W

$$\begin{bmatrix} a_o & a_1 & \cdots & a_{N-1} \\ a_1 & a_o & \cdots & a_{N-2} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ a_{N-1} & a_{N-2} & \cdots & a_o \end{bmatrix} \cdot \begin{bmatrix} W_o \\ W_1 \\ \cdot \\ \cdot \\ \cdot \\ W_{N-1} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ a_N \end{bmatrix} \qquad (1)$$

Where $a_o \ldots a_{N-1}$ represent the autocorrelation of the input time series.

Following the technique of PEACOCK & TREITEL (1969) this matrix equation is augmented in the way that the right-hand side vanishes, and a new equation is added. This leads to

$$\begin{bmatrix} a_o & a_1 & \cdots & a_{N-1} & a_N \\ a_1 & a_o & \cdots & a_{N-2} & a_{N-1} \\ \vdots & \vdots & & \vdots & \vdots \\ a_N \cdots & a_{N-1} & \cdots & a_1 & a_o \end{bmatrix} \begin{bmatrix} 1 \\ \Gamma_2 \\ \vdots \\ \Gamma_{N+1} \end{bmatrix} = \begin{bmatrix} P_{N+1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (2)$$

where

$$\Gamma = \begin{bmatrix} 1 \\ \Gamma_2 \\ \vdots \\ \Gamma_{N+1} \end{bmatrix} \equiv \begin{bmatrix} 1 \\ -W_o \\ \vdots \\ -W_{N-1} \end{bmatrix}$$

which is BURGS's (1967) equation for the prediction error filter at a distance of one time unit.

Since the input wavelet consists of N terms and the cross-correlation contains only the non zero term (right-hand side of (2)) this implies that the prediction operator shortens the output wavelet to a spike in which P_{N+1} is the mean square error or the output power. In the case of the autocorrelation function being a spike the time series involved contains maximum information.

Therefore Γ may be considered as a set of prediction filter weights which, when convolved with the input data, will generate a white noise series. The elements will be uncorrelated with each other, and the filter will have created the greates destruction of entropy which is possible.

The input power spectrum may be obtained by correcting the output power for the response of the filter. In the frequency domain this is in BURG's notation the maximum entropy of power, P_E:

$$P_E(f) = \frac{P_{N+1} \;/\; f_{NY}}{\left| 1 + \sum_{n=1}^{N} \Gamma_{N+1}\, e^{-i2\pi\, fn\Delta t} \right|^2} \quad (3)$$

Where $f_N = \frac{1}{2\Delta t}$ is the Nyquist frequency. The filter coefficients of Γ are obtained from eg.(2), in which the autocorrelation matrix is an N+1 by N+1 Toeplitzmatrix that must be semi-positive definite and its determinant non-negative. So the power spectrum will remain positive at all points. Eg.(2) can be solved by the LEVINSON-Algorithm.

Eg.(3) shows that $P_E(f)$ is maximum, when the spectrum of Γ_{N+1} is minimum. This occurs at the frequencies occupied by the signal, for the prediction error operator must remove the amplitudes at these frequencies to whiten the time series. This estimate of power does not give any information about the phase.

3. Problems and examples

The most important problem lies in determining the optimum length of the prediction error filter:
CHEN & STEGEN (1974) have found that a too short operator does not give a satisfactory resolution: Fig. 1 shows the peak being broad and the side lobes staying high in this case. When the number of terms is increased the resolution improves rapidly. But if the operator gets too long, the filter amplifies the effect of the noise and produces several spectral peaks with comparable density.

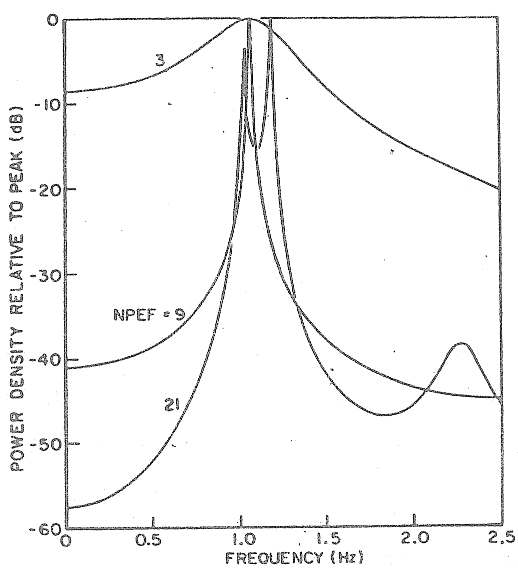

Fig. 1: Maximum entropy power spectra obtained from 24 data points sampled with $\Delta t=0.05$ s from a 1-Hz zero initial phase sine wave, superimposed with 5% white noise. The NPEF used for each spectrum is indicated (from CHEN et al. 1974).

Tab. 1: Quality of maximum entropy power spectrum of 24 data points superimposed with different intensities of white noise (from CHEN et al. 1974)

NPEF	Noise Level 5%	10%	20%	40%	80%	160%
3	No	No	No	No	No	No
6	Yes	Yes	No	No	No	No
9	Yes	Yes	Yes	No	No	No
12	Yes	Yes	Yes	Yes	No	No
15	Yes	Yes	Yes	Yes	No	No
18	No	No	Yes	Yes	Yes	No
21	No	No	Yes	No	No	No
24	No	No	No	No	No	No

Data points were sampled from a 1-Hz zero initial phase sine wave with $\Delta t=0.05$ s. Yes means that the spectrum is acceptable, and no means that it is unacceptable.

In addition Tab. 1 gives the influence at different noise intensity compared to filter length. Secondly, there is an effect of the number of data samples and the initial phase, especially when the data set covers less than one cycle of the sine wave. These effects influence the location of a spectral peak to a large extent (Figs. 2,3).

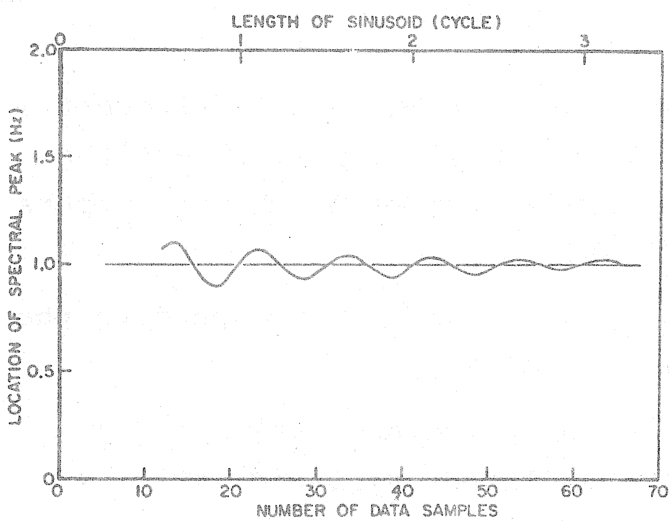

Fig. 2: The LOSP for various lengths of 1-Hz sinusoids. The sampling interval is 0.05 s. White noise background is 5%. The straight line at 1 Hz is the ideal LOSP (from CHEN et al. 1974).

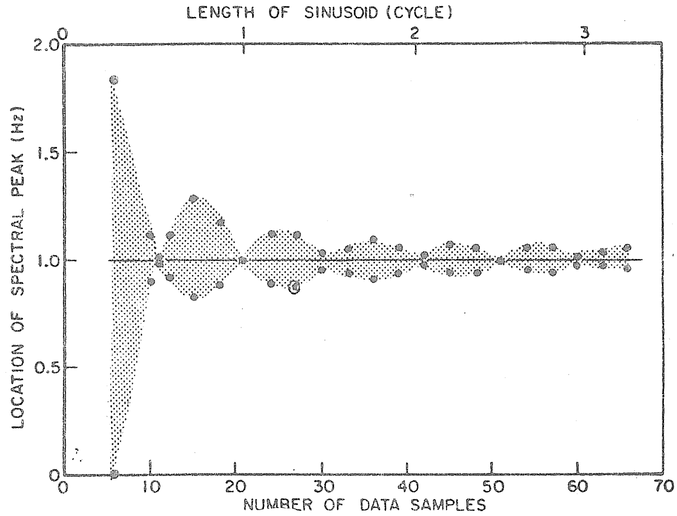

Fig. 3: Range of oscillation of the LOSP for all possible initial phases of various lengths of 1-Hz sinusoids. The sampling interval is 0.05 s (from CHEN et al. 1974).

A solution of the problem of determining the optimum length of the operator is given by the AKAIKE Final Prediction Error (FPE) Criterion (ULRYCH & BISHOP 1975):

$$(FPE)_N = \frac{M+N}{M-N} \quad S_N^2 \Longrightarrow \min \tag{4}$$

where N - length of prediction error filter
M - samples of time series
S_N^2 - mean square prediction error

N is chosen for $(FPE)_N$ being a minimum. But there are different results concerning the minimum: ULRYCH et al. (1975) accept the first relative minimum for the determination of N, whereas others take the absolute one (eg. HERBST 1976).

The square of the true amplitude is supposed to correspond to the integration of the spectral peak (LACOSS 1971).

ULRYCH et al. (1975) give an illustrative example analysing a generated time series of three equiamplitude sinusoids (Fig. 4 to 9): Fig. 4 gives the generated time series, and fig. 5 gives the sample of this series to be analysed. The result of the periodogram of this sample is contained in fig. 6, the maximum entropy spectrum is shown in fig. 7, the advantage of which is

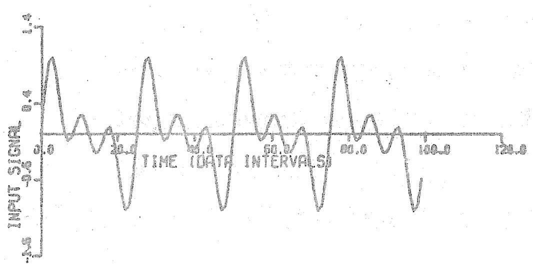

Fig. 4: Time series consisting of three quiamplitude sinusoids with frequencies equal to 0.04, 0.08, and 0.12 Hz (from ULRYCH et al. 1975).

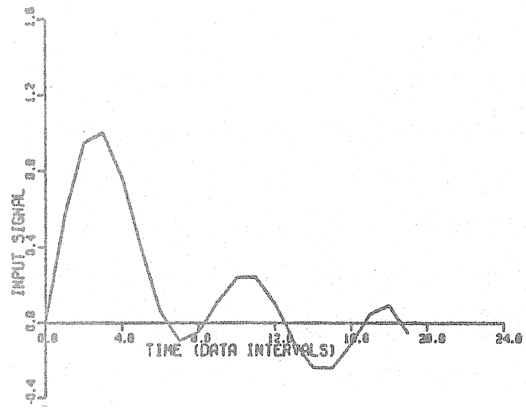

Fig. 5: A 20 sample point window of the trace shown in Figure 4 (from ULRYCH et al. 1975).

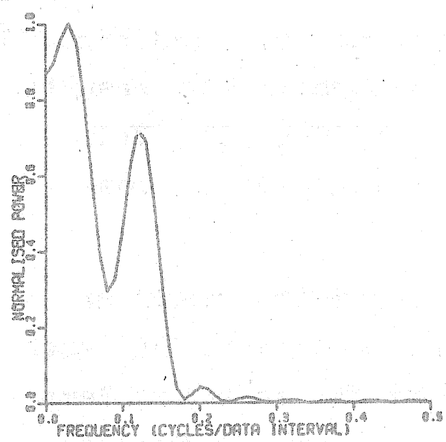

Fig. 6: Periodogram of the signal shown in Figure 5 (from ULRYCH et al. 1975).

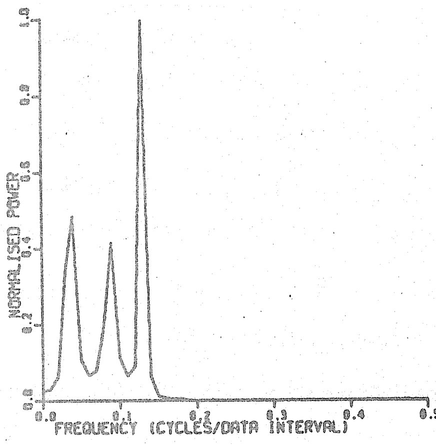

Fig. 7: Burg power spectrum of the signal shown in Figure 5 (from ULRYCH et al. 1975).

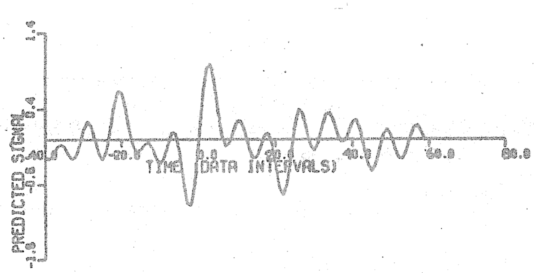

Fig. 8: Signal resulting from the forward and backward prediction of the signal shown in Figure 5 (from ULRYCH et al. 1975).

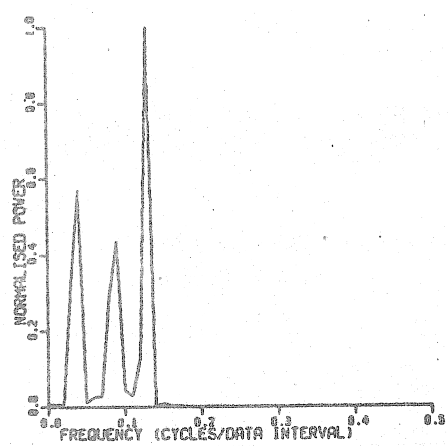

Fig. 9: Periodogram of the predicted signal (from ULRYCH et al. 1975).

obvious. Figs. 8 and 9 give much information on the properties of the maximum entropy method: The predicted signal differs quite a lot from the original series, and the periodogram of this predicted series is quite similar to the Burg power spectrum, the prediction time being about four times the sampled data. The integrated amplitude ratios are (normalized) 0.85: 0.70 : 1.00; the frequency shifts being less than 5%.

This example gives an impression of the confidence of the calculated amplitudes. An other example is given by HERBST (1976), who analysed the long-term drift superimposed on tilt measurements identifying this drift as coming from ground temperature (Fig. 10, 11, 12). He also shows that there is no resolution at all if the linear drift has not been removed before applying MEM. Additionally HERBST tried to determine the amplitudes and compared the results from MEM to correlation and regression analysis (Tab. 2). This comparison shows a fairly good agreement, but there is no confidence about the MEM-amplitudes.

Applying tilts to tidal records no acceptable results could be obtained: YARAMANCI (pers. communication) and the author did such analyses independently from each other and had the same problems in determining the amplitudes. Especially in the case of frequencies close together it is nearly impossible to make the integration.

Tab. 2: Results of the determination of amplitudes and phases (from HERBST 1976)

Component		Amplitude [10^{-3}''] from MEM	P1/P3	Amplitude [10^{-3}''] from Regressions-analysis	P1/P3	Phase [d]
EW	P1	24.5	1:5.1	31.6± 7.3	1:5.4	28
	P3	124.8		170.6± 26.7		10
NS	P1	25.8	1:4.4	21.5± 9.8	1:8.1	30
	P3	113.3		173.5± 15.1		80

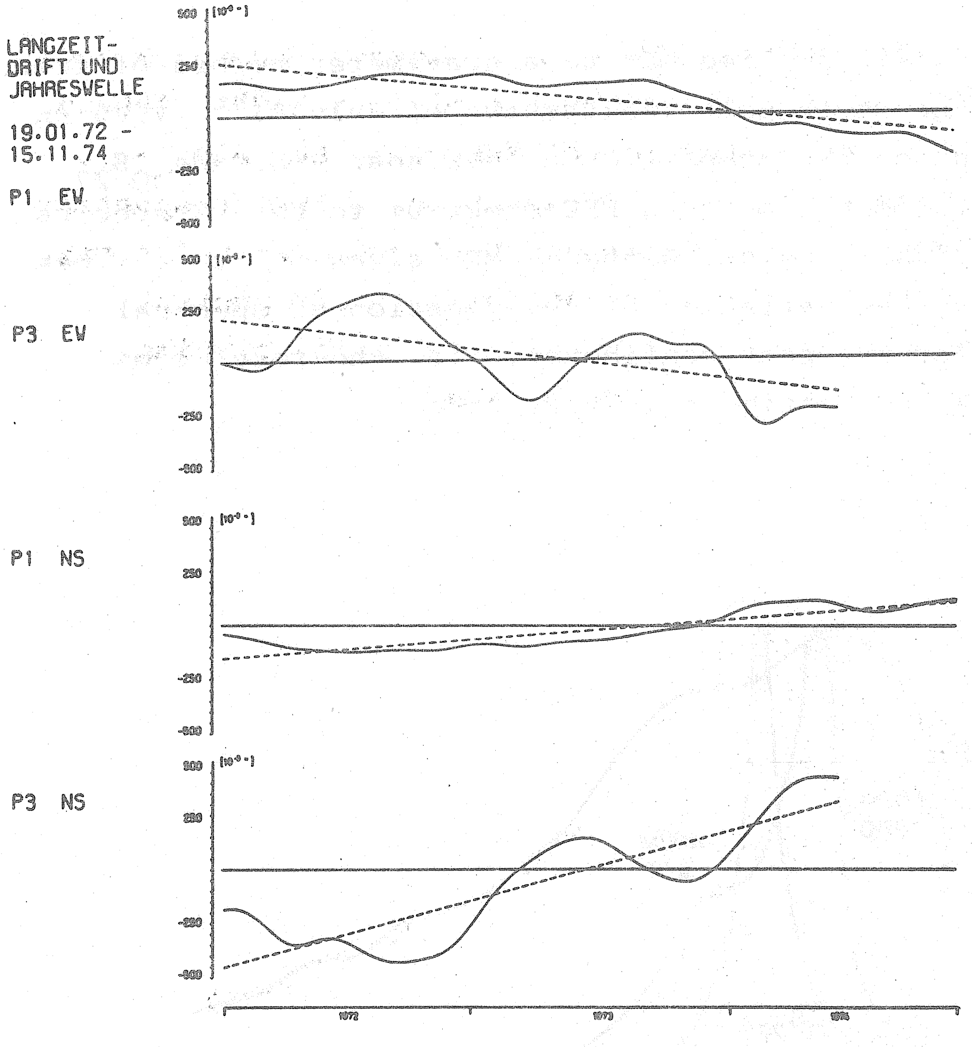

Fig. 10: Tilts with periods longer than 120 days recorded by two ASKANIA borhole tiltmeters P1 and P3 (depth 30 m and 15 m respectively) in EW and NS direction, --- linear drift (from FLACH et al. 1975).

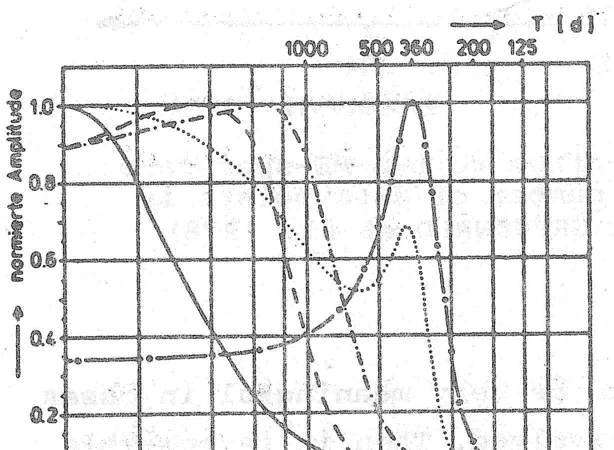

Fig. 11: ME spectra of the long-period tilts and the ground temperature recorded 1 m below surface (-o-o- ground temperature; ----EW-P1; ——NS-P1; .....EW-P3; -·-·-NS-P3)

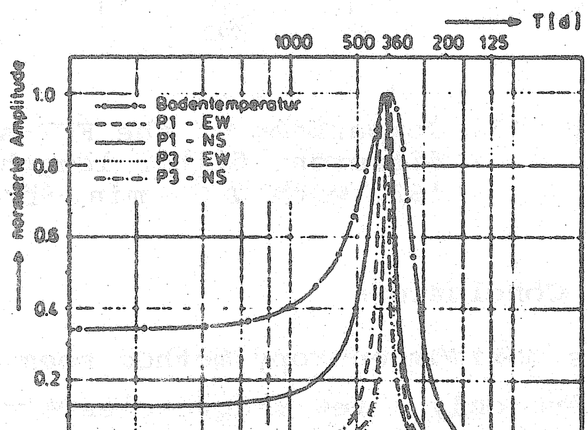

Fig. 12: ME spectra of the long-period tilts after lrift removed and ground temperat ıre.

(from HERBST 1976)

GRÜNEWALD et al. (1978) applied MEM to a gravimeter record of free oscillations at the south pole (earthquake of July 31st, 1970 in Columbia) and studied the properties of MEM near the mode $_0S_{27}$. Fig. 13 gives the comparison of a FFT-spectrum to two ME-spectra with different filter lengths. Although MEM gives a 2 to 4 times better resolution, the variation of the location of spectral peak is a proof for the poor confidence, i.e. the resolution given by MEM is an overestimate in this case.

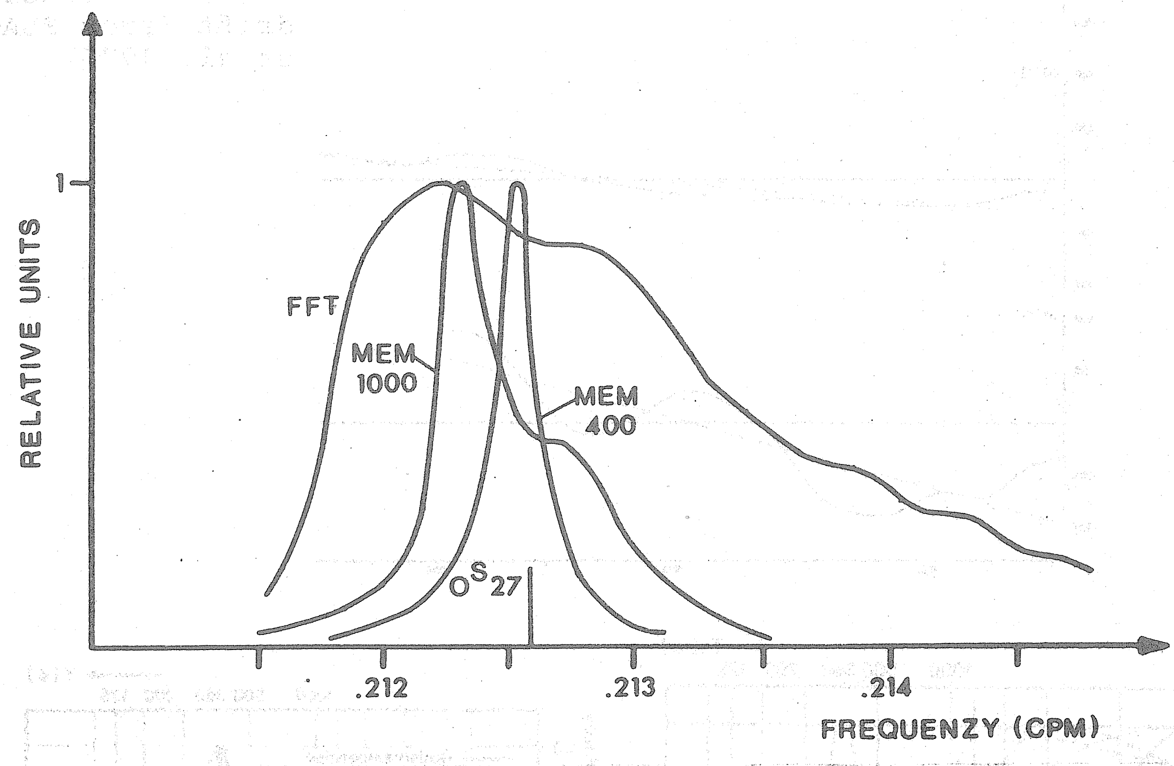

Fig. 13: Comparison of the FFT-spectrum to two ME-spectra of different filter length; number of data points is 1632 with $\Delta t = 1$ min. (from GRÜNEWALD et al. 1978).

4. Conclusions

The maximum entropy method seems to be very meaningful in cases where only a few frequencies are involved. Then it is possible to make experiments concerning the different lengths of the prediction error operator and different data sets to select the most plausible result.

Because of the uncertainty in the determination of the amplitude and the lack of information about the phase it is not a method to apply to tidal records like other spectral or least squares methods. MEM produces the information which is already known, the frequency, but not the information which is needed, namely amplitude and phase.

However tidal records are more and more used to study non-tidal signals, too. Therefore the maximum entropy method has to be considered as very important for such purposes.

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Data standardisation in tidal research

B. Ducarme

(ICET)

1. NATURE OF DATA

Hourly readings expressed in physical units: eg. mseca or µgal. For data expressed in other units a calibration table must be given in accordance. The data concerning associated phenomena (e. g. pressure or temperature) will be given in hourly readings and physical units.

2. PRESENTATION OF DATA

2.1 International Format

An International format has been adopted for more than ten years for the punched cards as follows

col 1
- 0: gravity meters
- 1: clinometers (NS component)
- 2: clinometers (EW component)
- 3: dilatometers
- 4: extensometers
- 5: clinometers not registrating along the principal directions } proposition
- 6: point positionning } proposition
- 7: chord values } proposition
- 8: oceanic tides } proposition
- 9: atmospheric and hydrological parameters

col 2-4 instrument number (numeric)

col 5-8 station number (integer value)

col 9-13 reduced julian date (integer part) (optional or blanks)

col 14-15 year

col 16-17 month

col 18-19 day

col 20 hour of the first reading of the card
0 = 00 hour UT
2 = 12 hour UT

col 21-25 first reading

col 26-30 second reading
....
....

col 76-80 twelfth reading

For convenience the readings are generally expressed as integers in tenth of the physical units for gravimeters and clinometers. An interruption in the data is marked by one special card with 9999 in col 1-4, the rest being left blank. In incomplete cards each missing reading will be replaced by 99999. The end of the data set for a given instrument in a given station is marked by two cards with 9999 in col 1-4, the rest being left blank.

2.2 Data on Magnetic Tape

A provisional convention has been proposed at the last meeting in Bonn (september 1977).

All the data concerning a specific instrument in a specific station will be written on the same file with records of 80 characters (card image) unblocked, under format control, without label, ASCII code.

The record sequence will be as follows

2.21 Title records or Heading:

This record will mention all characteristics of one station and one instrument needed for the data analysis and the presentation of the results. They will also give a calibration table when needed. For associated meteorological or hydrological data a clear description must be given.

These records will be formatted in A format. The end of the heading is given by a stop card with 9999 in column 1-4, the rest being left blank.

2.22 Data from the first instrument

The records will be written according to the card International format (see § 2.1).

2.23 Associated data

same presentation and same gaps

2.24 End of the tape - two end of file marks.

Some practical problems associated to the tidal analysis

P. Melchior, B. Ducarme

(ICET)

1. The clinometric results should be calculated in their real azimut. Practical formula are proposed in Ducarme (1975).
   Projection on NS and EW components are strictly valid only if observations have been made following two different azimuts (Chojnicki T., 1975).
2. The grouping of the waves is generally made according to Venedikov 1966 (table 1).
   A more refined grouping is proposed here (table 2) in order to correctly evaluate the main waves.
3. No standardisation of the extensometric results has been proposed yet. Referring to Melchior and Ducarme (1976) we propose to include systematically the ℓ/h ratio in the results of horizontal strainmeters (table 3) when they are not in a NS or EW exact azimut.

TABLE 1

CARTWRIGHT - TAYLFR - EDDFN POTENTIAL /484 WAVES/
CLASSICAL SEPARATION OF THE GROUPS FOLLOWING VENEDIKOV

ONE MONTH SEPEPATION

197	DIURNAL WAVES	
62	115.755-139.455	Q1
26	143.535-149.355	O1
22	152.656-158.464	M1
33	161.557-168.554	P1S1K1
22	172.656-177.465	J1
32	181.755-195.565	OO1

150	SFMI-DIURNAL WAVES	
39	215.955-23X.354	2N2
24	243.635-248.454	N2
26	252.756-258.554	M2
14	262.656-265.675	L2
47	267.455-295.585	S2K2

SIX MONTH SFPARATION

197	DIURNAL WAVES	
62	115.755-139.455	Q1
26	143.535-149.355	O1
32	152.656-158.464	M1
10	161.557-163.765	P1
23	164.554-168.554	S1K1
22	172.656-177.465	J1
32	181.755-195.565	OO1

150	SEMI-DIURNAL WAVES	
39	215.955-23X.354	2N2
24	243.635-248.454	N2
26	252.756-258.554	M2
14	262.656-265.675	L2
9	267.455-273.755	S2
38	274.554-295.585	K2
16	TER-DIURNAL WAVES	
16	335.755-375.575	M3

ONE YEAR SEPARATION

197	DIURNAL WAVES	
62	115.755-139.455	Q1
26	143.535-149.355	O1
32	152.656-158.464	M1
10	161.557-163.765	P1
3	164.554-164.566	S1
20	165.345-168.554	K1
22	172.656-177.465	J1
32	181.755-195.565	OO1

150	SEMI-DIURNAL WAVES	
39	215.955-23X.354	2N2
24	243.635-248.454	N2
26	252.756-258.554	M2
14	262.656-265.675	L2
9	267.455-273.755	S2
38	274.554-295.585	K2
16	TER-DIURNAL WAVFS	
16	335.755-375.575	M3

FINE STRUCTURE

197	DIURNAL WAVES	
62	115.755-139.455	Q1
26	143.535-149.355	O1
32	152.656-158.464	M1
3	161.557-162.556	PI1
7	163.535-163.765	P1
3	164.554-164.566	S1
11	165.345-165.665	K1
2	166.554-166.564	PSI1
7	167.355-168.554	PHI1
22	172.656-177.465	J1
32	181.755-195.565	OO1

150	SFMI-DIURNAL WAVES	
39	215.955-23X.354	2N2
24	243.635-248.454	N2
26	252.756-258.554	M2
14	262.656-265.675	L2
9	267.455-273.755	S2
38	274.554-295.585	K2
16	TER-DIURNAL WAVES	
16	335.755-375.575	M3

TABLE 2

CARTWRIGHT - TAYLER - EDDEN POTENTIAL /484 WAVES/
MAXIMUM SEPARATION FOR THE GROUPS

ONE MONTH SEPARATION

197	DIURNAL WAVES	
11	115.755-11X.454	SIGMQ1
21	124.756-129.566	SIGMA1
30	133.635-139.455	O1
26	143.535-149.355	O1
22	152.656-158.464	M1
33	161.557-168.554	P1S1K1
22	172.656-177.465	J1
18	181.755-186.554	OO1
14	191.655-195.565	NU1

150	SEMI-DIURNAL WAVES	
19	215.955-22X.454	EPS2
20	233.955-23X.354	2N2
24	243.635-248.454	N2
26	252.756-258.554	M2
14	262.656-265.675	L2
21	267.455-277.555	S2K2
16	282.656-285.655	ETA2
11	292.556-295.585	2K2

SIX MONTHS SEPARATION

197	DIURNAL WAVES	
11	115.755-11X.454	SIGMQ1
10	124.756-126.754	2Q1
11	127.455-129.565	SIGMA1
20	133.635-136.654	Q1
10	137.455-139.455	RO1
16	143.535-145.765	Q1
10	146.544-149.355	TAU1
15	152.656-155.675	NO1
7	156.555-158.464	KI1
10	161.557-163.765	P1
23	164.554-168.554	S1K1
8	172.656-174.555	TETA1
14	175.445-177.465	J1
7	181.755-183.655	SO1
11	184.554-186.554	OO1
14	191.655-195.565	NU1

150	SEMI-DIURNAL WAVES	
19	215.955-22X.454	EPS2
20	233.955-23X.354	2N2
24	243.635-248.454	N2
26	252.756-258.554	M2
14	262.656-265.675	L2
9	267.455-273.755	S2
12	274.554-277.555	K2
15	282.656-285.655	ETA2
11	292.556-215.585	2K2
16	TER-DIURNAL WAVES	
16	337.555-375.575	M3

ONE YEAR SEPARATION

197	DIURNAL WAVES	
11	115.755-11X.454	SIGMQ1
10	124.756-176.754	2Q1
11	127.455-129.565	SIGMA1
20	133.635-136.654	Q1
10	137.435-139.455	RO1
16	153.535-145.765	O1
10	146.544-149.355	TAU1
15	152.656-155.675	NO1
7	156.555-158.464	KI1
10	161.557-163.765	P1
3	164.554-164.566	S1
20	165.345-168.554	K1
8	172.656-174.555	TETA1
14	175.445-177.465	J1
7	181.755-183.655	SO1
11	184.554-186.554	OO1
14	191.655-195.565	NU1

150	SEMI-DIURNAL WAVES	
19	215.955-22X.454	EPS2
10	233.955-236.754	2N2
10	237.445-23X.354	MU2
13	243.635-245.755	N2
11	246.456-248.454	NU2
26	252.756-258.554	M2
5	262.656-264.555	LAMB2
9	265.445-265.675	L2
5	267.455-272.556	T2
4	273.545-273.755	S2
12	274.554-277.555	K2
15	282.656-285.655	ETA2
11	292.556-295.585	2K2
16	TER-DIURNAL WAVES	
16	337.555-375.575	M3

FINE STRUCTURE

197	DIURNAL WAVES	
11	115.755-11X.454	SIGMQ1
10	124.756-126.754	2Q1
11	127.455-129.565	SIGMA1
20	133.635-136.654	O1
10	137.435-139.455	RO1
16	143.535-145.765	O1
10	146.544-149.355	TAU1
15	152.656-155.675	NO1
7	156.555-158.464	KI1
3	161.557-162.556	PI1
7	163.535-163.765	P1
3	164.554-164.566	S1
11	165.345-165.665	K1
2	166.554-166.564	PSI1
7	167.355-168.554	PHI1
8	172.656-174.555	TETA1
14	175.445-177.465	J1
7	181.755-183.655	SO1
11	184.554-186.554	OO1
14	191.655-195.565	NU1

150	SEMI-DIURNAL WAVES	
19	215.955-22X.454	EPS2
10	233.955-236.754	2N2
10	237.445-23X.354	MU2
13	243.635-245.755	N2
11	246.456-248.454	NU2
26	252.756-258.554	M2
5	262.656-264.555	LAMB2
9	265.445-265.675	L2
5	267.455-272.556	T2
4	273.545-273.755	S2
12	274.554-277.555	K2
15	282.656-285.655	ETA2
11	292.556-295.585	2K2
16	TER-DIURNAL WAVES	
5	[illegible]35.755-347.455	MQ3
11	353.755-375.575	M3

TABLE 3

STATION 0254 WALFERDANGE EXTENSOMETRE HORIZONTAL GRAND DUCHE DE LUXEMBOURG

LABORATOIRE SOUTERRAIN DE GEODYNAMIQUE J.FLICK
MINE DE GYPSE
49 40 N 06 09 E H 295 M P 75 M D 270KM
BORD NE DU BASSIN DE PARIS. TRIAS /KEUPER/.
GISEMENT DE GYPSE DANS MARNES FAILLES A PROXIMITE
EXTENSOMETRE SUPER INVAR OZAWA / CAPTEUR CAPACITIF VAN RUYMBEKE / EAM
LONGUEUR 11M95 AZIMUT 37 80 NE
INSTALLATION P.MELCHIOR/A.VANDEWINKEL/M.VAN RUYMBEKE
MAINTENANCE J.FLICK

METHODE DES MOINDRES CARRES / FILTRES VENEDIKOV / LECTURES HORAIRES
POTENTIEL CARTWRIGHT TAYLER EDDEN / DEVELOPPEMENT COMPLET
CALCUL CENTRE INTERNATIONAL DES MAREES TERRESTRES /FAGS/ BRUXELLES
ORDINATEUR UNIVAC 7440/ SYSTEME BS1000 STANDARD 78/ 3/29
AZIMUT-37.80S-E

OZ12	77 7 31	77 8 24	77 8 26	77 8 30	77 9 11	77 9 13
OZ12	77 9 18	77 9 22	77 9 26	77 10 2	77 10 12	77 10 12
OZ12	77 10 21	77 11 22	77 11 25	77 12 9	77 12 14	77 12 22
OZ12	77 12 25	78 1 16	78 1 20	78 1 24	78 1 27	78 1 29
OZ12	78 2 1	78 2 1	78 2 4	78 2 26		

NOMBRE TOTAL DE JOURS 172 4128 LECTURES

GROUPE	SYMBOLE	AMPLITUDE EPOQUE CENTRALE	PHASE	FACT.AMPL.	EQM	DEPHASAGE	EQM	AMPLITUDE MOYENNE
1- 62	O1	0.8062	215.71	0.6707	0.1082	6.78	9.25	0.5940
63- 88	O1	3.6453	213.56	0.7909	0.0207	7.68	1.50	3.6402
89-110	M1	0.2413	182.40	0.6578	0.2743	16.65	23.89	0.2253
111-120	P1	1.9917	253.77	0.7270	0.0352	8.96	2.76	1.9655
121-143	S1K1	4.4879	180.57	0.6390	0.0137	12.31	1.23	4.4789
144-165	J1	0.3798	150.13	0.9686	0.2573	-13.79	15.20	0.3517
166-197	OO1	0.0884	251.71	0.7360	0.8789	-21.02	68.26	0.0754

GROUPE	SYMBOLE	AMPLITUDE	PHASE	L/H
MODELE THEORIQUE FARRELL		14.8552	17.38	0.137
1- 62	O1	9.9632	24.16	0.164
63- 88	O1	11.7491	25.06	0.167
89-110	M1	9.7715	34.03	0.195
111-120	P1	10.7997	26.34	0.172
121-143	S1K1	9.4929	29.68	0.182
144-165	J1	14.3885	3.58	0.042
166-197	OO1	10.9328	-3.64	-0.060

ERREUR Q.M. D 3.823008 E-9

198-236	2N2	0.4235	299.77	0.8174	0.0990	1.98	6.93	0.2712
237-260	N2	1.4293	311.56	0.7821	0.0204	-1.41	1.49	1.3319
261-286	M2	6.7454	304.92	0.7660	0.0039	-2.70	0.29	6.7565
287-300	L2	0.2956	132.80	0.8346	0.1022	9.04	7.02	0.2716
301-309	S2	3.1613	338.73	0.7702	0.0083	-2.69	0.62	3.1165
310-347	K2	0.7114	78.39	0.7915	0.0405	-9.13	2.92	0.6629

GROUPE	SYMBOLE	AMPLITUDE	PHASE	L/H
MODELE THEORIQUE FARRELL		9.3798	-33.60	0.137
198-236	2N2	7.6667	-31.62	0.129
237-260	N2	7.3361	-35.01	0.142
261-286	M2	7.1845	-36.30	0.147
287-300	L2	7.8286	-24.55	0.103
301-309	S2	7.2239	-36.29	0.147
310-347	K2	7.4242	-42.73	0.171

ERREUR O.M. SD 1.367579 E-9

O1/K1 1.2377 1-O1/1-K1 0.5792 M2/O1 0.9684

348-363	M3	0.2059	322.02	1.0982	1.3142	17.36	68.24	0.1709

GROUPE	SYMBOLE	AMPLITUDE	PHASE	L/H
MODELE THEORIQUE FARRELL		12.0595	-31.79	0.050
348-363	M3	13.2441	-14.43	0.022

ERREUR O.M. TD 7.883046

EPOQUE DE REFERENCE TJJ# 2443460.0

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Une méthode pour l'analyse des marées terrestres à partir d'enregistrements de longueur arbitraire.
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La projection des résultats d'analyse des composantes horizontales des marées sur les directions fondamentales.
Mat. i Prace Instytutu Geofiz. (Publ. Inst. Geoph. Pol. Ac. Sci.), vol 94, 1975.

Standardisation rules for the presentation of results in tidal research.

(Earth Tides Analysis Abbreviated Computer Output)

P. Melchior

(ICET)

Following the preliminary discussions held during the VIII International Symposium on Earth Tides in september 1977 we have prepared a new presentation of the results obtained by the least square method using the Venedikov filters 1966. The annexed listings are self explanatory.

Some details of the presentation are only valid for the specific analysis method used here but general rules should be adopted.

1st part: Title cards for the station and the instrument should contain

1° A maximum of data concerning the station including

- geographical coordinates (to 1" precision when possible)
- altitude, depth and distance to the nearest sea
- a short geological description
- responsible organisation
- people in charge of the station (installation and maintenance)

2° Enough data concerning the instrument including

- calibration method
- instrumental phase lag corrections
- for tiltmeters and strainmeters: azimut from N (+ = clockwise)
- responsible organisation

2d part: Title cards for the analysis method including

- Potential used
- filtering if any
- inertial correction if any

- computing center
- computer
- program version, date of the analysis

3rd part: Results of the analysis

a) It is compulsory to give for each wave the
   - group limits according to the Doodson argument (the first three digits being sufficient)
   - group name
   - observed amplitude for the main constituent in the group and associated mean square error (R.M.S.)
   - amplitude factor and associated mean square error (R.M.S.)
   - phase difference (observed minus calculated) and associated mean square error
   - residuals calculated with respect to Molodensky model 1

b) The m.s.e. on one hourly reading will be given according to the analysis method.

c) The results expressing physical quantities (e. g. amplitudes, standard deviations) will be given in the corresponding units:
   - microgal for the gravimetric tides
   - 0"001 (mseca) for the clinometric tides
   - 10^{-10} ℓ for the extensometric tides

BIBLIOGRAPHY

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TRANS WORLD PROFILF AUSTRALIA STATION ALICE SPRINGS
STATION 4209 ALICE SPRINGS VERTICAL COMPONENT NORTH.TERR.-AUSTRALIA

23 43 S 133 50 E H 590 M P 10 M D 1000 KM
IGSN71 978640.
SHALLOW TUNNEL INTO HILL, PRECAMBRIAN SCHISTS AND GNEISSES
BUREAU OF MINERAL RESOURCES, CANBERRA
GRAVIMETER GEODYNAMICS 084 P.MELCHIOR TRANS WORLD PROFILES
CALIBRATION BRUXELLES - FUNDAMENTAL STATION
INSTALLATION J.VAN SON
MAINTENANCE P.TAYLOR
GRANT AFOSR-73-2557 A PROJECT-TASK 8607-02

LEAST SQUARE ANALYSIS / VENEDIKOV FILTERS ON 48 HOURS / PROGRAMMING R.DUCARME
POTENTIAL CARTWRIGHT-TAYLER-EDDEN / COMPLETE DEVELOPMENT
COMPUTING CENTER INTERNATIONAL CENTER FOR EARTH TIDES/FAGS/ BRUSSELS
COMPUTER UNIVAC 1100/40 PROCESSED ON 78/ 5/24

INERTIAL CORRECTION PROPORTIONAL TO THE SQUARE OF ANGULAR SPEEDS
NORMALISATION FACTOR 1.18255
PHASE LAG 01 .73 M2 .76 01/M2 .96
CORRECTION FOR DIFFERENTIAL ATTENUATION M2/01 1.00976 /MODEL 2/

84	76 211/76 215	76 219/76 3 8	76 312/76 4 7	76 410/76 428	76 5 1/76 5 1
84	76 514/76 518	76 522/76 524	76 528/76 617	76 621/76 621	76 9 3/76 9 5
84	76 9 7/76 9 7	76 928/7610 2	7610 8/7610 8	761015/7611 6	7612 7/761223
84	761231/77 110	77 2 5/77 321	77 326/77 417	77 421/77 5 1	77 5 5/77 527
84	78 113/78 119	78 122/78 2 9	78 212/78 326	78 330/78 4 9	

M3 SIGNAL 1.43 NOISE .31 14 DAYS REJECTED

TIME INTERVAL 789.5 DAYS 8496 READINGS 24 BLOKS

WAVE GROUP ARGUMENT	N	WAVE	ESTIMATED AMPL.	R.M.S.	AMPL. FACTOR	R.M.S.	PHASE DIFF.	R.M.S.	RESIDUALS AMPL.	PHASE
115.-11X.	11	SIGMO1	.21	.08	1.2489	.4745	2.81	21.72	.02	34.9
124.-126.	10	2Q1	.81	.09	1.4023	.1551	6.60	6.35	.16	34.6
127.-129.	11	SIGMA1	.84	.09	1.1957	.1252	-10.62	5.99	.15	-85.9
133.-136.	20	Q1	5.33	.09	1.2168	.0197	1.97	.93	.31	36.4
137.-139.	10	RO1	.96	.08	1.1521	.1007	.07	5.00	.01	169.5
143.-145.	16	O1	26.67	.09	1.1667	.0038	.89	.19	.45	68.2
146.-149.	10	TAU1	.31	.06	1.0273	.1976	-11.69	11.02	.08	-126.4
152.-155.	15	NO1	1.98	.08	1.0990	.0442	-.21	2.31	.11	-176.1
156.-158.	7	KI1	.46	.08	1.3306	.2423	-23.40	10.43	.18	-83.2
161.-162.	3	PI1	.80	.10	1.2757	.1555	15.53	6.99	.22	77.8
163.-163.	7	P1	12.31	.13	1.1571	.0122	.40	.60	.09	66.9
164.-164.	3	S1	1.16	.19	4.6160	.7605	47.90	9.44	.99	60.4
165.-165.	11	K1	36.53	.15	1.1362	.0047	.58	.24	.37	97.2
166.-166.	2	PSI1	.56	.13	2.1702	.5138	-29.27	13.56	.32	-58.4
167.-168.	7	PHI1	.36	.10	.7881	.2088	10.21	15.17	.19	160.7
172.-174.	8	TETA1	.30	.08	.8627	.2362	26.32	15.69	.19	135.4
175.-177.	14	J1	2.17	.08	1.2086	.0465	-1.10	2.21	.09	-26.1
181.-183.	7	SO1	.46	.09	1.5386	.3033	-2.24	11.31	.11	-9.1
184.-186.	11	OO1	1.11	.14	1.1268	.1378	9.24	7.00	.18	105.0
191.-195.	14	NU1	.14	.11	.7375	.5904	-.80	45.88	.08	-178.6
215.-22X.	19	EPS2	.54	.07	1.1511	.1520	-1.45	7.62	.01	-107.7
233.-236.	10	2N2	1.84	.08	1.1569	.0483	-2.41	2.42	.08	-95.0
237.-23X.	10	MU2	2.19	.08	1.1367	.0415	1.15	2.10	.06	135.9
243.-245.	13	N2	14.07	.09	1.1682	.0064	-.45	.32	.15	-48.5
246.-248.	11	NU2	2.59	.08	1.1302	.0344	-2.56	1.76	.14	-121.6
252.-258.	26	M2	73.67	.08	1.1711	.0012	-.34	.06	.82	-32.2
262.-264.	5	LAMB2	.65	.08	1.3948	.1623	-7.29	6.73	.13	-38.4
265.-265.	9	L2	2.16	.09	1.2150	.0521	-1.05	2.47	.11	-22.2
267.-272.	5	T2	1.91	.10	1.1150	.0562	3.00	2.90	.13	128.6
273.-273.	4	S2	34.02	.10	1.1623	.0034	.59	.17	.36	79.7
274.-277.	12	K2	9.27	.11	1.1635	.0133	1.80	.66	.29	85.6
282.-285.	15	ETA2	.57	.10	1.2806	.2333	10.88	10.51	.12	68.1
292.-295.	11	2K2	.12	.11	1.0537	.9676	42.00	52.96	.09	118.1
335.-347.	5	M03	.33	.01	1.0740	.0474	.83	2.54	.01	63.9
353.-375.	11	M3	1.23	.01	1.0903	.0133	-.84	.70	.03	-33.9

STANDARD DEVIATION D 4.02 SD 4.60 TD .87 MICROGAL
STUDENT FACTOR T(S=95%,M> 334)=1.96

01/K1 1.0268 1-01/1-K1 1.2238 M2/01 1.0038
CENTRAL EPOCH TJJ= 2443214.0

Working Group conclusions for improved Analysis

Procedure and Presentation

At the first meeting of the working group, taking into account the information required for physical investigations (*ref. Baker: A review of the objectives of tidal analysis, p. 4571*), the following conclusions were reached:-

1- The analysis procedure should be based on the least squares method. *(ref. Yaramanci: Tidal analysis methods and the optimal linear system approximation, p. 4621)*

2- The full Cartwright-Tayler-Edden potential with some additional Doodson waves (total 505 waves) should be used in harmonic analysis methods. For nonharmonic analysis methods requiring the tidal potential in the time domain, one can use the Fourier synthesis of the potential used in the harmonic methods or preferably one can work directly in the time domain using existing programs *(ref. Munk and Cartwright; Broucke, Zurn and Slichter; Harrison)*

3- Realistic error limits based on residual spectra and time variant analysis should be given *(ref. Wenzel: Autocorrelation of earth tide observations with respect to error estimation, BIM 76. Schüller: Principles of the HYCON method, p. 4667. Yaramanci: The spectral interpretation of the error calculation in tidal analysis, p. 4676.)*
For the Chojnicki method, a new version of error estimation has been produced and should be used from now on. *(ref. Chojnicki: Supplementary precision estimation of results of tidal data adjustment, p. 4670).*

4- An indication of the time variation of the main tidal waves should be given. *(ref. Schüller: Principles of the HYCON method, p. 4667).*

5- Non-linear waves (e. g. shallow water waves) should be included *(ref. Baker: Non-equilibrium influences on the tidal signal, p. 4596. Yaramanci: Principles of the TIFA method, p. 4659).*

6- An attempt should be made to model departures from the equilibrium response to the potential
(ref. Baker: Non-equilibrium influences on the tidal signal, p. 4596
Yaramanci: Principles of the response method, p. 4649
Principles of the TIFA method, p. 4659)

7- An attempt should be made to model multi-channel inputs *(ref. Munk and Cartwright, Jenkins and Watts: Spectral analysis and its applications Holden Day San Franc. 1968).*

8- The frequency response of the numerical filters used and information concerning the data window should be given *(ref. Schüller: About the sensitivity of the Venedikov tidal parameter estimates to leakage effects, p. 4635 Jenkins and Watts, Bäth)*

9- A print out of the results and the errors in physical units (e.g. microgals, milliseconds), δ or γ factors where appropriate, phases, full details of the instrument, station, dates and azimuth is to be given in accordance with the proposed "Earth Tide Analysis Abbreviated Computer Output" *Ref. P. Melchior: Standardisation rules for the presentation of results in tidal research, p. 4720).*

10- A full description of the experiment and the experimental difficulties, calibration methods, azimuth accuracy etc. should be given.

11- For gravitymeters, dates and tidal parameters of M_2 and O_1 are to be given for previous measurements with the same instrument at the fundamental base stations *(ref. Baker: A review of the objectives of tidal analysis p. 4571).*

12- Plots of amplitude factors and phase differences as functions of frequency (i. e. the response functions) with error bars, should be provided.

13- Plots of the observed spectrum after removal of the low energy part of the signal and of the residual spectrum should be given (in physical units and with logarithmic scale for amplitudes).

14- Plots of the observed and the residual time series should be given as an available option.

15- A print-out of the results after the removal of an appropriate body tidal model (e. g. Molodensky I) should be given for all waves (amplitudes in µgal, mseca etc., phases).

16- A plot of the spectrum of the data window (normalised to 1) should be given to emphasise the problem of leakage particularly for the effects of gaps. *(ref. Schüller: About the sensitivity of the Venedikov tidal parameter estimates to leakage effects, p. 4635. Bartha: Remarks on the investigation of residual curves of Earth Tides records p. 4684).*

17- A standard data set for vertical and horizontal components has been produced by Dr. Wenzel for testing analysis methods. It is available upon request from ICET.

REFERENCES

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Elsevier Scientific Publishing Company, New York (1974)

Suggested topics for investigation

All the above topics should be further investigated and in particular the points 5, 6 and 7.

In addition the following points should be investigated:

1) The effects of gaps with respect to leakage

2) Are we allowed to interpolate and to what extent?

3) *Realistic* artificial noise should be produced so as to give one possible way of checking the performance of different methods.

4) The production of a standard subroutine for the computation of the theoretical tides in both the frequency and time domain based upon the adopted tidal potential.